The Online Appendix for "Quality, Variable Markups, and Welfare: A Quantitative General Equilibrium Analysis of Export Prices"

A Derivation of Demand Function

The utility of a consumer in country j takes the following form:

$$U_j = \left[\sum_i \int_{\omega \in \Omega_{ij}} \left(q_{ij}(\omega) x_{ij}^c(\omega) + \overline{x} \right)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{A.1}$$

subject to the following budget constraint:

$$\sum_i \int_{\omega \in \Omega_{ij}} p_{ij}(\omega) x_{ij}^c(\omega) d\omega \leq y_j \tag{A.2}$$

So that the Lagrange function can be written as: $\mathcal{L} = \left[\sum_i \int_{\omega \in \Omega_{ij}} \left(q_{ij}(\omega) x_{ij}^c(\omega) + \overline{x} \right)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} + \lambda \left(y_j - \sum_i \int_{\omega \in \Omega_{ij}} p_{ij}(\omega) x_{ij}^c(\omega) d\omega \right)$,where λ is the Lagrange multiplier, y_j denotes the consumer's income. Taking the first order condition with respect to $x_{ij}^c(\omega)$ yields:

$$\lambda p_{ij}(\omega) = U_j^{\frac{1}{\sigma}} \left(q_{ij}(\omega) x_{ij}^c(\omega) + \overline{x} \right)^{-\frac{1}{\sigma}} q_{ij}(\omega), \tag{A.3}$$

Following Jung, Simonovska and Weinberger (2019), we define $P_{j\sigma} = \left\{ \sum_i \int_{\omega \in \Omega_{ij}} \tilde{p}_{ij}(\omega)^{1-\sigma} d\omega \right\}^{\frac{1}{1-\sigma}}$, and $P_j = \sum_i \int_{\omega \in \Omega_{ij}} \tilde{p}_{ij}(\omega) d\omega$, where $\tilde{p}_{ij}(\omega) = p_{ij}(\omega) / q_{ij}(\omega)$ is the quality adjusted price. The first order condition (A.3) can be rewritten as:

$$q_{ij}(\omega) x_{ij}^c(\omega) + \overline{x} = U_j \left(\lambda \tilde{p}_{ij}(\omega) \right)^{-\sigma} \tag{A.4}$$

Plugging equation (A.4) into equation (A.1), we have:

$$\lambda = \frac{1}{P_{j\sigma}}$$

Then substituting the above equation into equation (A.4) yield the solution for $x_{ij}^c(\omega)$:

$$q_{ij}(\omega) x_{ij}^c(\omega) = \left[\frac{\tilde{p}_{ij}(\omega)}{P_{j\sigma}} \right]^{-\sigma} U_j - \bar{x}, \tag{A.5}$$

Plugging the previous equation (A.5) into the budget constraint, we have:

$$\begin{aligned} y_j &= \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) q_{ij}(\omega) x^c_{ij}(\omega) d\omega \\ &= \sum_i \int_{\omega\in\Omega_{ij}} \left[\frac{\tilde{p}_{ij}(\omega)}{P_{j\sigma}}\right]^{-\sigma} U_j \tilde{p}_{ij}(\omega) d\omega - \bar{x} \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) d\omega \\ &= U_j P_{j\sigma} - \bar{x} P_j, \end{aligned}$$

Hence, we have:

$$U_j = \frac{y_j + \bar{x} P_j}{P_{j\sigma}} \tag{A.6}$$

Combing the previous equation (A.6) with equation (A.5) implies:

$$x_{ij}(\omega) = x^c_{ij}(\omega) L_j = \frac{L_j}{q_{ij}(\omega)} \left[\frac{y_j + \bar{x} P_j}{P^{1-\sigma}_{j\sigma}} \left(\frac{p_{ij}(\omega)}{q_{ij}(\omega)}\right)^{-\sigma} - \bar{x}\right] \tag{A.7}$$

B Log Utility Function

The representative consumer in country j's demand satisfies:

$$x_{ij}(\omega) = x^c_{ij}(\omega) L_j = \frac{\bar{x} L_j}{q_{ij}(\omega)} \left[\frac{\psi_j}{\tilde{p}_{ij}(\omega)} - 1\right] \tag{B.1}$$

where $\tilde{p}_{ijs}(\omega) = \frac{p_{ij}(\omega)}{q_{ij}(\omega)}$ and $\psi_j = \frac{y_j + \bar{x} P_j}{\bar{x} N_j}$. The aggregate prices satisfies $P_j = \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) d\omega$. Now, sales and profit for a given variety exported from i to j are as follows,

$$r_{ij}(\omega) = \bar{x} L_j \tilde{p}_{ij}(\omega) \left[\frac{\psi_j}{\tilde{p}_{ij}(\omega)} - 1\right] \tag{B.2}$$

$$\pi_{ij}(\omega) = \bar{x} L_j \left[\tilde{p}_{ij}(\omega) - \tilde{c}_{ij}(\omega)\right] \left[\frac{\psi_j}{\tilde{p}_{ij}(\omega)} - 1\right] \tag{B.3}$$

where $\tilde{c}_{ij}(\omega) = \frac{c_{ij}(\omega)}{q_{ij}(\omega)}$ is the quality-adjusted marginal cost. Given the quality adjusted marginal cost, firms maximize their profits. This implies that the optimal price of the good satisfies:

$$\tilde{p}_{ij}(\omega) = \sqrt{\psi_j \tilde{c}_{ij}(\omega)}$$

We assume that the marginal cost of producing a variety of final good with quality q_{ij} by a firm with productivity φ is given by:

$$c_{ij}(\varphi, \varepsilon) = \left(T_{ij} w_i + \frac{w_i \tau_{ij}}{\varphi} q^{\eta}_{ij}\right) \varepsilon$$

where τ_{ij} is ad valorem trade cost and T_{ij} is a specific transportation cost from country i to country j. Maximizing the profit is equivalent to minimizing the quality-adjusted cost $\tilde{c}_{ij}(\omega)$

by the envelop theorem. Choosing the quality to minimize the quality-adjusted marginal cost implies that the optimal level of quality for a firm with productivity φ is:

$$q_{ij}(\varphi,\varepsilon) = \left(\frac{T_{ij}\varphi}{(\eta-1)\,\tau_{ij}}\right)^{\frac{1}{\eta}} \tag{B.4}$$

and hence the quality adjusted marginal cost of production now is:

$$\tilde{c}_{ij}\left(\varphi,\varepsilon\right) = \left(\frac{\eta}{\eta-1}T_{ij}w_i\right)^{\frac{\eta-1}{\eta}}\left(\frac{\varphi}{\eta w_i\tau_{ij}}\right)^{-\frac{1}{\eta}}\varepsilon \tag{B.5}$$

At the productivity cutoff $\varphi^*_{ij}\left(\varepsilon\right)$, we have $\tilde{p}^*_{ij}\left(\varphi,\varepsilon\right) = \tilde{c}^*_{ij}\left(\varphi,\varepsilon\right) = \psi_j$, which implies that the productivity cutoff $\varphi^*_{ij}\left(\varepsilon\right)$ takes the following form:

$$\varphi^*_{ij}\left(\varepsilon\right) = \varphi^*_{ij}\varepsilon^{\eta} = \frac{\eta^{\eta}}{(\eta-1)^{\eta-1}}T_{ij}^{\eta-1}\tau_{ij}w_i^{\eta}\left(\psi_j\right)^{-\eta}\varepsilon^{\eta},$$

In the log utility function, price could be written as:

$$p_{ij}(\varphi,\varepsilon) = \left[\frac{\varphi}{\varphi^*_{ij}\left(\varepsilon\right)}\right]^{\frac{1}{2\eta}}\frac{\eta}{\eta-1}T_{ij}\varepsilon.$$

Different from the CES utility function, now the markup function could be expressed explicitly as $\left[\frac{\varphi}{\varphi^*_{ij}(\varepsilon)}\right]^{\frac{1}{2\eta}}$.

C Derivation for P_j, $P_{j\sigma}$, X_{ij} and π_i

To derive the aggregate variables, we define $t_{ij} = \tilde{p}_{ij}\left(\omega\right)/p^*_j$. Following the insight of Arkolakis et al. (2019) and Jung, Simonovska and Weinberger (2019), this will make the integration not country specific. From equations (9) and (11), we have:

$$\frac{\tilde{c}_{ij}\left(\varphi,\varepsilon\right)}{\tilde{p}^*_j} = \frac{\tilde{c}_{ij}\left(\varphi,\varepsilon\right)}{\tilde{c}^*_{ij}\left(\varphi,\varepsilon\right)} = \left(\frac{\varphi}{\varphi^*_{ij}\left(\varepsilon\right)}\right)^{-\frac{1}{\eta}} \tag{C.1}$$

Combining the above equation with equation (6) we have:

$$\sigma\left(\frac{\varphi}{\varphi^*_{ij}\left(\varepsilon\right)}\right)^{-\frac{1}{\eta}} = t_{ij}^{\sigma+1} + (\sigma-1)\,t_{ij} \tag{C.2}$$

which implies that t_{ij} is a monotonically decreasing function of φ. Note that t_{ij} will lies between $(0,1]$ since $\varphi \in \left[\varphi^*_{ij}\left(\varepsilon\right),\infty\right)$. Totally differentiating both sides gives us:

$$d\varphi = -\eta\sigma^{\eta}\varphi^*_{ij}\left(\varepsilon\right)\frac{(\sigma+1)\,t_{ij}^{\sigma} + (\sigma-1)}{\left[t_{ij}^{\sigma+1} + (\sigma-1)\,t_{ij}\right]^{1+\eta}}dt_{ij} \tag{C.3}$$

First, we derive $P_{j\sigma}$. By definition, we have:

$$\begin{aligned} P_{j\sigma} &= \left\{ \sum_i N_{ij} \int_0^\infty \int_{\varphi_{ij}^*(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} \mu_{ij}(\varphi,\varepsilon) f(\varepsilon) d\varphi d\varepsilon \right\}^{\frac{1}{1-\sigma}} \\ &= \tilde{p}_j^* \left\{ \sum_i N_{ij} \int_0^\infty \left[\int_{\varphi_{ij}^*(\varepsilon)}^\infty t_{ij}^{1-\sigma} \mu_{ij}(\varphi,\varepsilon) d\varphi \right] f(\varepsilon) d\varepsilon \right\}^{\frac{1}{1-\sigma}} \end{aligned} \tag{C.4}$$

Plugging in the expression of conditional density $\mu_{ij}(\varphi,\varepsilon)$ into equation (C.4) and then we transform the integration variable from φ to t_{ij} by using the relationship between φ and t_{ij}, the inner integration with respect to productivity can be written as:

$$\int_{\varphi_{ij}^*(\varepsilon)}^\infty t_{ij}^{1-\sigma} \mu_{ij}(\varphi,\varepsilon) d\varphi = \frac{\eta\theta}{\sigma^{\eta\theta}} \int_0^1 t_{ij}^{1-\sigma} \left[t_{ij}^{\sigma+1} + (\sigma-1) t_{ij} \right]^{\eta\theta-1} \left[(\sigma+1) t_{ij}^\sigma + (\sigma-1) \right] dt_{ij}$$

which is a constant, and we denote it as β_σ. Thus,

$$P_{j\sigma} = \beta_\sigma^{\frac{1}{1-\sigma}} \tilde{p}_j^* N_j^{\frac{1}{1-\sigma}}$$

Second, we derive P_j. By definition, we have

$$\begin{aligned} P_j &= \sum_i N_{ij} \int_0^\infty \int_{\varphi_{ij}^*(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon) \mu_{ij}(\varphi,\varepsilon) f(\varepsilon) d\varphi d\varepsilon \\ &= \tilde{p}_j^* \sum_i N_{ij} \int_0^\infty \left[\int_{\varphi_{ij}^*(\varepsilon)}^\infty t_{ij} \mu_{ij}(\varphi,\varepsilon) d\varphi \right] f(\varepsilon) d\varepsilon \\ &= \beta \tilde{p}_j^* N_j \end{aligned}$$

In the last equality, we use the same variable transformation method as before where β is a constant, defined by:

$$\beta = \frac{\eta\theta}{\sigma^{\eta\theta}} \int_0^1 t_{ij} \left[t_{ij}^{\sigma+1} + (\sigma-1) t_{ij} \right]^{\eta\theta-1} \left[(\sigma+1) t_{ij}^\sigma + (\sigma-1) \right] dt_{ij}$$

To derive the equations (C.5) and (C.6), we plug in $\tilde{p}_j^* = \left(\frac{w_j + \bar{x} P_j}{\bar{x} P_{j\sigma}^{1-\sigma}} \right)^{\frac{1}{\sigma}}$ into $P_{j\sigma}$ and P_j, we have:

$$\begin{aligned} P_{j\sigma} &= \beta_\sigma^{\frac{1}{1-\sigma}} \left(\frac{w_j + \bar{x} P_j}{\bar{x} P_{j\sigma}^{1-\sigma}} \right)^{\frac{1}{\sigma}} N_j^{\frac{1}{1-\sigma}} \\ P_j &= \beta \left(\frac{w_j + \bar{x} P_j}{\bar{x} P_{j\sigma}^{1-\sigma}} \right)^{\frac{1}{\sigma}} N_j, \end{aligned}$$

which provide us with 2 equations to solve for $P_{j\sigma}$ and P_j. Solving the system yields:

$$\bar{x}P_j = \frac{\beta}{\beta_\sigma - \beta} w_j \tag{C.5}$$

$$\bar{x}P_{j\sigma} = \frac{\beta_\sigma^{\frac{1}{1-\sigma}}}{\beta_\sigma - \beta} N_j^{\frac{\sigma}{1-\sigma}} w_j \tag{C.6}$$

Next, we derive bilateral trade flow X_{ij}, which is given by:

$$\begin{aligned} X_{ij} &= N_{ij} \int_0^\infty \left[\int_{\varphi_{ij}^*(\varepsilon)}^\infty r_{ij}(\varphi, \varepsilon) \mu_{ij}(\varphi, \varepsilon) d\varphi \right] f(\varepsilon) d\varepsilon \\ &= N_{ij} \left(\bar{x}\tilde{p}_j^* L_j \right) \int_0^\infty \left[\int_{\varphi_{ij}^*(\varepsilon)}^\infty t_{ij} \left(t_{ij}^{-\sigma} - 1 \right) \mu_{ij}(\varphi, \varepsilon) d\varphi \right] f(\varepsilon) d\varepsilon \\ &= (\beta_\sigma - \beta) \bar{x}\tilde{p}_j^* L_j N_{ij} = X_j \frac{N_{ij}}{N_j} \end{aligned}$$

where $X_j = \sum_i X_{ij}$ is total absorption.

Finally, we derive firm's expected average profit π_i, which satisfies:

$$\begin{aligned} \pi_i &= \frac{1}{J_i} \sum_j N_{ij} \int_0^\infty \int_{\varphi_{ij}^*(\varepsilon)}^\infty \pi_{ij}(\varphi, \varepsilon) \mu_{ij}(\varphi) f(\varepsilon) d\varphi d\varepsilon \\ &= \frac{1}{J_i} \beta_\pi \sum_j \bar{x}\tilde{p}_j^* L_j N_{ij} = \frac{1}{J_i} \frac{\beta_\pi}{\beta_\sigma - \beta} \sum_j X_{ij} \\ &= \frac{1}{J_i} \frac{\beta_\pi}{\beta_\sigma - \beta} \sum_j \frac{N_{ij}}{N_j} X_j \end{aligned}$$

where

$$\beta_\pi = \frac{\eta\theta}{\sigma^{\eta\theta}} \int_0^1 \frac{\left(t_{ij}^{\sigma+1} - t_{ij} \right) \left(t_{ij}^{-\sigma} - 1 \right)}{\sigma} \left[t_{ij}^{\sigma+1} + (\sigma - 1) t_{ij} \right]^{\eta\theta - 1} \left[(\sigma + 1) t_{ij}^\sigma + (\sigma - 1) \right] dt_{ij}$$

D Proof of Propositions

D.1 Proof of Proposition 1

The percentage change of U_j satisfies:

$$d \ln U_j = \frac{\sigma}{\sigma - 1} \left(d \ln w_j - d \ln \tilde{p}_j^* \right) \tag{D.1}$$

Based on equations (11), (13) and (21), we can rewrite N_{ij} as:

$$N_{ij} = \frac{\kappa\beta_\pi}{f\beta_X} b_i L_i \left[\frac{\eta^\eta}{(\eta - 1)^{\eta - 1}} T_{ij}^{\eta - 1} \tau_{ij} w_i^\eta \left(\tilde{p}_j^* \right)^{-\eta} \right]^{-\theta} \tag{D.2}$$

where $\beta_X = \beta_\sigma - \beta$ is a constant. This implies that

$$\lambda_{jj} = \frac{X_{jj}}{\sum_i X_{ij}} = \frac{N_{jj}}{\sum_i N_{ij}} = \frac{b_j L_j \left(T_{jj}^{\eta-1} \tau_{jj} w_j^{\eta}\right)^{-\theta}}{\sum_i b_i L_i \left(T_{ij}^{\eta-1} \tau_{ij} w_i^{\eta}\right)^{-\theta}} \tag{D.3}$$

Consider the foreign shocks: $(b_i, L_i, T_{ij}, \tau_{ij})$ is changed to $(b_i', L_i', T_{ij}', \tau_{ij}')$ for $i \neq j$ such that $b_j = b_j', L_j = L_j', T_{jj} = T_{jj}', \tau_{jj} = \tau_{jj}'$. Totally differentiating the previous equation implies:

$$d \ln \lambda_{jj} = \sum_i \lambda_{ij} \left[\theta\eta \left(d \ln w_i - d \ln w_j\right) - d \ln \xi_{ij}\right] \tag{D.4}$$

where $d \ln \xi_{ij}$ reflects any foreign shock, which satisfies:

$$d \ln \xi_{ij} = -\theta \left(\eta - 1\right) d \ln T_{ij} - \theta d \ln \tau_{ij} + d \ln b_i + d \ln L_i$$

The expression of $\tilde{p}_j^*$, together with equation (C.5) and (C.6), imply that:

$$d \ln \tilde{p}_j^* = \frac{1}{\sigma} d \ln w_j + \frac{\sigma - 1}{\sigma} d \ln P_{j\sigma} = d \ln w_j - \sum_i \lambda_{ij} d \ln N_{ij} \tag{D.5}$$

Totally differentiating the expression of N_{ij} and substituting the percentage change of N_{ij} into the previous equation, we have:

$$\begin{aligned} d \ln \tilde{p}_j^* &= d \ln w_j - \sum_i \lambda_{ij} d \ln N_{ij} \\ &= d \ln w_j + \sum_i \lambda_{ij} \left[\theta\eta \left(d \ln w_i - d \ln \tilde{p}_j^*\right) - d \ln \xi_{ij}\right] \\ &= \frac{1}{1 + \eta\theta} d \ln w_j + \frac{1}{1 + \eta\theta} \sum_i \lambda_{ij} \left[\theta\eta d \ln w_i - d \ln \xi_{ij}\right] \end{aligned} \tag{D.6}$$

Hence, the percentage change in welfare satisfies:

$$\begin{aligned} d \ln U_j &= \frac{\sigma}{\sigma - 1} \left(d \ln w_j - d \ln \tilde{p}_j^*\right) \\ &= -\frac{\sigma}{\sigma - 1} \frac{1}{1 + \eta\theta} \sum_i \lambda_{ij} \left[\theta\eta \left(d \ln w_i - d \ln w_j\right) - d \ln \xi_{ij}\right] \\ &= -\frac{\sigma}{\sigma - 1} \frac{1}{1 + \eta\theta} d \ln \lambda_{jj} \end{aligned} \tag{D.7}$$

Integrating the previous expression between the initial equilibrium (before the shock) and the new equilibrium (after the shock), we finally get

$$\widehat{U}_j = \left(\widehat{\lambda}_{jj}\right)^{-\frac{\sigma}{\sigma-1} \frac{1}{1+\eta\theta}} \tag{D.8}$$

It shows that the changes in welfare at country j can be inferred from changes in the share of domestic expenditure, λ_{jj}, using the parameter, $-\frac{\sigma}{\sigma-1} \frac{1}{1+\eta\theta}$.

D.2 Proof of Proposition 2

We consider an arbitrary change in trade costs from τ_{ij} to τ'_{ij} and T_{ij} to T'_{ij}. The share of expenditure on domestic goods in the initial and new equilibrium, respectively, are given by:

$$\lambda_{jj} = \frac{X_{jj}}{\sum_i X_{ij}} = \frac{b_j L_j \left(T_{jj}^{\eta-1}\tau_{jj} w_j^{\eta}\right)^{-\theta}}{\sum_i b_i L_i \left(T_{ij}^{\eta-1}\tau_{ij} w_i^{\eta}\right)^{-\theta}} \tag{D.9}$$

$$\lambda'_{jj} = \frac{b_j L_j \left(T_{jj}^{\eta-1}\tau_{jj} \left(w'_j\right)^{\eta}\right)^{-\theta}}{\sum_i b_i L_i \left(\left(T'_{ij}\right)^{\eta-1}\tau'_{ij} \left(w'_i\right)^{\eta}\right)^{-\theta}} \tag{D.10}$$

Combing the previous two equations, we obtain:

$$\widehat{\lambda}_{jj} = \frac{\left(\widehat{w}_j\right)^{-\eta\theta}}{\sum_i \lambda_{ij} \left[\left(\widehat{T}_{ij}\right)^{\eta-1} \widehat{\tau}_{ij}\right]^{-\theta} \left(\widehat{w}_i\right)^{-\eta\theta}} \tag{D.11}$$

Labor market clearing condition implies that:

$$w_i L_i = \sum_j \lambda_{ij} w_j L_j = \sum_j \frac{b_i L_i \left[T_{ij}^{\eta-1}\tau_{ij}\right]^{-\theta} w_i^{-\eta\theta}}{\sum_{i'} b_{i'} L_{i'} \left[T_{i'j}^{\eta-1}\tau_{i'j}\right]^{-\theta} w_{i'}^{-\eta\theta}} w_j L_j \tag{D.12}$$

After τ_{ij} becomes τ'_{ij} and T_{ij} becomes T'_{ij}, the previous equation becomes:

$$w'_i L_i = \sum_j \frac{b_i L_i \left[\left(T'_{ij}\right)^{\eta-1} \tau'_{ij}\right]^{-\theta} \left(w'_i\right)^{-\eta\theta}}{\sum_{i'} b_{i'} L_{i'} \left[\left(T'_{i'j}\right)^{\eta-1} \tau'_{i'j}\right]^{-\theta} \left(w'_{i'}\right)^{-\eta\theta}} w'_j L_j$$

We can rearrange the previous expression as:

$$\widehat{w}_i w_i L_i = \sum_j \frac{\lambda_{ij} \left[\widehat{T}_{ij}^{\eta-1}\widehat{\tau}_{ij}\right]^{-\theta} \left(\widehat{w}_i\right)^{-\eta\theta}}{\sum_{i'} \lambda_{i'j} \left[\widehat{T}_{i'j}^{\eta-1}\widehat{\tau}_{i'j}\right]^{-\theta} \left(\widehat{w}_{i'}\right)^{-\eta\theta}} \widehat{w}_j w_j L_j$$

which implies the equation (27).

E Global Measure of Welfare Gains

E.1 Derivation of Equation (25) in Proposition 1

The welfare measure can be written as follows:

$$U_j = \left[\sum_i \int_{\omega\in\Omega_{ij}} \left(q_{ij}\left(\omega\right) x_{ij}^c\left(\omega\right) + \bar{x}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} = \frac{w_j + \bar{x} P_j}{P_{j\sigma}} \tag{E.1}$$

which together with the expression of $\bar{x}P_j = \frac{\beta}{\beta_\sigma - \beta} w_j$ and $\bar{x}P_{j\sigma} = \frac{\beta_\sigma^{\frac{1}{1-\sigma}}}{\beta_\sigma - \beta} N_j^{\frac{\sigma}{1-\sigma}} w_j$, implies that

$$U_j = \bar{x}\beta_\sigma^{\frac{\sigma}{\sigma-1}} N_j^{\frac{\sigma}{\sigma-1}}, \tag{E.2}$$

By definition, $N_j = \sum_i N_{ij}$, we thus have the following relationship

$$\hat{N}_j = \sum_i \lambda_{ij} \hat{N}_{ij}, \tag{E.3}$$

and combining the equation (E.2), we have

$$\hat{U}_j = \left(\sum_i \lambda_{ij} \hat{N}_{ij} \right)^{\frac{\sigma}{\sigma-1}}, \tag{E.4}$$

The equation (17) implies that $\lambda_{jj} = \frac{N_{jj}}{N_j} = \frac{N_{jj}}{\sum_i N_{ij}}$, so

$$\hat{N}_j = \sum_i \lambda_{ij} \hat{N}_{ij} = \frac{\hat{N}_{jj}}{\hat{\lambda}_{jj}}, \tag{E.5}$$

substituting into the last $\hat{U}_j$ equation, we have

$$\hat{U}_j = \left(\frac{\hat{\lambda}_{jj}}{\hat{N}_{jj}} \right)^{-\frac{\sigma}{\sigma-1}}, \tag{E.6}$$

We thus have

$$\hat{N}_{jj} = \left(\hat{\varphi}_{jj}^*\right)^{-\theta} = \left(\frac{\hat{w}_j}{\hat{\widetilde{p}}_j^*} \right)^{-\theta\eta} = \left(\hat{N}_j\right)^{-\theta\eta} = \left(\frac{\hat{N}_{jj}}{\hat{\lambda}_{jj}} \right)^{-\theta\eta} = \left(\hat{\lambda}_{jj}\right)^{\frac{\theta\eta}{1+\theta\eta}} \tag{E.7}$$

where the first equality stems from the equation (13), the second equality stems from the equation (12), the third equality stems from the equation (17), the fourth equality stems from the equation (E.5). The previous equation (E.6), together with the equation (E.7), implies that:

$$\hat{U}_j = \left(\frac{\hat{\lambda}_{jj}}{\hat{N}_{jj}} \right)^{-\frac{\sigma}{\sigma-1}} = \left(\frac{\hat{\lambda}_{jj}}{\left(\hat{\lambda}_{jj}\right)^{\frac{\theta\eta}{1+\theta\eta}}} \right)^{-\frac{\sigma}{\sigma-1}} = \left(\hat{\lambda}_{jj}\right)^{-\frac{\sigma}{\sigma-1}\frac{1}{1+\theta\eta}}$$

E.2 Equivalent Variation as Global Measure of Welfare

Formally, the exact welfare change in country j is computed as $e\left(\mathbf{p}_j, U_j'\right)/w_j - 1$, where $\mathbf{p}_j$ and w_j are the set of good prices and the wage in the initial equilibrium, respectively, and U_j' is the utility level in the counterfactual equilibrium. The expenditure function in country j takes the following form:

$$e_j = \sum_i \int_{\omega\in\Omega_{ij}} p_{ij}(\omega) x^c_{ij}(\omega) d\omega \tag{E.8}$$

subject to the following budget constraint:

$$\left[\sum_i \int_{\omega\in\Omega_{ij}} \left(q_{ij}(\omega)x^c_{ij}(\omega)+\overline{x}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \geq U_j \tag{E.9}$$

Taking the first order condition with respect to $x^c_{ij}(\omega)$ yields:

$$p_{ij}\left(\omega\right) = \lambda U_j^{\frac{1}{\sigma}} \left(q_{ij}(\omega)x^c_{ij}(\omega)+\overline{x}\right)^{-\frac{1}{\sigma}} q_{ij}\left(\omega\right), \tag{E.10}$$

where λ is the Lagrange multiplier. The previous equation can be rewritten as:

$$q_{ij}(\omega)x^c_{ij}(\omega)+\overline{x} = U_j\left(\tilde{p}_{ij}\left(\omega\right)/\lambda\right)^{-\sigma} \tag{E.11}$$

where $\tilde{p}_{ij}\left(\omega\right) = p_{ij}\left(\omega\right)/q_{ij}\left(\omega\right)$ is the quality adjusted price. Plugging equation (E.11) into equation (E.9), we have:

$$\lambda = P_{j\sigma} = \left[\sum_i \int_{\omega\in\Omega_{ij}} \left(\tilde{p}_{ij}\left(\omega\right)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$

Then substituting the above equation into equation (E.11) yields the solution for $x^c_{ij}(\omega)$:

$$q_{ij}(\omega)x^c_{ij}(\omega) = \left[\frac{\tilde{p}_{ij}\left(\omega\right)}{P_{j\sigma}}\right]^{-\sigma} U_j - \bar{x}, \tag{E.12}$$

Plugging the previous equation (E.12) into the object function, we have:

$$\begin{aligned} e_j &= \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) q_{ij}(\omega) x^c_{ij}(\omega) d\omega \\ &= \sum_i \int_{\omega\in\Omega_{ij}} \left[\frac{\tilde{p}_{ij}\left(\omega\right)}{P_{j\sigma}}\right]^{-\sigma} U_j \tilde{p}_{ij}(\omega) d\omega - \bar{x}\sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) d\omega \\ &= P_{j\sigma}U_j - \bar{x}P_j, \end{aligned}$$

Hence, the exact welfare change in country j is computed as

$$\begin{aligned} e\left(\mathbf{p}_j, U'_j\right)/w_j - 1 &= \frac{P_{j\sigma}U'_j - \bar{x}P_j - \left(P_{j\sigma}U_j - \bar{x}P_j\right)}{P_{j\sigma}U_j - \bar{x}P_j} \\ &= \frac{P_{j\sigma}U_j}{P_{j\sigma}U_j - \bar{x}P_j} \frac{U'_j - U_j}{U_j} \end{aligned}$$

where $P_{j\sigma}U_j = \frac{\beta_\sigma}{\beta_\sigma - \beta} w_j$ and $\bar{x}P_j = \frac{\beta}{\beta_\sigma - \beta} w_j$ in equilibrium. Hence, the exact welfare change in country j satisfies

$$e\left(\mathbf{p}_j,U_j'\right)/w_j-1=\frac{\beta_\sigma}{\beta_\sigma-\beta}\frac{U_j'-U_j}{U_j}=\frac{\beta_\sigma}{\beta_\sigma-\beta}\widehat{U}_j$$

F Multi Sector Extension

F.1 Derivation of Multi Sector Model

Household utility in country j can be written as:

$$U_j=\prod_s C_{js}^{\alpha_s}, \tag{F.1}$$

with

$$C_{js}=\left[\sum_i\int_{\omega\in\Omega_{ijs}}\left(q_{ijs}(\omega)x_{ijs}^c(\omega)+\overline{x}_s\right)^{\frac{\sigma_s-1}{\sigma_s}}d\omega\right]^{\frac{\sigma_s}{\sigma_s-1}}, \tag{F.2}$$

The representative consumer in country j's demand satisfies:

$$x_{ijs}^c(\omega)=\frac{\overline{x}_s}{q_{ijs}(\omega)}\left\{\left[\frac{\tilde{p}_{ijs}(\omega)}{\tilde{p}_{js}^*}\right]^{-\sigma_s}-1\right\} \tag{F.3}$$

where $\tilde{p}_{ijs}(\omega)=\frac{p_{ijs}(\omega)}{q_{ijs}(\omega)}$ and $\tilde{p}_{js}^*=\left[\frac{\alpha_s\left(\sum_s\bar{x}_sP_{js}+y_j\right)}{\bar{x}_sP_{j\sigma s}^{1-\sigma_s}}\right]^{\frac{1}{\sigma_s}}$. The aggregate prices satisfy $P_{js}=\sum_i\int_{\omega\in\Omega_{ijs}}\tilde{p}_{ijs}(\omega)\,d\omega$ and $P_{j\sigma s}=\left\{\sum_i\int_{\omega\in\Omega_{ijs}}\tilde{p}_{ijs}(\omega)^{1-\sigma}d\omega\right\}^{\frac{1}{1-\sigma}}$. Now, quantity, sales, and profit for a given variety exported from i to j in sector s are as follows,

$$x_{ijs}(\omega)=\frac{\bar{x}_sL_j}{q_{ijs}(\omega)}\left[\left(\frac{\tilde{p}_{ijs}(\omega)}{p_{js}^*}\right)^{-\sigma_s}-1\right] \tag{F.4}$$

$$r_{ijs}(\omega)=\bar{x}_sL_j\tilde{p}_{ijs}(\omega)\left[\left(\frac{\tilde{p}_{ijs}(\omega)}{p_{js}^*}\right)^{-\sigma_s}-1\right] \tag{F.5}$$

$$\pi_{ijs}(\omega)=\bar{x}_sL_j\left[\tilde{p}_{ijs}(\omega)-\tilde{c}_{ijs}(\omega)\right]\left[\left(\frac{\tilde{p}_{ijs}(\omega)}{p_{js}^*}\right)^{-\sigma_s}-1\right] \tag{F.6}$$

where $\tilde{c}_{ijs}(\omega)=\frac{c_{ijs}(\omega)}{q_{ijs}(\omega)}$ is the quality-adjusted marginal cost. Given the quality adjusted marginal cost, firms maximize their profits. This implies that the optimal price of the good satisfies:

$$\sigma\frac{\tilde{c}_{ijs}(\omega)}{p_{js}^*}=\left(\frac{\tilde{p}_{ijs}(\omega)}{p_{js}^*}\right)^{\sigma+1}+(\sigma-1)\frac{\tilde{p}_{ijs}(\omega)}{p_{js}^*} \tag{F.7}$$

We assume that the marginal cost of producing a variety of final good with quality q_{ijs} by a firm with productivity φ is given by:

$$c_{ijs}(\varphi,\varepsilon)=\left(T_{ijs}w_i+\frac{w_i\tau_{ijs}q_{ijs}^{\eta_s}}{\varphi}\right)\varepsilon$$

where τ_{ijs} is ad valorem trade cost and T_{ijs} is a specific transportation cost from country i to country j in sector s. Productivity φ follows the Pareto distribution with c.d.f. $G_i(\varphi) = 1 - b_{is}\varphi^{-\theta_s}$, and ε follows the log-normally distribution with the variance σ_s in sector s. Maximizing the profit is equivalent to minimizing the quality-adjusted cost $\tilde{c}_{ijs}(\omega)$ by the envelop theorem. Choosing the quality to minimize the quality-adjusted marginal cost implies that the optimal level of quality for a firm with productivity φ is:

$$q_{ijs}(\varphi, \varepsilon) = \left(\frac{T_{ijs}\varphi}{(\eta_s - 1)\,\tau_{ijs}}\right)^{\frac{1}{\eta_s}} \tag{F.8}$$

and hence the quality adjusted marginal cost of production now is:

$$\tilde{c}_{ijs}(\varphi, \varepsilon) = \left(\frac{\eta_s}{\eta_s - 1} T_{ijs} w_i\right)^{\frac{\eta_s - 1}{\eta_s}} \left(\frac{\varphi}{\eta_s w_i \tau_{ijs}}\right)^{-\frac{1}{\eta_s}} \varepsilon \tag{F.9}$$

At the productivity cutoff $\varphi^*_{ijs}(\varepsilon)$, we have $p^*_{ijs}(\varphi, \varepsilon) = c^*_{ijs}(\varphi, \varepsilon) = p^*_{js}$, which implies that the productivity cutoff $\varphi^*_{ijs}(\varepsilon)$ takes the following form:

$$\varphi^*_{ijs}(\varepsilon) = \varphi^*_{ijs}\varepsilon^{\eta_s} = \frac{\eta_s^{\eta_s}}{(\eta_s - 1)^{\eta_s - 1}} T_{ijs}^{\eta_s - 1} \tau_{ijs} w_i^{\eta_s} \left(\tilde{p}^*_{js}\right)^{-\eta_s} \varepsilon^{\eta_s},$$

Based on the similar derivation in the one-sector model in Section 3, we know that the exporting firm mass N_{ijs}, the aggregate price P_{js} and $P_{j\sigma s}$, the trade flow X_{ijs}, the expected average profit π_{is} and the potential firm mass J_{is} in sector s satisfy:

$$N_{ijs} = \kappa_s J_{is} b_{is} \left(\varphi^*_{ijs}\right)^{-\theta_s} \tag{F.10}$$

$$\bar{x}_s P_{js} = \beta_s \tilde{p}^*_{js} N_{js} \tag{F.11}$$

$$\bar{x}_s P_{j\sigma s} = \beta_{\sigma s}^{\frac{1}{1-\sigma_s}} \tilde{p}^*_{js} N_{js}^{\frac{1}{1-\sigma_s}} \tag{F.12}$$

$$X_{ijs} = \beta_{Xs} \bar{x}_s \tilde{p}^*_{js} N_{ijs} L_j \tag{F.13}$$

$$\pi_{is} = \beta_{\pi s} \sum_j \bar{x}_s \kappa_s b_{is} \left(\varphi^*_{ijs}\right)^{-\theta_s} \tilde{p}^*_{js} L_j \tag{F.14}$$

$$J_{is} = \frac{\beta_{\pi s}}{\beta_{Xs}} \frac{\alpha_s L_i}{f_s} \tag{F.15}$$

where κ_s, β_s, $\beta_{\sigma s}$, $\beta_{\pi s}$ and β_{Xs} are constant. Now, the expression of choke price $\tilde{p}^*_{js}$, together with the equation (F.11) and (F.12), implies[39]

$$\bar{x}_s P_{js} = \gamma_s w_j \tag{F.16}$$

$$\bar{x}_s P_{j\sigma s} = \frac{\gamma_s}{\beta_s} \beta_{\sigma s}^{\frac{1}{1-\sigma_s}} N_{js}^{\frac{\sigma_s}{1-\sigma_s}} w_j \tag{F.17}$$

$$\tilde{p}_{js}^* = \frac{\gamma_s}{\beta_s} \frac{w_j}{N_{js}} \tag{F.18}$$

where γ_s are determined by $\beta_s \alpha_s \left(\sum_s \gamma_s + 1\right) = \beta_{\sigma s} \overline{x}_s^{\sigma_s} \gamma_s$.

F.2 Proof of Proposition 3

The percentage change of U_j satisfies:

$$d \ln U_j = \sum_s \frac{\alpha_s \sigma_s}{\sigma_s - 1} \left(d \ln w_j - d \ln \tilde{p}_{js}^*\right) \tag{F.19}$$

Based on equations (11), (13) and (21), we can rewrite N_{ij} as:

$$N_{ijs} = \frac{\kappa \beta_{\pi s}}{\beta_{X s} f_s} \alpha_s b_{is} L_i \left(\frac{\eta_s^{\eta_s}}{(\eta_s - 1)^{\eta_s - 1}} T_{ijs}^{\eta_s - 1} \tau_{ijs} w_i^{\eta_s} \left(\tilde{p}_{js}^*\right)^{-\eta_s} \right)^{-\theta_s} \tag{F.20}$$

which implies that

$$\lambda_{jjs} = \frac{X_{jjs}}{\sum_i X_{ijs}} = \frac{N_{jjs}}{\sum_i N_{ijs}} = \frac{b_{js} L_j \left(T_{jjs}^{\eta - 1} \tau_{jjs} w_j^{\eta}\right)^{-\theta}}{\sum_i b_{is} L_i \left(T_{ijs}^{\eta - 1} \tau_{ijs} w_i^{\eta}\right)^{-\theta}} \tag{F.21}$$

Consider the foreign shocks: $(b_{is}, L_i, T_{ijs}, \tau_{ijs})$ is changed to $(b'_{is}, L'_i, T'_{ijs}, \tau'_{ijs})$ for $i \neq j$ such that $b_{js} = b'_{js}, L_j = L'_j, T_{jjs} = T'_{jjs}, \tau_{jjs} = \tau'_{jjs}$. Totally differentiating the previous equation implies:

$$d \ln \lambda_{jjs} = \sum_i \lambda_{ijs} \left[\theta \eta \left(d \ln w_i - d \ln w_j\right) - d \ln \xi_{ijs}\right] \tag{F.22}$$

[39]We can get them by first conjecturing $\overline{x}_s P_{js} = \gamma_s w_j$, where γ_s is sector level constant. Then $\sum_s \bar{x}_s P_{js} = \left(\sum_s \gamma_s\right) w_j$, which implies the price cut-off $\tilde{p}_{js}^*$ can be written as:

$$\left(\tilde{p}_{js}^*\right)^{\sigma_s} = \frac{\alpha_s \left(\sum_s \gamma_s + 1\right) w_j}{\overline{x}_s P_{j\sigma s}^{1-\sigma_s}} = \frac{\beta_s^{1-\sigma_s} \alpha_s \left(\sum_s \gamma_s + 1\right)}{\beta_{\sigma s} \overline{x}_s^{\sigma_s} \gamma_s^{1-\sigma_s}} \left(\frac{w_j}{N_{js}}\right)^{\sigma_s}$$

Hence, we have

$$\bar{x}_s P_{js} = \beta_s \left(\sigma_s, \theta_s, \eta_s\right) \tilde{p}_{js}^* N_{js} = \left[\frac{\beta_s \alpha_s \left(\sum_s \gamma_s + 1\right)}{\beta_{\sigma s} \overline{x}_s^{\sigma_s} \gamma_s^{1-\sigma_s}}\right]^{\frac{1}{\sigma_s}} w_j = \gamma_s w_j$$

Hence, γ_s is determined by

$$\beta_s \alpha_s \left(\sum_s \gamma_s + 1\right) = \beta_{\sigma s} \overline{x}_s^{\sigma_s} \gamma_s$$

Hence, we have equations (F.16), (F.17) and (F.18).

where $d\ln\xi_{ijs}$ reflects any foreign shock, which satisfies:

$$d\ln\xi_{ijs} = -\theta_s(\eta_s - 1)\,d\ln T_{ijs} - \theta_s d\ln\tau_{ijs} + d\ln b_{is} + d\ln L_i$$

The expression of $\tilde{p}_j^*$, together with equation (C.5) and (C.6), imply that:

$$d\ln\tilde{p}_{js}^* = \frac{1}{\sigma_s}d\ln w_j + \frac{\sigma_s - 1}{\sigma_s}d\ln P_{j\sigma s} = d\ln w_j - \sum_i \lambda_{ijs} d\ln N_{ijs} \tag{F.23}$$

Totally differentiating the expression of N_{ij} and substituting the percentage change of N_{ij} into the previous equation, we have:

$$\begin{aligned}
d\ln\tilde{p}_{js}^* &= d\ln w_j - \sum_i \lambda_{ijs} d\ln N_{ijs} \\
&= d\ln w_j + \sum_i \lambda_{ijs}\left[\eta_s\theta_s\left(d\ln w_i - d\ln\tilde{p}_{js}^*\right) - d\ln\xi_{ijs}\right] \\
&= \frac{1}{1+\eta_s\theta_s}d\ln w_j + \frac{1}{1+\eta_s\theta_s}\sum_i \lambda_{ijs}\left[\eta_s\theta_s d\ln w_i - d\ln\xi_{ijs}\right]
\end{aligned} \tag{F.24}$$

Hence, the percentage change in welfare satisfies:

$$\begin{aligned}
d\ln U_j &= \sum_s \frac{\alpha_s\sigma_s}{\sigma_s - 1}\left(d\ln w_j - d\ln\tilde{p}_{js}^*\right) \\
&= -\sum_s \frac{\alpha_s\sigma_s}{\sigma_s - 1}\frac{1}{1+\eta_s\theta_s}\sum_i \lambda_{ijs}\left[\eta_s\theta_s\left(d\ln w_i - d\ln w_j\right) - d\ln\xi_{ijs}\right] \\
&= -\sum_s \frac{\alpha_s\sigma_s}{\sigma_s - 1}\frac{1}{1+\eta_s\theta_s}d\ln\lambda_{jjs}
\end{aligned} \tag{F.25}$$

Integrating the previous expression between the initial equilibrium (before the shock) and the new equilibrium (after the shock), we finally get

$$\widehat{U}_j = \prod_s \left(\widehat{\lambda}_{jjs}\right)^{-\frac{\alpha_s\sigma_s}{\sigma_s-1}\frac{1}{1+\eta_s\theta_s}} \tag{F.26}$$

It shows that the changes in welfare at country j can be inferred from changes in the share of domestic expenditure, λ_{jjs}, using the parameter, $\frac{\alpha_s\sigma_s}{\sigma_s-1}\frac{1}{1+\eta_s\theta_s}$.

G Fixed Quality Case without T_{ij}

We prove the welfare implication of our model without q_{ij} and T_{ij}. From the demand system, we have the representative consumer in country j's demand given by:

$$x_{ij}(\omega) = L_j\left[\frac{y_j + \bar{x}P_j}{P_{j\sigma}^{1-\sigma}}p_{ij}(\omega)^{-\sigma} - \bar{x}\right] \tag{G.1}$$

where $P_j = \sum_i \int_{\omega\in\Omega_{ij}} p_{ij}(\omega)d\omega$ and $P_{j\sigma} = \left\{\sum_i \int_{\omega\in\Omega_{ij}} p_{ij}(\omega)^{1-\sigma}d\omega\right\}^{\frac{1}{1-\sigma}}$. Now, quantity, sales, and profit for a given variety exported from i to j are as follows,

$$x_{ij}(\omega) = \bar{x}L_j\left[\left(\frac{p_{ij}(\omega)}{p_j^*}\right)^{-\sigma} - 1\right] \tag{G.2}$$

$$r_{ij}(\omega) = \bar{x}L_j p_{ij}(\omega)\left[\left(\frac{p_{ij}(\omega)}{p_j^*}\right)^{-\sigma} - 1\right] \tag{G.3}$$

$$\pi_{ij}(\omega) = \bar{x}L_j\left[p_{ij}(\omega) - c_{ij}(\omega)\right]\left[\left(\frac{p_{ij}(\omega)}{p_j^*}\right)^{-\sigma} - 1\right] \tag{G.4}$$

where $p_j^* = \left(\frac{y_j+\bar{x}P_j}{\bar{x}P_{j\sigma}^{1-\sigma}}\right)^{\frac{1}{\sigma}}$ is the choke price. Given the quality adjusted marginal cost, firms maximize their profits. This implies that the optimal price of the good satisfies:

$$\sigma\frac{c_{ij}(\omega)}{p_j^*} = \left(\frac{p_{ij}(\omega)}{p_j^*}\right)^{\sigma+1} + (\sigma-1)\frac{p_{ij}(\omega)}{p_j^*} \tag{G.5}$$

For the production, we assume that the marginal cost of production is

$$c_{ij} = \frac{w_i\tau_{ij}}{\varphi}\varepsilon$$

where φ follows the Pareto distribution with c.d.f. $G_i(\varphi) = 1 - b_i\varphi^{-\theta}$ and ε is drawn from a log normal distribution. At the productivity cutoff φ_{ij}^* to sell goods from country i to country j, we have $p_{ij}^*(\varphi) = c_{ij}^*(\varphi) = p_j^*$, which implies:

$$\varphi_{ij}^* = \frac{w_i\tau_{ij}}{p_j^*}\varepsilon \tag{G.6}$$

Based on the similar derivation in Section 3, we know that the exporting firm mass N_{ij}, the aggregate price P_j and $P_{j\sigma}$, the trade flow X_{ij}, the expected average profit π_i and the potential firm mass J_i satisfy:

$$N_{ij} = \kappa' J_i b_i\left(\varphi_{ij}^*\right)^{-\theta} \tag{G.7}$$

$$\bar{x}P_j = \beta' p_j^* N_j \tag{G.8}$$

$$\bar{x}P_{j\sigma} = \beta'_\sigma p_j^* N_j^{\frac{1}{1-\sigma}} \tag{G.9}$$

$$X_{ij} = \beta'_X \bar{x} p_j^* N_{ij} L_j \tag{G.10}$$

$$\pi_i = \beta'_\pi \sum_j \bar{x}\kappa' b_i\left(\varphi_{ij}^*\right)^{-\theta} p_j^* L_j \tag{G.11}$$

$$J_i = \frac{\beta'_\pi}{\beta'_X}\frac{L_i}{f} \tag{G.12}$$

where κ', β', β'_σ, β'_X and β'_π are constant. The expression of choke price p_j^*, together with the equation (G.8) and (G.9), implies

$$\bar{x}P_j = \frac{\beta'}{\beta'_\sigma - \beta'} w_j \tag{G.13}$$

$$\bar{x}P_{j\sigma} = \frac{(\beta'_\sigma)^{\frac{1}{1-\sigma}}}{\beta'_\sigma - \beta'} N_j^{\frac{\sigma}{1-\sigma}} w_j \tag{G.14}$$

$$p_j^* = \frac{1}{\bar{x}\left(\beta'_\sigma - \beta'\right)} \frac{w_j}{N_j} \tag{G.15}$$

Now, the welfare still satisfy:

$$U_j = \beta_u \left(\frac{w_j}{p_j^*}\right)^{\frac{\sigma}{\sigma-1}}$$

where $\beta_u = \bar{x}^{\frac{1}{1-\sigma}} \left(\frac{\beta'_\sigma}{\beta'_\sigma - \beta'}\right)^{\frac{\sigma}{\sigma-1}}$ is a constant. The percentage change of U_j satisfies:

$$d\ln U_j = \frac{\sigma}{\sigma - 1}\left(d\ln w_j - d\ln p_j^*\right) \tag{G.16}$$

Now, λ_{jj} satisfies:

$$\lambda_{jj} = \frac{N_{jj}}{\sum_i N_{ij}} = \frac{b_j L_j \left(\tau_{jj} w_j\right)^{-\theta}}{\sum_i b_i L_i \left(\tau_{ij} w_i\right)^{-\theta}} \tag{G.17}$$

Consider the foreign shocks: (b_i, L_i, τ_{ij}) is changed to (b'_i, L'_i, τ'_{ij}) for $i \neq j$ such that $b_j = b'_j, L_j = L'_j, T_{jj} = T'_{jj}, \tau_{jj} = \tau'_{jj}$. Totally differentiating the previous equation implies:

$$d\ln \lambda_{jj} = \sum_i \lambda_{ij}\left[\theta\left(d\ln w_i - d\ln w_j\right) - d\ln \xi_{ij}\right] \tag{G.18}$$

where $d\ln \xi_{ij}$ reflects any foreign shock, which satisfies:

$$d\ln \xi_{ij} = -\theta d\ln \tau_{ij} + d\ln b_i + d\ln L_i$$

The expression of p_j^* imply that:

$$d\ln p_j^* = d\ln w_j - \sum_i \lambda_{ij} d\ln N_{ij} \tag{G.19}$$

Totally differentiating the expression of N_{ij} and substituting the percentage change of N_{ij} into the previous equation, we have:

$$\begin{aligned} d\ln p_j^* &= d\ln w_j + \sum_i \lambda_{ij}\left[\theta\left(d\ln w_i - d\ln p_j^*\right) - d\ln \xi_{ij}\right] \\ &= \frac{1}{1+\theta} d\ln w_j + \frac{1}{1+\theta}\sum_i \lambda_{ij}\left[\theta d\ln w_i - d\ln \xi_{ij}\right] \end{aligned} \tag{G.20}$$

Hence, the percentage change in welfare satisfies:

$$d \ln U_j = \frac{\sigma}{\sigma - 1} \left(d \ln w_j - d \ln p_j^* \right)$$
$$= -\frac{\sigma}{\sigma - 1} \frac{1}{1 + \theta} d \ln \lambda_{jj}$$

Integrating the previous expression between the initial equilibrium (before the shock) and the new equilibrium (after the shock), we finally get

$$\widehat{U}_j = \left(\widehat{\lambda}_{jj} \right)^{-\frac{\sigma}{\sigma-1} \frac{1}{1+\theta}} \tag{G.21}$$

It shows that the changes in welfare at country j can be inferred from changes in the share of domestic expenditure, λ_{jj}, using the parameter, $-\frac{\sigma}{\sigma-1} \frac{1}{1+\theta}$.

H No Variable Markup Case with $\bar{x} = 0$

We prove the welfare implication of our model with a constant markup. From the demand system, we have the representative consumer in country j's demand given by:

$$x_{ij}(\omega) = \frac{w_j L_j}{q_{ij}(\omega) P_{j\sigma}^{1-\sigma}} \left(\frac{p_{ij}(\omega)}{q_{ij}(\omega)} \right)^{-\sigma} \tag{H.1}$$

where $P_{j\sigma} = \left\{ \sum_i \int_{\omega \in \Omega_{ij}} \tilde{p}_{ij}(\omega)^{1-\sigma} d\omega \right\}^{\frac{1}{1-\sigma}}$. To make our derivation compact, we define $\tilde{p}_{ij}(\omega) = p_{ij}(\omega) / q_{ij}(\omega)$. We thus can write quantity, sales, and profit for a given variety exported from i to j as follows,

$$x_{ij}(\omega) = \frac{w_j L_j}{q_{ij}(\omega)} \frac{\tilde{p}_{ij}(\omega)^{-\sigma}}{P_{j\sigma}^{1-\sigma}} \tag{H.2}$$

$$r_{ij}(\omega) = w_j L_j \frac{\tilde{p}_{ij}(\omega)^{1-\sigma}}{P_{j\sigma}^{1-\sigma}} \tag{H.3}$$

$$\pi_{ij}(\omega) = w_j L_j \left[\tilde{p}_{ij}(\omega) - \tilde{c}_{ij}(\omega) \right] \frac{\tilde{p}_{ij}(\omega)^{-\sigma}}{P_{j\sigma}^{1-\sigma}} \tag{H.4}$$

where $\tilde{c}_{ij}(\omega) = c_{ij}(\omega) / q_{ij}(\omega)$ is the quality adjusted marginal cost, where $c_{ij}(\omega)$ is the marginal cost of production. Given the quality adjusted marginal cost, firms maximize their profits. This implies that the optimal price of the good satisfies:

$$\tilde{p}_{ij}(\omega) = \frac{\sigma - 1}{\sigma} \tilde{c}_{ij}(\omega) \tag{H.5}$$

In a similar spirit as in Feenstra and Romalis (2014), the marginal cost of producing a variety of final good with quality q_{ij} by a firm with productivity φ is:

$$c_{ij}(\varphi,\varepsilon) = \left(T_{ij}w_i + \frac{w_i\tau_{ij}q_{ij}^{\eta}}{\varphi}\right)\varepsilon$$

where φ follows the Pareto distribution with c.d.f. $G_i(\varphi) = 1 - b_i\varphi^{-\theta}$ and ε is drawn from a log normal distribution with zero mean and variance σ_ε^2. From the first-order condition associated with the previous marginal cost equation, the optimal level of quality for a firm with productivity φ is:

$$q_{ij}(\varphi,\varepsilon) = \left[\frac{T_{ij}\varphi}{(\eta-1)\,\tau_{ij}}\right]^{\frac{1}{\eta}} \tag{H.6}$$

and hence the quality adjusted marginal cost of production, the quality adjusted marginal cost and the export profit could be rewritten as:

$$\tilde{c}_{ij}(\varphi,\varepsilon) = \frac{c_{ij}(\varphi,\varepsilon)}{q_{ij}(\varphi,\varepsilon)} = \left(\frac{\eta}{\eta-1}T_{ij}w_i\right)^{\frac{\eta-1}{\eta}}\left(\frac{\varphi}{\eta w_i\tau_{ij}}\right)^{-\frac{1}{\eta}}\varepsilon \tag{H.7}$$

$$\tilde{p}_{ij}(\omega) = \frac{\sigma-1}{\sigma}\left(\frac{\eta}{\eta-1}T_{ij}w_i\right)^{\frac{\eta-1}{\eta}}\left(\frac{\varphi}{\eta w_i\tau_{ij}}\right)^{-\frac{1}{\eta}}\varepsilon \tag{H.8}$$

$$\pi_{ij}(\omega) = \frac{1}{\sigma}w_jL_j\frac{\tilde{p}_{ij}(\omega)^{1-\sigma}}{P_{j\sigma}^{1-\sigma}} \tag{H.9}$$

There is also an export fixed cost $f_{ij}w_i$, which need to pay before the exporting. As a result, only a fraction of firms will export and export produtivity cutoff satisfies:

$$\varphi_{ij}^* = \left[\frac{\sigma-1}{\sigma}\left(\frac{\eta}{\eta-1}T_{ij}w_i\right)^{\frac{\eta-1}{\eta}}(\eta w_i\tau_{ij})^{\frac{1}{\eta}}\,\varepsilon\left(\frac{\sigma w_i f_{ij}P_{j\sigma}^{1-\sigma}}{w_jL_j}\right)^{\frac{1}{\sigma-1}}\right]^{\eta} \tag{H.10}$$

With these definitions in mind, the aggregate price statistics, $P_{j\sigma}$, can be rewritten as:

$$P_{j\sigma} = \left\{\frac{\eta\theta\kappa}{\eta\theta-(\sigma-1)}\sum_i b_iJ_i\left(\frac{\sigma-1}{\sigma}\left(\frac{\eta}{\eta-1}T_{ij}w_i\right)^{\frac{\eta-1}{\eta}}(\eta w_i\tau_{ij})^{\frac{1}{\eta}}\right)^{-\theta\eta}\left(\frac{\sigma w_i f_{ij}}{w_jL_j}\right)^{\frac{\sigma-1-\theta\eta}{\sigma-1}}\right\}^{-\frac{1}{\theta\eta}} \tag{H.11}$$

where κ is a constant. The bilateral trade flow, X_{ij}, would satisfy:

$$X_{ij} = N_{ij}\int_0^{\infty}\int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi,\varepsilon)\,\mu_{ij}(\varphi,\varepsilon)\,f(\varepsilon)\,d\varphi d\varepsilon \tag{H.12}$$

$$= \frac{\eta\theta\kappa}{\eta\theta-(\sigma-1)}b_iJ_iw_jL_j\frac{\left(\frac{\sigma-1}{\sigma}\left(\frac{\eta}{\eta-1}T_{ij}w_i\right)^{\frac{\eta-1}{\eta}}(\eta w_i\tau_{ij})^{\frac{1}{\eta}}\right)^{-\theta\eta}\left(\frac{\sigma w_i f_{ij}}{w_jL_j}\right)^{\frac{\sigma-1-\theta\eta}{\sigma-1}}}{P_{j\sigma}^{-\theta\eta}} \tag{H.13}$$

Firm's profits equals to the total fixed cost paid, which yields the free entry condition:

$$w_i f = \frac{1}{\sigma}\frac{1}{J_i}\sum_j X_{ij} = \frac{1}{\sigma}\frac{w_i L_i}{J_i} \tag{H.14}$$

where the last equality stems from that total income equals to total expenditure. Hence, the potential firm mass is

$$J_i = \frac{L_i}{\sigma f}$$

Now, the percentage change of U_j satisfies:

$$d\ln U_j = d\ln w_j - d\ln P_{j\sigma} \tag{H.15}$$

Now, λ_{jj} satisfies:

$$\lambda_{jj} = \frac{X_{jj}}{\sum_i X_{ij}} = \frac{b_j L_j \left(\left(T_{jj}^{\eta-1}\tau_{jj}\right)^{\frac{1}{\eta}} w_j\right)^{-\theta\eta} \left(w_j f_{jj}\right)^{\frac{\sigma-1-\theta\eta}{\sigma-1}}}{\sum_i b_i L_i \left(\left(T_{ij}^{\eta-1}\tau_{ij}\right)^{\frac{1}{\eta}} w_i\right)^{-\theta\eta} \left(w_i f_{ij}\right)^{\frac{\sigma-1-\theta\eta}{\sigma-1}}} \tag{H.16}$$

Consider the foreign shocks: τ_{ij}, T_{ij}, f_{ij} are changed to τ'_{ij}, T'_{ij}, f'_{ij} for $i \neq j$, respectively, such that $\tau_{jj} = \tau'_{jj}$ $T_{jj} = T'_{jj}$ and $f_{jj} = f'_{jj}$. Totally differentiating the previous equation implies:

$$d\ln\lambda_{jj} = \sum_i \lambda_{ij}\left[\left(\frac{\sigma}{\sigma-1}\theta\eta - 1\right)(d\ln w_i - d\ln w_j) - d\ln\xi_{ij}\right] \tag{H.17}$$

where $d\ln\xi_{ij}$ reflects any foreign shock, which satisfies:

$$d\ln\xi_{ij} = -\theta\eta\left(\frac{1}{\eta}d\ln\tau_{ij} + \frac{\eta-1}{\eta}d\ln T_{ij} + \left(\frac{1}{\sigma-1} - \frac{1}{\theta\eta}\right)d\ln f_{ij}\right) \tag{H.18}$$

The expression of $P_{j\sigma}$ implies that:

$$d\ln P_{j\sigma} = \sum_i \lambda_{ij}\left[d\ln w_i + \left(\frac{1}{\sigma-1} - \frac{1}{\theta\eta}\right)(d\ln w_i - d\ln w_j) - \frac{1}{\theta\eta}d\ln\xi_{ij}\right] \tag{H.19}$$

Hence, the percentage change in welfare satisfies:

$$\begin{aligned} d\ln U_j &= -\sum_i \lambda_{ij}\left[\left(\frac{\sigma}{\sigma-1} - \frac{1}{\theta\eta}\right)(d\ln w_i - d\ln w_j) - \frac{1}{\theta\eta}d\ln\xi_{ij}\right] \\ &= -\frac{1}{\theta\eta}d\ln\lambda_{jj} \end{aligned}$$

Integrating the previous expression between the initial equilibrium (before the shock) and the new equilibrium (after the shock), we finally get

$$\widehat{U}_j = \left(\widehat{\lambda}_{jj}\right)^{-\frac{1}{\theta\eta}} \tag{H.20}$$

It shows that the changes in welfare at country j can be inferred from changes in the share of domestic expenditure, λ_{jj}, using the parameter, $-\frac{1}{\theta\eta}$.

I Derivation for Welfare Comparison

I.1 Quality Case with T_{ij}

The representative consumer has preferences of:

$$U_j = \left[\sum_i \int_{\omega\in\Omega_{ij}} \left(q_{ij}(\omega)x^c_{ij}(\omega) + \overline{x}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} = \frac{y_j + \overline{x}P_j}{P_{j\sigma}} = \frac{\beta_\sigma}{\beta_\sigma - \beta}\frac{w_j}{P_{j\sigma}} \tag{I.1}$$

where $P_{j\sigma} = \left\{\sum_i J_i \int_0^\infty \int_{\varphi^*_{ij}(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} g_i(\varphi) f(\varepsilon)\, d\varphi d\varepsilon\right\}^{\frac{1}{1-\sigma}}$. Totally differentiating the previous equation, we have:

$$\begin{aligned} d\ln U_j &= d\ln w_j - d\ln P_{j\sigma} \\ &= d\ln w_j - \sum_i \lambda_{ij}\left(\frac{1}{\sigma-1} d\ln\left[J_i \int_0^\infty \int_{\varphi^*_{ij}(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} g_i(\varphi) f(\varepsilon)\, d\varphi d\varepsilon\right]\right) \end{aligned}$$

where

$$\begin{aligned} &\frac{1}{\sigma-1} d\ln\left[J_i \int_0^\infty \int_{\varphi^*_{ij}(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} g_i(\varphi) f(\varepsilon)\, d\varphi d\varepsilon\right] \\ = &-\frac{\int_0^\infty \int_{\varphi^*_{ij}(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} d\ln \tilde{p}_{ij}(\varphi,\varepsilon) g_i(\varphi) f(\varepsilon)\, d\varphi d\varepsilon}{\int_0^\infty \int_{\varphi^*_{ij}(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} g_i(\varphi) f(\varepsilon)\, d\varphi d\varepsilon} \\ &+\frac{1}{\sigma-1} d\ln J_i \\ &+\frac{1}{\sigma-1}\frac{\int_0^\infty \left(\tilde{p}^*_j\right)^{1-\sigma} g_i\left(\varphi^*_{ij}(\varepsilon)\right) \varphi^*_{ij}(\varepsilon) f(\varepsilon)\, d\varepsilon}{\int_0^\infty \int_{\varphi^*_{ij}(\varepsilon)}^\infty \tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma} g_i(\varphi) f(\varepsilon)\, d\varphi d\varepsilon} d\ln \varphi^*_{ij} \end{aligned}$$

where the first term is the effects of changes in the prices of existing varieties calculated in ACDR; the second term is the effects of a change in potential firm entrants; the third term is the impact on welfare associated with the change in cutoff. Same as ACDR, the effects of changes in potential firm entrants, $d\ln J_i = 0$. However, the third term, the impact from a change in cutoff, is not infinitesimal, which should be larger than the gap between GT_j^{Bench} and $GT_j^{con\ mkp}$. The welfare change in our benchmark model are given by $-\frac{\sigma}{\sigma-1}\frac{1}{1+\theta\eta}\widehat{\lambda}_{jj}$ and the welfare change under the model without markup is given by $-\frac{\widehat{\lambda}_{jj}}{\theta\eta}$. Hence, their gap equals to

$$
\begin{aligned}
& -\frac{\sigma}{\sigma-1}\frac{1}{1+\theta\eta}\widehat{\lambda}_{jj}-\left(-\frac{\widehat{\lambda}_{jj}}{\theta\eta}\right) \\
= & -\frac{\theta\eta-(\sigma-1)}{\theta\eta\,[\sigma-1]}\frac{1}{1+\eta\theta}d\ln\lambda_{jj}
\end{aligned}
$$

In the following, we will prove that the third term is larger than this gap, $-\frac{\theta\eta-(\sigma-1)}{\theta\eta[\sigma-1]}\frac{1}{1+\eta\theta}d\ln\lambda_{jj}$. Hence, if we only focus on the first term by ignoring the extensive margin, the gain from trade in our benchmark model, GT_j^{bench}, is less than $GT_j^{con\ mkp}$. However, if including extensive, the gain from trade in our benchmark model, GT_j^{bench}, should be larger than $GT_j^{con\ mkp}$.

Proof: The third term could be rewritten as:

$$
\begin{aligned}
& \frac{1}{\sigma-1}\frac{\int_0^\infty\left(\tilde{p}_j^*\right)^{1-\sigma}g_i\left(\varphi_{ij}^*(\varepsilon)\right)\varphi_{ij}^*(\varepsilon)f(\varepsilon)\,d\varepsilon}{\int_0^\infty\int_{\varphi_{ij}^*(\varepsilon)}^\infty\tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma}g_i(\varphi)f(\varepsilon)\,d\varphi d\varepsilon}d\ln\varphi_{ij}^* \\
= & \frac{1}{\sigma-1}\frac{\int_0^\infty g_i\left(\varphi_{ij}^*(\varepsilon)\right)\varphi_{ij}^*(\varepsilon)f(\varepsilon)\,d\varepsilon}{\beta\int_0^\infty\left[1-G_{ij}\left(\varphi_{ij}^*(\varepsilon)\right)\right]f(\varepsilon)\,d\varepsilon}d\ln\varphi_{ij}^* \\
= & \frac{1}{\sigma-1}\frac{\theta}{\beta}d\ln\varphi_{ij}^*
\end{aligned}
$$

where $\beta=\int_{\varphi_{ij}^*(\varepsilon)}^\infty\left[\frac{\tilde{p}_{ij}(\varphi,\varepsilon)}{\tilde{p}_j^*}\right]^{1-\sigma}\frac{g_i(\varphi)}{1-G_{ij}\left(\varphi_{ij}^*(\varepsilon)\right)}d\varphi$ is constant. Consider that $\frac{\tilde{p}_{ij}(\varphi,\varepsilon)}{\tilde{p}_j^*}>\frac{\tilde{c}_{ij}(\varphi,\varepsilon)}{\tilde{p}_j^*}=\left(\frac{\varphi}{\varphi_{ij}^*(\varepsilon)}\right)^{-\frac{1}{\eta}}$, we know that β could satisfy

$$
\begin{aligned}
\beta & < \int_{\varphi_{ij}^*(\varepsilon)}^\infty\left[\left(\frac{\varphi}{\varphi_{ij}^*(\varepsilon)}\right)^{-\frac{1}{\eta}}\right]^{1-\sigma}\theta\left(\varphi_{ij}^*(\varepsilon)\right)^{\theta+1}\varphi^{-\theta-1}d\frac{\varphi}{\varphi_{ij}^*(\varepsilon)} \\
& = \int_{\varphi_{ij}^*(\varepsilon)}^\infty\theta\left(\frac{\varphi}{\varphi_{ij}^*(\varepsilon)}\right)^{-\frac{\theta\eta-(\sigma-1)}{\eta}-1}d\left(\frac{\varphi}{\varphi_{ij}^*(\varepsilon)}\right)=\frac{\theta\eta}{\theta\eta-(\sigma-1)}
\end{aligned}
$$

The expression of $N_{ij}=J_i\int_0^\infty\left[1-G_{ij}\left(\varphi_{ij}^*(\varepsilon)\right)\right]f(\varepsilon)\,d\varepsilon$ implies that:

$$
d\ln\varphi_{ij}^*=-\frac{1}{\theta}d\ln N_{ij}
$$

which implies that the impact of cutoff on welfare satisfies:

$$
\begin{aligned}
& -\frac{1}{\sigma-1}\sum_i\lambda_{ij}\frac{\int_0^\infty\left(\tilde{p}_j^*\right)^{1-\sigma}g_i\left(\varphi_{ij}^*(\varepsilon)\right)\varphi_{ij}^*(\varepsilon)f(\varepsilon)\,d\varepsilon}{\int_0^\infty\int_{\varphi_{ij}^*(\varepsilon)}^\infty\tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma}g_i(\varphi)f(\varepsilon)\,d\varphi d\varepsilon}d\ln\varphi_{ij}^* \\
= & -\frac{1}{\sigma-1}\frac{\theta}{\beta}\sum_i\lambda_{ij}d\ln\varphi_{ij}^*>-\frac{\theta\eta-(\sigma-1)}{\eta\,[\sigma-1]}\sum_i\lambda_{ij}d\ln\varphi_{ij}^* \\
= & \frac{\theta\eta-(\sigma-1)}{\theta\eta\,[\sigma-1]}\sum_i\lambda_{ij}d\ln N_{ij}=-\frac{\theta\eta-(\sigma-1)}{\theta\eta\,[\sigma-1]}\frac{1}{1+\eta\theta}d\ln\lambda_{jj}
\end{aligned}
$$

This implies that the impact on welfare associated with the change in cutoff should be larger than $-\frac{\theta\eta-(\sigma-1)}{\theta\eta[\sigma-1]}\frac{1}{1+\eta\theta}d\ln\lambda_{jj}$.

I.2 Fixed Quality Case without T_{ij}

The representative consumer has preferences of:

$$U_j = \left[\sum_i \int_{\omega\in\Omega_{ij}} \left(x_{ij}^c(\omega)+\overline{x}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} = \frac{\beta_\sigma}{\beta_\sigma-\beta}\frac{w_j}{P_{j\sigma}} \tag{I.2}$$

where $P_{j\sigma} = \left\{\sum_i J_i \int_{\varphi_{ij}^*}^{\infty} p_{ij}\left(\varphi\right)^{1-\sigma} g_i\left(\varphi\right)d\varphi\right\}^{\frac{1}{1-\sigma}}$. Totally differentiating the previous equation, we have:

$$\begin{aligned} d\ln U_j &= d\ln w_j - d\ln P_{j\sigma} \\ &= d\ln w_j - \sum_i \lambda_{ij}\left(\frac{1}{\sigma-1}d\ln\left[J_i\int_{\varphi_{ij}^*}^{\infty} p_{ij}\left(\varphi\right)^{1-\sigma} g_i\left(\varphi\right)d\varphi\right]\right) \end{aligned}$$

where

$$\begin{aligned} &\frac{1}{\sigma-1}d\ln\left[J_i\int_{\varphi_{ij}^*}^{\infty} p_{ij}\left(\varphi\right)^{1-\sigma} g_i\left(\varphi\right)d\varphi\right] \\ = \;& -\frac{\int_{\varphi_{ij}^*}^{\infty} p_{ij}\left(\varphi\right)^{1-\sigma} d\ln p_{ij}\left(\varphi\right) g_i\left(\varphi\right)d\varphi}{\int_{\varphi_{ij}^*}^{\infty} p_{ij}\left(\varphi\right)^{1-\sigma} g_i\left(\varphi\right)d\varphi} \\ & + \frac{1}{\sigma-1}d\ln J_i \\ & + \frac{1}{\sigma-1}\frac{\left(\tilde{p}_j^*\right)^{1-\sigma} g_i\left(\varphi_{ij}^*\right)\varphi_{ij}^*}{\int_{\varphi_{ij}^*}^{\infty} p_{ij}\left(\varphi\right)^{1-\sigma} g_i\left(\varphi\right)d\varphi}d\ln\varphi_{ij}^* \end{aligned}$$

where the first term is the effects of changes in the prices of existing varieties calculated in ACDR; the second term is the effects of a change in potential firm entrants; the third term is the impact on welfare associated with the change in cutoff. Same as ACDR, the effects of changes in potential firm entrants, $d\ln J_i = 0$. However, the third term, the impact from a change in cutoff, is not infinitesimal, which should be larger than the gap between $GT_j^{no\ q}$ and $GT_j^{no\ q,\ con\ mkp}$. The welfare changes under variable markups but no Washington Apples mechanism are given by $GT_j^{no\ q} = -\frac{\sigma}{\sigma-1}\frac{1}{1+\theta}\widehat{\lambda}_{jj}$ and the welfare change under the model without both endogenous quality and variable markup is given by $GT_j^{no\ q,\ con\ mkp} = -\frac{\widehat{\lambda}_{jj}}{\theta}$. Hence, their gap equals to

$$
\begin{aligned}
& -\frac{\sigma}{\sigma-1}\frac{1}{1+\theta}\widehat{\lambda}_{jj}-\left(-\frac{\widehat{\lambda}_{jj}}{\theta}\right) \\
= & -\frac{\theta-(\sigma-1)}{\theta\left[\sigma-1\right]}\frac{1}{1+\theta}d\ln\lambda_{jj}
\end{aligned}
$$

In the following, we will prove that the third term is larger than this gap, $-\frac{\theta-(\sigma-1)}{\theta[\sigma-1]}\frac{1}{1+\theta}d\ln\lambda_{jj}$. Hence, if we only focus on the first term by ignoring the extensive margin, the gain from trade under variable markups but no Washington Apples mechanism, $GT_j^{no\ q}$, is less than $GT_j^{no\ q,\ con\ mkp}$. However, if including extensive margin, the gain from trade under variable markups but no Washington Apples mechanism, $GT_j^{no\ q}$, should be larger than $GT_j^{no\ q,\ con\ mkp}$.

Proof: The third term could be rewritten as:

$$
\begin{aligned}
& \frac{1}{\sigma-1}\frac{\left(\tilde{p}_j^*\right)^{1-\sigma}g_i\left(\varphi_{ij}^*\right)\varphi_{ij}^*}{\int_{\varphi_{ij}^*}^{\infty}p_{ij}\left(\varphi\right)^{1-\sigma}g_i\left(\varphi\right)d\varphi}d\ln\varphi_{ij}^* \\
= & \frac{1}{\sigma-1}\frac{\theta}{\beta}d\ln\varphi_{ij}^*
\end{aligned}
$$

where $\beta=\int_{\varphi_{ij}^*}^{\infty}\left[\frac{p_{ij}(\varphi)}{p_j^*}\right]^{1-\sigma}\frac{g_i(\varphi)}{1-G_{ij}\left(\varphi_{ij}^*\right)}d\varphi$ is constant. Consider that $\frac{p_{ij}(\varphi)}{p_j^*}>\frac{c_{ij}(\varphi)}{p_j^*}=\left(\frac{\varphi}{\varphi_{ij}^*}\right)^{-1}$, we know that β could satisfy

$$
\beta<\int_{\varphi_{ij}^*}^{\infty}\theta\left(\frac{\varphi}{\varphi_{ij}^*}\right)^{-(\theta-(\sigma-1))-1}d\left(\frac{\varphi}{\varphi_{ij}^*}\right)=\frac{\theta}{\theta-(\sigma-1)}
$$

The expression of $N_{ij}=J_i\left[1-G_{ij}\left(\varphi_{ij}^*\right)\right]$ implies that:

$$
d\ln\varphi_{ij}^*=-\frac{1}{\theta}d\ln N_{ij}
$$

which implies that the impact of cutoff on welfare satisfies:

$$
\begin{aligned}
& -\frac{1}{\sigma-1}\sum_i\lambda_{ij}\frac{\left(\tilde{p}_j^*\right)^{1-\sigma}g_i\left(\varphi_{ij}^*\right)\varphi_{ij}^*}{\int_{\varphi_{ij}^*}^{\infty}p_{ij}\left(\varphi\right)^{1-\sigma}g_i\left(\varphi\right)d\varphi}d\ln\varphi_{ij}^* \\
= & -\frac{1}{\sigma-1}\frac{\theta}{\beta}\sum_i\lambda_{ij}d\ln\varphi_{ij}^*>-\frac{\theta-(\sigma-1)}{\sigma-1}\sum_i\lambda_{ij}d\ln\varphi_{ij}^* \\
= & -\frac{\theta-(\sigma-1)}{\theta\left[\sigma-1\right]}\frac{1}{1+\theta}d\ln\lambda_{jj}
\end{aligned}
$$

This implies that the impact on welfare associated with the change in cutoff should be larger than $-\frac{\theta-(\sigma-1)}{\theta[\sigma-1]}\frac{1}{1+\theta}d\ln\lambda_{jj}$.

J Supplementary Table: Welfare Comparison for All Countries

country	Bench	no q	con mkp	no q, con mkp
AUS	4.131	26.684	1.747	6.077
AUT	6.391	38.485	2.721	9.347
BEL	10.731	56.618	4.630	15.521
BRA	1.114	7.910	0.467	1.651
CAN	5.925	36.196	2.519	8.676
CHE	7.154	42.082	3.053	10.444
CHN	1.636	11.425	0.686	2.421
DEU	3.934	25.566	1.662	5.789
DNK	5.955	36.348	2.532	8.720
ESP	3.703	24.242	1.564	5.453
FIN	3.805	24.827	1.607	5.601
FRA	3.478	22.929	1.468	5.124
GBR	4.706	29.857	1.993	6.912
GRC	4.294	27.595	1.816	6.313
HKG	10.800	56.864	4.661	15.618
IDN	2.565	17.403	1.080	3.788
IND	1.037	7.384	0.435	1.537
IRL	7.951	45.638	3.401	11.583
ITA	2.273	15.565	0.956	3.359
JPN	1.292	9.125	0.542	1.914
KOR	2.314	15.820	0.973	3.418
MEX	4.513	28.805	1.910	6.632
MYS	6.530	39.154	2.781	9.547
NLD	5.977	36.453	2.541	8.750
NOR	5.187	32.420	2.200	7.609
POL	3.453	22.779	1.457	5.087
PRT	4.643	29.514	1.966	6.820
RUS	2.445	16.650	1.029	3.612
SAU	4.688	29.763	1.986	6.887
SGP	13.372	65.218	5.819	19.208
SWE	4.714	29.899	1.996	6.923
THA	4.962	31.231	2.103	7.283
TUR	2.436	16.595	1.025	3.599
TWN	5.045	31.672	2.139	7.404
USA	2.130	14.647	0.895	3.148
ZAF	2.112	14.533	0.888	3.122

K Supplementary Figure

Figure 9: Sales and Markup Distribution

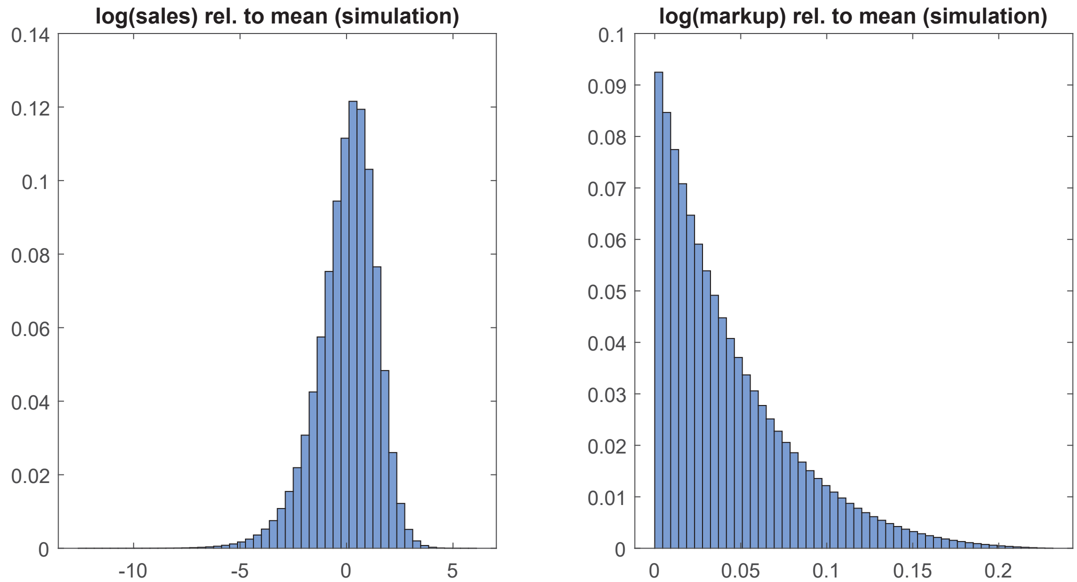


Figure 10: The relationship between market size and firm-level variables (prices, sales, and quality)

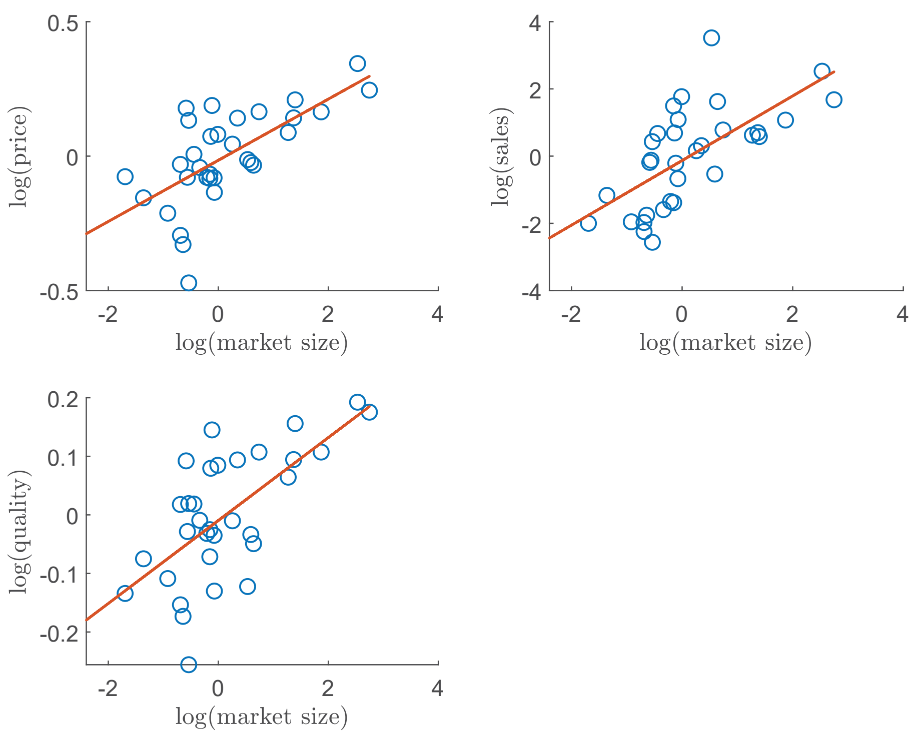
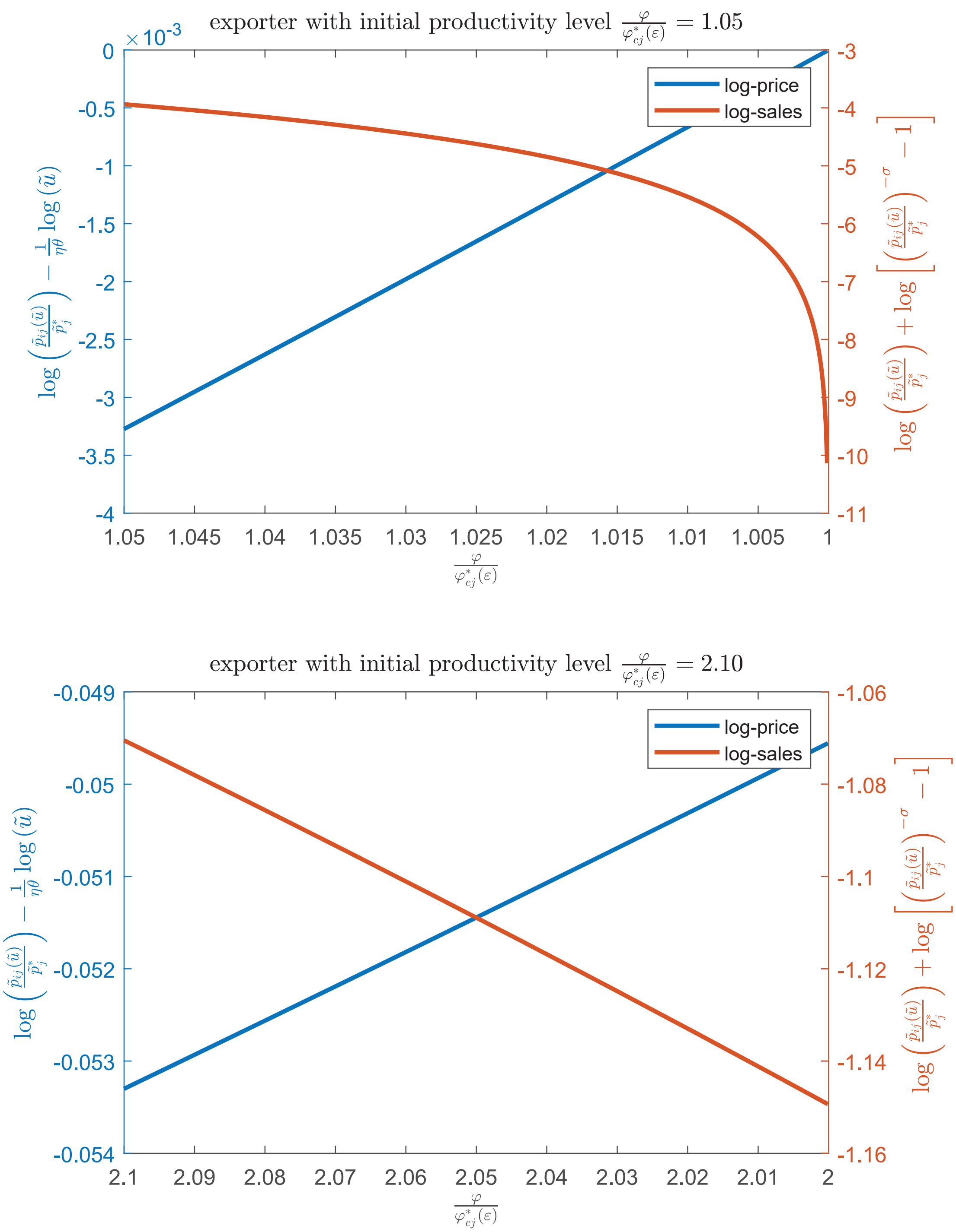


Figure 11: Illustration: the Changes in Prices and Sales by Low- vs. High-productivity Firms after Trade Cost Shock



Explanatory notes on Figure 11:

The upper panel plots a low-productivity firm whose productivity is only 5% above the cutoff productivity before the trade shock, i.e., $\frac{\varphi}{\varphi^*_{cj}(\varepsilon)} = 1.05$. When trade cost increases by

5% (either from τ or T), $\frac{\varphi}{\varphi^*_{cj}(\varepsilon)}$ goes to 1. Then, this producer starts to become a marginal exporter. The left y-axis plots the change of log(price), and the right y-axis plots the change of log(sales). Clearly, the variation in price changes is very small whereas the change in sales is large. Next, we turn to a initially high-productivity firm with $\frac{\varphi}{\varphi^*_{cj}(\varepsilon)} = 2.10$ shown in the lower panel. When it is hit by 5% increase in trade cost, the changes in log(price) is similar comparing with the low-productivity exporter in the upper panel, but the change in log(sales) is much smaller for this high-productivity firm.