

# Online Appendix for “Equilibrium Allocations under Alternative Waitlist Designs: Evidence from Deceased Donor Kidneys Kidneys”

Nikhil Agarwal, Itai Ashlagi, Michael Rees, Paulo Somaini, Daniel Waldinger.

## A Detailed Estimation Results

### *Positive Crossmatch Probability*

Not all accepted offers result in transplantation because additional testing may yield a positive crossmatch indicating that the patient is likely to develop an immune response to the donor’s kidney. These transplants are not carried out, and if possible the organ is placed with another patient. To account for positive crossmatches when computing value functions and conducting counterfactual simulations, we estimate a probit model to predict the probability that a patient has a positive crossmatch with an organ they have accepted. The specification includes interactions between the patient’s CPRA and the number of HLA mismatches with the donor, in addition to controls for patient age and number of years on dialysis. We use a subset of the variables included in the CCP model to avoid overfitting. Coefficient estimates and standard errors are displayed in Table A.I. The results are intuitive and consistent with medical knowledge. For example, higher CPRA is associated with a higher positive crossmatch probability, as are more tissue-type dissimilarities (as measured by DR or HLA mismatches). This is consistent with the view that patients with more sensitized immune systems may be more likely to test positive against foreign antibodies, even if they have not tested positive in the past.

### *Maximum Number of Offers and Discards*

Some organs are not offered to all compatible patients in NYRT. This usually occurs either because an organ becomes unsuitable for transplantation or because the organ is accepted by a patient in another OPO. We call these events “timeouts.”

We model the maximum number of offers that can be made for a given organ using a censored exponential hazards model. Duration is the number of observed offers. Censoring occurs if the organ is placed, or if it is discarded after being offered to all compatible NYRT patients. The hazard function is

Table A.I: Positive Crossmatch Model

CPRA	1.0
0 or 1 HLA Mismatches	-1.0
2 or 3 HLA Mismatches	0.0
0 DR Mismatches	-0.0
CPRA x 1{0 or 1 HLA Mismatches}	-0.0
CPRA x 1{2 or 3 HLA Mismatches}	-0.0
CPRA - 0	-0.0
CPRA - 0.8 if CPRA > 0.8	-3.0
Log Dialysis Time at Registration (Years)	-0.0
Log Dialysis Time at Registration x 1{Over 5 Years}	1.0
Patient Age at Registration (Years)	0.0
Age at Registration - 35 if Age > 35	-0.0
Constant	-0.0
Observations	

$$\lambda_o(z) = \lambda_o \exp(z\beta) \quad (\text{A.1})$$

where  $z$  are characteristics of the donor,  $\beta$  is a vector of coefficients, and  $\lambda_o$  is the constant baseline hazard rate. We allow the hazard to depend on geography and indicators of donor quality. Specifically, we control for whether the donor is an expanded criteria donor (ECD); the donor's cause of death (DCD); and whether the donor was recovered in NYRT, as well as interactions among these variables. The estimated timeout hazards are inputs in the counterfactual exercises.

Kidneys that reach the maximum number of offers can be discarded or allocated to a patient outside NYRT. We model the probability that a donor's unallocated kidneys are discarded using a probit model that includes the same set of covariates used to estimate the maximum number of potential offers. This part of the model does not influence allocation and incentives for patients in NYRT. It is used to properly account for changes in discards for kidneys not allocated to patients in NYRT.

Table A.II: Survival Model Estimates

	Gompertz (1)	Weibull (2)	Cox (3)
Diabetic Patient	0.0812 (0.0336)	0.0739 (0.0336)	0.0850 (0.0336)
Bloodtype A Patient	0.159 (0.0437)	0.127 (0.0436)	0.165 (0.0438)
Bloodtype O Patient	0.00394 (0.0392)	0.00400 (0.0392)	0.00385 (0.0392)
Calculated Panel Reactive Antibodies (CPRA)	-0.000126 (0.00150)	-0.000211 (0.00150)	-0.000275 (0.00150)
CPRA = 0	0.190 (0.0738)	0.179 (0.0738)	0.181 (0.0739)
CPRA - 80 if CPRA>=80	-0.0230 (0.00650)	-0.0204 (0.00650)	-0.0225 (0.00650)
Age (at Registration)	-0.0418 (0.0150)	-0.0363 (0.0151)	-0.0361 (0.0151)
Age - 18 if Age>=18	0.0399 (0.0184)	0.0356 (0.0186)	0.0348 (0.0186)
Age - 35 if Age>=35	-0.00988 (0.00966)	-0.0121 (0.00966)	-0.0104 (0.00966)
Age - 50 if Age>=50	0.0236 (0.00729)	0.0231 (0.00728)	0.0242 (0.00729)
Age - 65 if Age>=65	0.0241 (0.00927)	0.0233 (0.00926)	0.0238 (0.00929)
Prior Transplant	0.0513 (0.0552)	0.0590 (0.0550)	0.0546 (0.0552)
Body Mass Index (BMI)	-0.0155 (0.00639)	-0.0145 (0.00639)	-0.0156 (0.00640)
Missing BMI	-0.0680 (0.199)	0.0736 (0.199)	-0.104 (0.200)
BMI >= 18.5	-0.0382 (0.106)	-0.0450 (0.106)	-0.0356 (0.106)
BMI >= 25	0.00882 (0.0492)	0.00346 (0.0492)	0.00918 (0.0492)
BMI >= 30	0.0509 (0.0595)	0.0429 (0.0595)	0.0513 (0.0595)
Total Serum Albumin	-0.163 (0.0549)	-0.160 (0.0550)	-0.156 (0.0548)
Missing Total Serum Albumin	-0.533 (0.189)	-0.461 (0.189)	-0.490 (0.189)
Total Serum Albumin >= 3.7	-0.0645 (0.0591)	-0.0630 (0.0592)	-0.0681 (0.0591)
Total Serum Albumin >= 4.4	0.0512 (0.0510)	0.0405 (0.0509)	0.0505 (0.0510)
On Dialysis at Registration	-0.149 (0.113)	-0.169 (0.113)	-0.142 (0.113)
Log Years on Dialysis at Registration	-0.00139 (0.0185)	0.00451 (0.0185)	-0.00291 (0.0185)
Log Years on Dialysis at Registration x 1{Over 5 Years}	0.187 (0.110)	0.181 (0.110)	0.181 (0.110)
Constant	-5.870 (0.342)	-5.308 (0.352)	
Gompertz Shape Parameter	0.0000922 (0.0000210)		
Weibull Shape Parameter		-0.0785 (0.0143)	
Observations	9623	9623	9623

## Detailed CCP Estimates

Table A.III: Conditional Choice Probability of Acceptance (Detailed)

	Base Specification		Unobserved Heterog.		Waiting Time + UH	
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	-3.70	(0.02)	-4.47	(0.03)	-4.49	(0.05)
Patient Diabetic	-0.06	(0.01)	-0.05	(0.02)	-0.03	(0.02)
Calculated Panel Reactive Antibody (CPRA)	0.60	(0.05)	0.68	(0.06)	0.58	(0.09)
CPRA >= 0.8	0.27	(0.05)	0.10	(0.06)	0.12	(0.08)
CPRA = 0	-0.10	(0.02)	-0.02	(0.03)	-0.02	(0.03)
CPRA - 0.8 if CPRA >= 0.8	-0.37	(0.37)	-0.37	(0.48)	-0.56	(0.50)
Patient had Prior Transplant	0.38	(0.02)	0.36	(0.02)	0.14	(0.03)
Donor Age < 18	0.27	(0.10)	-0.09	(0.19)	-0.04	(0.20)
Donor Age 18-35	0.59	(0.12)	-0.06	(0.19)	0.02	(0.19)
Donor Age 50+	-0.83	(0.16)	-0.77	(0.21)	-0.87	(0.22)
Donor Cause of Death Anoxia	-0.04	(0.02)	-0.12	(0.06)	-0.10	(0.06)
Donor Cause of Death Stroke	0.01	(0.02)	0.02	(0.06)	0.04	(0.07)
Donor Cause of Death CNS	0.17	(0.09)	-0.16	(0.32)	-0.16	(0.36)
Donor Creatinine 0.5-1.0	-0.06	(0.03)	0.02	(0.11)	-0.01	(0.11)
Donor Creatinine 1.0-1.5	0.01	(0.03)	0.00	(0.11)	-0.04	(0.10)
Donor Creatinine >= 1.5	-0.13	(0.03)	-0.21	(0.11)	-0.23	(0.11)
Donor Pancreas Offered	0.36	(0.03)	0.54	(0.09)	0.56	(0.09)
Expanded Criteria Donor (ECD)	-0.14	(0.02)	-0.53	(0.08)	-0.53	(0.10)
Donation from Cardiac Death (DCD)	-0.10	(0.02)	-0.51	(0.06)	-0.50	(0.09)
Donor Male	0.01	(0.01)	0.05	(0.05)	0.06	(0.04)
Donor History of Hypertension	0.01	(0.02)	-0.01	(0.05)	-0.01	(0.05)
Perfect Tissue Type Match	2.33	(0.31)	2.92	(0.43)	2.89	(0.44)
2 A Mismatches	-0.08	(0.02)	0.00	(0.02)	0.00	(0.02)
2 B Mismatches	0.06	(0.02)	0.02	(0.03)	0.03	(0.03)
2 DR Mismatches	-0.06	(0.02)	-0.05	(0.02)	-0.05	(0.02)
ABO Compatible	-0.35	(0.05)	-0.40	(0.09)	-0.41	(0.09)
Regional Offer	-1.38	(0.06)	-2.90	(0.19)	-2.92	(0.19)
National Offer	-1.54	(0.04)	-3.05	(0.12)	-3.11	(0.11)
Non-NYRT Donor, NYRT Match Run	1.23	(0.02)	2.02	(0.05)	2.08	(0.05)
Patient Blood Type A	-0.17	(0.02)	-0.28	(0.07)	-0.28	(0.07)
Patient Blood Type O	-0.32	(0.02)	-0.38	(0.06)	-0.39	(0.06)
Patient on Dialysis at Registration	-0.02	(0.02)	-0.10	(0.02)	-0.09	(0.02)
Patient Age at Registration	0.04	(0.01)	0.10	(0.01)	0.10	(0.01)
Patient Age - 18 if Age >= 18	-0.05	(0.01)	-0.11	(0.01)	-0.11	(0.01)
Patient Age - 35 if Age >= 35	0.01	(0.00)	0.02	(0.01)	0.02	(0.01)
Patient Age - 50 if Age >= 50	0.00	(0.00)	0.00	(0.00)	0.00	(0.00)
Patient Age - 65 if Age >= 65	-0.01	(0.00)	0.00	(0.01)	-0.01	(0.01)
Log Waiting Time (years)					0.09	(0.06)
Log Waiting Time x 1{Over 1 Year}					-0.15	(0.07)
Log Waiting Time x 1{Over 2 Years}					-0.13	(0.12)
Log Waiting Time x 1{Over 3 Years}					0.30	(0.11)
Patient BMI at Departure					-0.07	(0.03)
Patient BMI - 18.5 if BMI >= 18.5	0.03	(0.03)	0.07	(0.04)	0.06	(0.04)
Patient BMI - 25 if BMI >= 25	0.02	(0.01)	0.02	(0.01)	0.02	(0.01)
Patient BMI - 30 if BMI >= 30	-0.01	(0.01)	-0.02	(0.01)	-0.02	(0.01)
Patient Serum Albumin	-0.02	(0.03)	-0.01	(0.03)	-0.01	(0.03)
Serum Albumin - 3.7 if >= 3.7	-0.04	(0.05)	-0.07	(0.06)	-0.06	(0.06)
Serum Albumin - 4.4 if >= 4.4	0.12	(0.05)	0.16	(0.06)	0.16	(0.06)
Log Dialysis Time at Registration (Years)	0.04	(0.00)	0.05	(0.01)	0.05	(0.01)
Log Dialysis Time at Registration x 1{Over 5 years}	0.49	(0.03)	0.44	(0.04)	0.43	(0.04)
Perfect Tissue Type Match x Prior Transplant	-0.44	(0.19)	-0.39	(0.27)	-0.29	(0.27)
Perfect Tissue Type Match x Diabetic Patient	0.03	(0.16)	0.06	(0.23)	0.06	(0.23)
Perfect Tissue Type Match x Patient Age	-0.01	(0.01)	-0.02	(0.01)	-0.02	(0.01)
Perfect Tissue Type Match x CPRA	0.85	(0.35)	1.35	(0.48)	1.53	(0.48)
Perfect Tissue Type Match x 1{CPRA above 80%}	-0.50	(0.30)	-0.30	(0.40)	-0.38	(0.41)
Perfect Tissue Type Match x ECD Donor	-0.63	(0.16)	-0.72	(0.23)	-0.72	(0.23)
Perfect Tissue Type Match x DCD Donor	-0.46	(0.33)	-1.03	(0.47)	-1.05	(0.47)
Perfect Tissue Type Match x NYRT Donor	0.44	(0.18)	-0.02	(0.26)	-0.02	(0.26)
Perfect Tissue Type Match x ABO Compatible	0.02	(0.17)	0.09	(0.24)	0.08	(0.24)
NYRT Donor x 1{2 A Mismatches}	0.16	(0.03)	0.06	(0.04)	0.05	(0.04)
NYRT Donor x 1{2 B Mismatches}	-0.02	(0.03)	-0.05	(0.04)	-0.05	(0.04)
NYRT Donor x 1{2 DR Mismatches}	-0.03	(0.03)	-0.01	(0.04)	-0.01	(0.03)
NYRT Donor x 1{Donor Age < 18}	-0.05	(0.06)	0.18	(0.22)	0.19	(0.25)
NYRT Donor x 1{Donor Age 18-35}	0.13	(0.04)	0.24	(0.15)	0.25	(0.15)

NYRT Donor x 1{Donor Age 50+}	-0.45	(0.03)	-0.69	(0.13)	-0.68	(0.12)
Patient Age x 1{Donor Age < 18}	-0.01	(0.00)	0.00	(0.00)	0.00	(0.00)
Patient Age x 1{Donor Age 18-35}	-0.02	(0.00)	0.00	(0.01)	0.00	(0.01)
Patient Age x 1{Donor Age 50+}	0.02	(0.00)	0.02	(0.01)	0.02	(0.01)
Patient Age - 35 if Age >= 35 x 1{Donor Age 18-35}	0.02	(0.01)	0.00	(0.01)	0.00	(0.01)
Patient Age - 35 if Age >= 35 x 1{Donor Age 50+}	-0.01	(0.01)	0.00	(0.01)	-0.01	(0.01)
Log Waiting Time x Prior Transplant					0.23	(0.02)
Log Waiting Time x Patient Diabetic					-0.03	(0.02)
Log Waiting Time x Patient Age					0.00	(0.00)
Log Waiting Time x CPRA					0.08	(0.05)
Log Waiting Time x 1{CPRA >= 80}					0.00	(0.05)
Log Waiting Time x Patient Serum Albumin					-0.01	(0.01)
Log Waiting Time x Patient BMI at Departure					0.00	(0.00)
Log Waiting Time x 1{Patient Blood Type A}					0.01	(0.03)
Log Waiting Time x 1{Patient Blood Type O}					-0.01	(0.03)
Patient BMI Missing					-1.27	(0.61)
Patient Serum Albumin Missing					-0.05	(0.12)
Donor Unobservable Std. Dev.			1.02	(0.03)	1.04	(0.04)
Idiosyncratic Shock Std. Dev.	1.00		1.00		1.00	
Acceptance Rate	0.140%		0.140%		0.140%	
Number of Offers	2713043		2713043		2713043	
Number of Donors	5642		5642		5642	
Number of Patients	9494		9494		9494	

Table A.IV: Out-of-sample Model Validation

Relative Mean-Squared Prediction Error of CCP Estimator		
	Estimation Sample	Validation Sample
Sparse Specification	87%	88%
Baseline Specification	81%	86%
Richer Specification	77%	89%
Richest Specification	73%	136%

Note: Validation sample includes offers made between January 1, 2014 and June 30, 2014. The relative mean squared error normalizes the MSE relative to a baseline estimator that predicts a constant CCP in each period. The sparse specification reduces the interactions and knots in the piecewise linear terms included in  $\chi(\cdot)$  from our baseline specification so that we estimate about one fourth of the co-efficients. The richer specification increases the number of interactions and knots in the piecewise linear terms by a factor of four from the baseline, and the last specification further increases the number of terms by another factor of three.

## B Counterfactuals

### B.1 Computation Details

#### B.1.1 Counterfactual Scoring Mechanisms

Our algorithm to compute steady-state equilibria for counterfactual scoring mechanisms uses a discrete time grid  $t = t_0, \dots, t_l, t_{l+1}, \dots, T$ , arbitrary initial beliefs  $\pi^0$ , and a sample of patients and donors as inputs. In the baseline results, the type space is given by a random sample of 300 patients and 500 donors drawn from our dataset. We discretize time into quarters for the first 15 years after registration, then every 2 years until year 25, and every 25 years thereafter. These results are not sensitive to a larger set of patient and donor types or finer time partitions. Details are provided in section C.

An equilibrium is computed by iterating through the following steps until convergence:

1. Compute the value function  $V_x^k(t_l)$ , given beliefs  $\pi^{k-1}$ , via backwards induction from  $V_x^k(t_{l+1})$ . This calculation also yields patient strategies  $\sigma_x^k(\Gamma, t) = 1 \left\{ \Gamma \geq V_x^k(t) \right\}$  and departure rates  $\kappa_x^k(t)$ .
2. Compute the queue composition  $m^k$  given departure rates  $\kappa_x^k(t)$ .
3. Compute  $\pi^k(t; x, z)$  using the queue composition and the accept/reject strategies  $\sigma_x^k(\Gamma, t)$ .
4. For step  $k > 1$ : Terminate if the largest change in value functions and queue length/composition between iterations –  $\sup_{x,l} |V_x^k(t_l) - V_x^{k-1}(t_l)|$ ,  $\sup_{x,l} |m_x^k(t_l; x) - m_x^{k-1}(t_l)|$ , and  $N^k - N^{k-1}$  – are uniformly below a tolerance level. Otherwise, repeat steps 1-4.

If this algorithm terminates, the resulting accept/reject rules yield an equilibrium (up to the threshold tolerance). Because the equilibrium we compute may not be unique, we tried different starting values for  $\pi^0$ . Our experiments at the estimated parameters did not find multiple equilibria. Pseudo-code is provided below.

#### Value Function Computation (Backwards Induction):

For a small  $h$ , the value function derived in equation (3) can be approximated as

$$(\rho + \delta_x(t)) V_x(t) \approx \lambda \int \pi_x(t; z) \int \max \{0, \Gamma(t; x, z) + \varepsilon - V_x(t)\} dGdF + \frac{V_x(t+h) - V_x(t)}{h}$$

Because the right-hand side is monotonically decreasing in  $V_x(t)$ , there is a unique value of  $V_x(t)$  that satisfies the equation. We will use this expression to obtain the value function by backwards induction. At iteration  $k$ , given  $V_x^k(t_{l+1})$  we use the bisection method to calculate the value of  $v$  that solves:

$$(\rho + \delta_x(t_l))v = \lambda \int \pi_x^k(t_l; z) \int \max\{0, \Gamma(t_l; x, z) + \varepsilon - v\} dGdF + \frac{V_x^k(t_{l+1}) - v}{t_{l+1} - t_l} \quad (\text{B.2})$$

Because this problem can be written as finding  $v = f(v)$  where  $f(\cdot)$  is strictly decreasing, we can take any initial guess  $v_0$  and set the lower bound to  $\min(v_0, f(v_0))$  and the upper bound to  $\max(v_0, f(v_0))$ . We use the initial guess  $v_0 = V_x^k(t_{l+1})$ .

**Offer Probabilities,  $\pi_{x,z}(t)$ :**

Section B.1.2 derives a computationally tractable approximation to offer probabilities given a scoring rule  $s$ , a large waitlist  $N^*$ , and an acceptance policy function. The expression in equation (B.5) below can be simplified and solved for analytically. We use that solution in our algorithm.

**Waitlist Size/Composition,  $m, N$ :**

We use  $\kappa_x(t)$  and  $\gamma_x$  to update the queue composition. Solving the ODE in Definition 1, part 3(a), we get that for any  $h > 0$ ,

$$m_x(t+h) = m_x(t) \exp\left(-\int_0^h \kappa_x(t+\tau) d\tau\right),$$

where  $m_x(0) = \lambda_x$ . Approximating  $\kappa_x(t+\tau) = \kappa_x(t+h)$  for all  $\tau \in (0, h)$ , we have that

$$m_x(t_{l+1}) = m_x(t_l) \exp(-\kappa_x(t_{l+1})(t_{l+1} - t_l)). \quad (\text{B.3})$$

Finally, we scale the output so that  $m_x(t_l)$  is a probability measure.

The size of the waitlist,  $N$ , is determined by part 3(b) of Definition 1.

### B.1.2 Approximating Offer Probabilities

Fix a particular agent  $i$  with priority score  $s$ . Ties are broken randomly, so wlog consider each agent's tiebreaker to be drawn from a uniform distribution. Let  $1 - \alpha_i$  be the tie-breaker for agent  $i$ .

An offer may be the last one because it may be accepted or because the kidney may expire after the offer. This model, specified in equation (A.1), yields a probability,  $p_0 = \lambda_o(z)$ , the

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**Algorithm 1** Steady State Equilibrium
 

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1: Inputs: Patient and donor characteristics, scoring rule  $s$ , parameters  $\Gamma, \delta, \rho$ , and patient
   age grid  $\{t_0, \dots, t_L = T\}$ . Let  $t_x^0$  be the arrival time for patient of type  $x$ .
2: Outputs:  $V^*, \pi^*, m^*, N^*$ 
3: Initialize  $k = 0$  and beliefs  $\pi_x^k(t)$  for all  $x$  and  $t \in \{t_0, \dots, t_L\}$ 
4: repeat
5:    $V^k \leftarrow$  Backwards Induction( $\pi^k$ )
6:    $\kappa_x^k(t_l) \leftarrow \delta_x(t_l) + \lambda \sum_z \pi_{x,z}^k(t_l) \mathbb{P}(\Gamma(t_l; x, z) + \varepsilon > V_x^k(t_l))$ 
7:    $m^k, N^k \leftarrow$  Forward Simulation( $\kappa^k$ ) ▷ Waitlist Composition
8:    $\pi^k \leftarrow$  Compute Offer Probabilites( $V^k, m^k, N^k$ ) ▷ Offer Probabilities
9:    $k \leftarrow k + 1$ 
10: until  $k > 1, \|V^k - V^{k-1}\|_\infty < \epsilon, \|m^k - m^{k-1}\|_\infty < \epsilon$ , and  $N^k = N^{k-1}$  ▷ Convergence
11:  $V^* \leftarrow V^k, m^* \leftarrow m^k, N^* \leftarrow N^k, \pi^* \leftarrow \pi^k$ 
12: function BACKWARDS INDUCTION( $\pi$ )
13:   for all  $x$  do
14:     Set  $V_x(T) = 0$ 
15:     for all  $x$  and  $t_l = t_{L-1}$  to  $t_x^0$  do
16:       Compute  $V_x(t_l)$  by solving for  $v$  in equation (B.2)
17:     end for
18:   end for
19:   return  $V_x(t_l)$  for all  $x$  and  $t_l \in \{t_x^0, \dots, T\}$ 
20: end function
21: function FORWARD SIMULATION( $\kappa$ )
22:   for all  $x$  do
23:      $m_x(t_x^0) \leftarrow \lambda_x$ 
24:     for all  $t_l = t_{x+1}^0$  to  $T$  do
25:        $m_x(t_{l+1}) \leftarrow m_x(t_l) \exp(-\kappa_x(t_l)(t_{l+1} - t_l))$ 
26:     end for
27:   end for
28:    $N^k \leftarrow \sum_{x,t_l} m_x^k(t_l) \kappa_x^k(t_l)$  ▷ Waitlist Size: Definition 1, part 3(b)
29:    $m_x(t_l) \leftarrow m_x(t_l) / N^k$  for all  $x$  and  $t_l$ 
30:   return  $m_x(t_l)$  for  $t_l \in \{t_x^0, \dots, T\}$  and  $N^k$ 
31: end function
32: function COMPUTE OFFER PROBABILITIES( $m, V, N$ )
33:    $p^a(t_l; x, z) \leftarrow \mathbb{P}(\Gamma(t_l; x, z) + \varepsilon > V_x(t_l))$  for all  $x, t_l$ 
34:   for all  $s = \max s(t_l; x, z)$  to  $\min s(t_l; x, z)$  do
35:     Compute  $\pi$  using equation (B.5)
36:   end for
37:   return  $\pi^k$ 
38: end function

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probability of a timeout before the next offer for an object of type  $z$ . For simplicity, we fix  $z$  and drop it from the notation.

An agent receives an offer if the total number of acceptances and timeouts after offers to agents with a higher priority score than agent  $i$  is strictly less than the number of copies of the object available. Consider waitlists that are composed of  $N$  agents randomly drawn from distribution  $m$ . The probability that each drawn agent is ordered above  $i$  and that the kidney is either accepted by the agent or times out is

$$p(s, \alpha) = m_H(s) p_H(s) + m_E(s) \alpha p_E(s).$$

The first term represents the case when an agent with a higher priority (group  $H$ ) is drawn. The probability of the kidney becoming unavailable conditional on an agent drawn from a higher priority group is:

$$p_H(s) = p_0 + (1 - p_0) \frac{1}{m_H(s)} \sum_{t,x} m(t; x) 1\{s(t; x) > s\} \mathbb{P}(\Gamma(t; x) + \varepsilon > V_x(t)).$$

The second term is the probability that an agent with priority score  $s$  is drawn. The term  $p_E(s)$ , representing the case when an agent in the same priority group is drawn, is defined analogously as  $p_H(s)$ .

Therefore, the number of times a kidney would become unavailable after being offered to an agent ordered above  $i$  is a binomial random variable  $X$  with parameters  $N$  and  $p(s, \alpha)$ . An object is available to agent  $i$  if  $X < q$ , where  $q$  is the number of copies of the object. Hence, the probability that  $i$  receives an offer is given by

$$\int_0^1 \mathbb{P}(X < q | s, \alpha) d\alpha, \tag{B.4}$$

where we have integrated over the tie-breaker  $\alpha$ , and explicit conditioning on  $N$  is subsumed for simplicity.

For large  $N$  and small  $p(s, \alpha)$ , the distribution of  $X$  approaches the distribution of a Poisson random variable with parameter  $Np(s, \alpha)$ . Therefore, the expression in equation (B.4) yields the following expression for  $\pi_x(t)$ :

$$\pi_x(t) = \int_0^1 \sum_{q' < q} \frac{e^{-Np(s, \alpha)} (Np(s, \alpha))^{q'}}{q'!} d\alpha,$$

where we use the Poisson approximation to re-write  $\mathbb{P}(X < q | s, \alpha)$ . As a reminder, the

object type  $z$  is dropped from the notation for simplicity as it is fixed, although the offer probabilities depend on it. This integral can be solved for in closed form for  $q \in \{1, 2\}$ :

$$\pi_x(t) = \frac{e^{-Np(s,0)} - e^{-Np(s,1)}}{N(p(s,1) - p(s,0))} + 1 \{q = 2\} \frac{(1 + Np(s,0))e^{-Np(s,0)} - (1 + Np(s,1))e^{-Np(s,1)}}{N(p(s,1) - p(s,0))}. \quad (\text{B.5})$$

## B.2 Optimal Assignments and Optimal Offer Rates

The objective functions for these two problems are identical. It is given by:

$$\sum \frac{1}{\bar{V}_x^{\mathcal{M}_0}(\lambda_0)} \left[ \frac{\gamma_x}{\rho} V_x(0) + \sum_l N m_x(t_l) (t_{l+1} - t_l) V_x(t_l) \right],$$

where  $\bar{V}_x^{\mathcal{M}_0}(\lambda_0)$  is defined in equation (11) and  $V$  are choice variables interpreted as in the rest of the paper. The constraints on the two problems differ and each has a separate, third choice variable. For the optimal assignment mechanism, we choose assignment policies  $\mu$ . For the optimal offer mechanism, we choose offer rates  $\pi$ . We describe these variables and constraints below. The nonlinear problem is solved using the KNITRO optimizer interfaced with MATLAB.

### B.2.1 Optimal Assignments

This allocation maximizes the objective function above by assigning an object of type  $z$  to agents currently on the list. The social planner knows the payoffs  $\Gamma_{xzt}$  as well as the idiosyncratic shocks  $\varepsilon$ . The planner also knows the steady state distribution of agents waiting for an assignment but not the future arrivals of objects or agents. The choice variable is the probability  $\mu_{xzt}$  with which a compatible object of type  $z$  is allocated to an agent of type  $x$  who has waited for  $t$  periods. Given  $\mu$ , the assignment is made to compatible agents of type  $x$  that have waited for  $t$  periods and have the highest draws of  $\varepsilon$ . Choosing  $\mu$  is equivalent to choosing a cutoff  $\underline{\varepsilon}_{xzt}$  such that  $\mu_{xzt} = \mathbb{P}(a(\varepsilon; x, z, t) = 1) = \int 1\{\varepsilon > \underline{\varepsilon}_{xzt}\} dG$ , where the integral is taken with respect to  $\varepsilon$ .

There are three constraints:

1. Value Function: The agent's net present value  $V_x(\cdot)$  from the expected stream of as-

signments under the policy  $\mu_{xzt}$  is defined by

$$\left(1 + \left(\rho + \delta_x(t_l) + \lambda \sum_z f_z \mu_{xzt_l} c_{xz}\right) (t_{l+1} - t_l)\right) V_x(t_l) = (t_{l+1} - t_l) \lambda w_x(t_l) + V_x(t_{l+1}),$$

where

$$w_x(t) = \sum_z f_z c_{xz} \int (\Gamma_{xzt} + \varepsilon) 1\{\varepsilon > \underline{\varepsilon}_{xzt}\} dG,$$

$f_z$  is the probability that the object type is  $z$ , integrals are over  $\varepsilon$ , and  $c_{xz}$  is the known (estimated) compatibility probability. These expressions for  $V$  and  $w$  are obtained by solving the value function from following the policy of accepting offers with  $\varepsilon$  above  $\underline{\varepsilon}_{xzt}$ , with offers made whenever an object arrives. The term  $w_x(t)$  denotes the expected value to an agent of type  $x$  conditional on an object arriving.

2. Feasibility: The total mass of type  $z$  objects that are assigned upon arrival must not exceed the mass of objects that arrive. Specifically, for each  $z$ , we impose the constraint

$$\sum_{x,l} Nm_x(t_l) (t_{l+1} - t_l) c_{xz} \mu_{xzt_l} \leq q_z.$$

The left hand side is the cumulative product of the (discretized) masses of each type of agent on the waitlist,  $Nm_x(t_l) (t_{l+1} - t_l)$ , multiplied by the assignment probabilities  $c_{xz} \mu_{xzt_l}$  for each agent. This quantity cannot exceed the mass of objects that arrive,  $q_z$ .

3. Steady-State Composition: The measure of agents of type  $x$  that have waited for  $t$  periods is in steady state. This constraint is analogous to equation (B.3) above. Specifically, for each  $x$  and  $l > 0$ , we have that

$$Nm_x(t_{l+1}) = Nm_x(t_l) \exp\left(-\left(\delta_x(t_l) + \lambda \sum_z f_z c_{xz} \mu_{xzt_l}\right) (t_{l+1} - t_l)\right),$$

$$Nm_x(t_0) = \gamma_x.$$

The term  $\lambda \sum_z f_z c_{xz} \mu_{xzt_l}$  is the cumulative assignment rate across objects for an agent of type  $x$  at time  $t_l$ . This, when added to  $\delta_x(t_{l+1})$ , yields the total departure rate.

In addition, we impose that each  $\mu_{xzt}$  belongs to unit interval.

### B.2.2 Optimal Offer Rates

This problem maximizes the objective function above by choosing a probability of offering an object of type  $z$  to agents currently on the list. The social planner has full information about the payoffs  $\Gamma_{xzt}$ , but does not know the idiosyncratic shocks  $\varepsilon$ . She knows the steady-state distribution of agents waiting for an assignment but not the future arrivals of objects or agents. The choice variable in this problem is the probability  $\pi_{xzt}$  with which an arriving object of type  $z$  is offered to an agent of type  $x$  who has waited for  $t$  periods. Agents optimally choose which offers to accept given  $\pi$ .

As before, there are three constraints:

1. Value Function: The agent's net present value  $V_x(\cdot)$  from the expected stream of assignments under the policy  $\pi_{xzt}$  is defined by

$$(1 + (\rho + \delta_x(t_l))(t_{l+1} - t_l)) V_x(t_l) = (t_{l+1} - t_l) \lambda w_x(t_l) + V_x(t_{l+1}),$$

where

$$w_x(t) = \sum_z f_z \pi_{xzt} c_{xz} \int \max\{0, \Gamma_{xzt} + \varepsilon - V_x(t)\} dG,$$

$f_z$  is the probability that the object type is  $z$ , and integrals are taken with respect to  $\varepsilon$ . As in the optimal assignment problem,  $w_x(t)$  is the expected value to an agent of type  $x$  conditional on an object arriving. However, in this problem, the agent makes optimal decisions and offers do not depend on the payoff shocks. Therefore, an assignment occurs only if the agent is offered the object and the agent accepts. Acceptance occurs if the payoff shock exceeds  $V_x(t) - \Gamma_{xzt}$ .

2. Feasibility: The total mass of type  $z$  objects assigned must not exceed the mass of objects that arrive. Specifically, for each  $z$ , we impose the constraint

$$\sum_{x,l} \tilde{m}_x(t_l) (t_{l+1} - t_l) \pi_{xzt_l} \left[ c_{xz} \int 1 \{\Gamma_{xzt_l} + \varepsilon > V_x(t_l)\} dG + p_{0,z} \right] \leq q_z,$$

where the integral is over  $\varepsilon$ . This constraint is analogous to the feasibility constraint in the optimal assignment problem. The difference is that the assignment rate  $c_{xz}\mu_{xzt}$  is replaced by the term

$$\pi_{xzt_l} \left[ c_{xz} \int 1 \{\Gamma_{xzt_l} + \varepsilon > V_x(t_l)\} dG + p_{0,z} \right].$$

The term  $\pi_{zxt_l}$  denotes the probability that an agent of type  $x$  receives an offer for an object of type  $z$  after she has waited for  $t_l$  periods. The term in brackets is the probability that any such offer is the last offer for the object that can be made. It is the sum of the probability that object is compatible and transplanted,

$$c_{xz} \int 1 \{ \Gamma_{xzt_l} + \varepsilon > V_x(t_l) \} dG,$$

and the probability that no more offers can be made after the current one. This term arises from the technological constraint on the number of offers that can be made for an object. The model used to determine  $p_{0,z}$  is described in Appendix A.

This constraint only restricts the expected number of assignments. Therefore, the offer rates  $\pi_{xzt}$  may not be implementable for a specific sequence of donor and patient arrivals.

3. Steady-State Composition: The measure of agents of type  $x$  that have waited for  $t$  periods is in steady state. Specifically, for each  $x$  and  $l > 0$ , we have

$$\begin{aligned} \tilde{m}_x(t_{l+1}) &= \tilde{m}_x(t_l) \exp \left( - \left( \delta_x(t_{l+1}) + \lambda \sum_z f_z \mu_{xzt_l} \right) (t_{l+1} - t_l) \right), \\ \tilde{m}_x(t_l) &= \gamma_x, \end{aligned}$$

where

$$\mu_{xzt} = \pi_{zxt} c_{xz} \int 1 \{ \Gamma_{xzt} + \varepsilon > V_x(t) \} dG.$$

This constraint differs from its analogue in the optimal assignment problem because here the assignment probability  $\mu_{xzt}$  depends on agents' acceptance decision.

In addition, we impose that each  $\pi_{xzt}$  belongs to unit interval.

## C Robustness and Supplementary Evidence

### C.1 Robustness

Table C.V: Outcomes under Alternative Modeling Assumptions

	EV <sub>x</sub>	Waitlist			Transplanted Donors			EV <sub>x</sub> Decomp.		ΔV <sub>x</sub> (0) > -5%
		Queue Length	Reduction in Discard Rate	Years on Waitlist	Age	Head Trauma	Hyper-tensive	Obs.	Unobs.	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: Patient Unobserved Heterogeneity										
Pre-2014 Priorities	--	5166.5	--	2.80	44.2	15.6%	46.1%	--	--	--
Post-2014 Priorities	-1.1%	5108.3	0.3%	2.78	44.2	15.5%	46.1%	-0.3%	-0.8%	94.7%
First Come First Served	1.7%	5317.5	-1.8%	2.84	44.3	15.8%	46.1%	1.0%	0.7%	91.0%
Last Come First Served	-50.9%	3399.7	20.9%	4.28	46.9	14.4%	51.1%	-45.2%	-5.7%	14.0%
Panel B: Annual Discount Factor of 10 Percent										
Pre-2014 Priorities	--	5150.4	--	2.76	44.8	16.4%	47.0%	--	--	--
Post-2014 Priorities	-1.3%	5084.4	0.4%	2.74	44.8	16.2%	47.0%	0.0%	-1.3%	96.7%
First Come First Served	2.6%	5304.5	-2.0%	2.81	44.9	16.5%	47.0%	1.2%	1.4%	90.3%
Last Come First Served	-59.1%	3465.1	18.6%	4.13	46.9	14.9%	50.8%	-53.4%	-5.7%	9.7%
Panel C: Larger Patient and Donor Type Space										
Pre-2014 Priorities	--	4440.0	--	2.58	44.9	23.2%	43.0%	--	--	--
Post-2014 Priorities	-0.9%	4365.4	0.6%	2.54	44.9	23.2%	43.0%	0.1%	-1.0%	98.8%
First Come First Served	1.5%	4567.0	-1.7%	2.61	44.7	23.4%	42.8%	0.7%	0.8%	93.4%
Last Come First Served	-43.0%	2385.8	22.6%	2.69	47.0	20.8%	48.2%	-35.2%	-7.8%	5.0%
Panel D: No Limit on Maximum Number of Offers										
Pre-2014 Priorities	--	4597.0	--	2.55	45.0	15.8%	47.4%	--	--	--
Post-2014 Priorities	-0.9%	4553.0	0.4%	2.54	45.0	15.8%	47.3%	0.1%	-1.0%	98.3%
First Come First Served	2.8%	4607.5	-0.2%	2.56	44.7	15.9%	47.1%	1.3%	1.5%	94.3%
Last Come First Served	-28.2%	2876.8	20.8%	2.65	46.1	14.7%	49.5%	-21.1%	-7.1%	45.0%

**Patient Unobserved Heterogeneity:** We re-estimated the model allowing for two unobserved patient types. Specifically, we re-parametrized the CCPs as follows:

$$P_{ijt} = G(\alpha_i + \chi(x_i, z_j, t)\theta + \eta_j),$$

where  $\alpha_i \in \{\alpha_1, \alpha_2\}$  with the parameters  $\alpha_1$  and  $\alpha_2$  and the share of  $\alpha_1$  to be estimated. This parameterization allows patients to have systematically higher or lower values of all transplants relative to their outside options. We abstract away from the initial conditions problem, setting the proportion of each patient type in our sample to the population average.

This latter assumption is appropriate for patients that registered during our sample period, but ignores selection that should arise for patients that registered prior to the sample of offers we consider.

Estimating this model requires another data augmentation step. This step draws each agent  $i$ 's type given their observed decisions and the parameters  $\alpha_1$ ,  $\alpha_2$ , and  $\pi_1$ . Conditional on  $(\alpha_1, \alpha_2, \pi_1)$ , the posterior probability that  $\alpha_i = \alpha_1$  is proportional to the likelihood of observing the decisions made by agent  $i$  multiplied by  $\pi_1$ . This likelihood is the product of the cumulative density functions of normal distributions. The parameters  $\alpha_1$ ,  $\alpha_2$ , and  $\pi_1$  are then updated using conjugate priors. We specify diffuse normal priors for  $\alpha_1$  and  $\alpha_2$  and a Dirichlet prior for  $\pi_1$  (see Section 3.4, [Gelman et al., 2014](#)). As recommended in [Gelman et al. \(2014\)](#), we check for re-ordering and impose the restriction that  $\alpha_1 > \alpha_2$ .

Table [C.V](#) Panel A presents the results for the steady states of benchmark mechanisms considered in the main text.

**Discount Factor:** As discussed in Section 3, the discount fraction  $\rho$  is not identified and is set to 5 percent per year. Here, we evaluate sensitivity of our results to using an annual discount rate of 10 percent. Only Steps 3 and 4 in Section 4.2 must be revised to obtain estimates with an alternative discount rate. Panel B of Table [C.V](#) presents the counterfactual results.

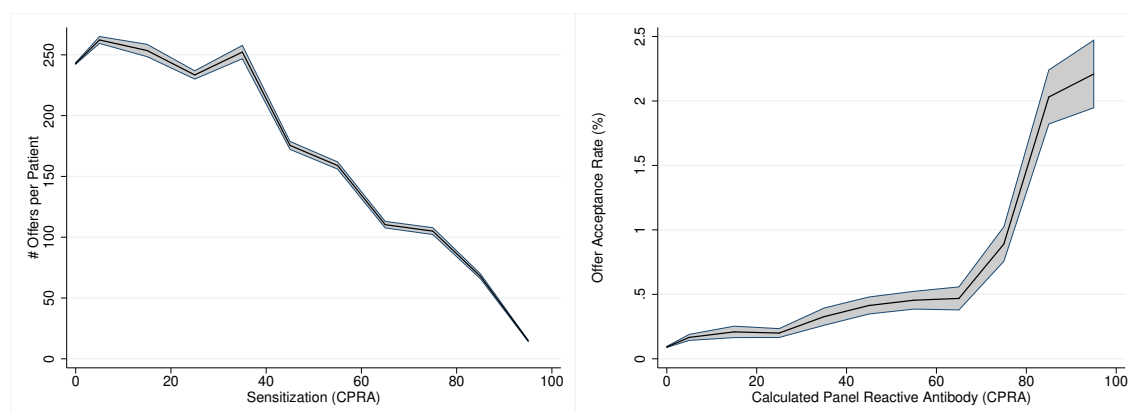
**Larger Samples:** The main text limits the number of types used in counterfactual calculations to 300 patient types and 500 donor types. To assess whether the results are sensitive to the specific sample and number of types, we re-calculated the counterfactuals involving scoring mechanism by drawing 1,000 patient types and 1,500 donor types. Panel C of Table [C.V](#) presents the results.

**Unlimited Offers:** Our results could be sensitive to the limit on the number of offers, especially if improvements in technology that allow the OPO to make many more offers obviates the need for finding better mechanisms. Panel D of Table [C.V](#) presents results calculated when this limit is removed.

## C.2 Supplementary Evidence

Results analogous to Figure 1 and Table 3 in Agarwal et al. (2018) are presented in Figures [C.1](#) and Table [C.VI](#) respectively.

Figure C.1: Offer and Acceptance Rate by CPRA



Note: Sample includes all offers made to NYRT patients between 2010 and 2013, including offers that did not meet pre-set donor screening criteria. Positive crossmatches are counted as acceptances. In each figure, the black line plots the mean among offers to patients in each CPRA bin, and the shaded region represents pointwise 95 percent confidence intervals.

Table C.VI: Evidence of Response to Dynamic Incentives

	Dependent Variable: Offer Accepted				
	(1)	(2)	(3)	(4)	(5)
Calculated Panel Reactive Antibodies (CPRA)	0.0148 (0.000764)	0.00869 (0.000889)	0.00822 (0.000879)	0.00757 (0.000831)	0.00770 (0.000833)
Variables Affecting Priority		X	X	X	X
Patient Characteristics			X	X	X
Donor and Match Characteristics				X	X
Interaction between CPRA and # HLA Mismatches					X
Mean Acceptance Rate	0.142%	0.142%	0.142%	0.142%	0.142%
Observations	2713043	2713043	2713043	2713043	2713043
R-squared	0.003	0.006	0.009	0.104	0.104

Notes: Estimates from a linear probability model of offer acceptance on patient Calculated Panel Reactive Antibodies (CPRA). The sample is offers made to NYRT patients between 2010 and 2013, including offers that did not meet pre-set screening criteria. CPRA is measured on a [0,1] scale at the time of the offer. Column 1 controls for a CPRA=0 indicator. Column 2 adds controls affecting patient priority: indicators for CPRA $\geq$ 0.2, CPRA $\geq$ 0.8, and age $<$ 18, as well as waiting time indicators and linear controls for 1-3, 3-5, and  $>$ 5 years. Column 3 adds other patient characteristics. Column 4 adds controls for donor and match characteristics. Column 5 adds interactions between CPRA indicators and # HLA mismatches. Patient characteristics are indicators for age 18-35, 35-50, and 50-65; indicators for diabetes, blood type, and the patient's transplant center; linear controls and indicators for dialysis time 1-3, 3-5, 5-10, and  $>$ 10 years; and an indicator for health status at listing. Donor controls are linear age; linear creatinine with indicators for 0.6-1.8 and  $>$ 1.8; the donor's mean acceptance rate; and indicators for diabetes, donation after cardiac death (DCD), and expanded criteria donor (ECD). Match characteristics are linear # HLA mismatches; indicators for zero HLA mismatch, 0 and 1 DR mismatch, identical blood type, offer year, and local donor; linear controls for (+) and (-) age difference; and interactions between local and zero-HLA mismatch, and local and donor age, donor over 40 and pediatric patient, donor over 55 and patient age 18-35, and donor over 60 and patient age 35-50 and over 50. Standard errors, clustered by donor, are in parentheses.



## D Additional Theoretical Results

### D.1 Existence of Steady-State Equilibria

This section proves that a steady-state equilibrium exists for sequential offer mechanisms that use a scoring rule. We make the following assumptions:

**Assumption 1.** (i) *The exogenous arrival rates  $\lambda$  and  $\gamma_x$  are finite*

(ii) *The exogenous departure rate  $\delta(\tau; x)$  is bounded below by  $\underline{\delta} > 0$  and bounded above by  $\bar{\delta}$ , uniformly for  $t \in [0, T)$  and all  $x \in \chi$*

(iii) *The conditional probability density function  $f_{\Gamma|t,x,z}$  exists, and is uniformly bounded*

(iv) *The conditional moment,  $\mathbb{E}[|\Gamma| | \tau, x, z] = \int |\Gamma| dF_{\Gamma|t,x,z}$  where  $\Gamma = \Gamma(x, z, \tau) + \varepsilon$ , is uniformly bounded in  $t, x, z$*

(v) *The family of functions  $g(t; x, z, \bar{\Gamma}) = F_{\Gamma|t,x,z}(\bar{\Gamma})$  indexed by  $\bar{\Gamma}, x, z$  is Lipschitz continuous in  $t$  with a common constant*

(vi) *The object arrival rate  $\lambda$  is strictly less than the total agent arrival rate  $\sum \gamma_x$*

(vii) *The set of scores  $S = \{s(t; x, z) : (t, x, z) \in [0, T] \times \chi \times \zeta\}$  is finite.*

Most empirical models will satisfy the continuity and boundedness assumptions above. The two substantive assumptions are parts (vi) and (vii). Part (vi) assumes that the objects that need to be assigned are scarce in order to ensure that the queue is unlikely to be empty. Part (vii) restricts the mechanisms for which we prove existence. The assumption is used to ensure that the set of all functions  $\pi_{xz}(t)$  is sufficiently small (more precisely, compact). Other assumptions that yield this conclusion would also suffice.

Our main result proves existence of a steady-state equilibrium.

**Theorem 1.** *Suppose Assumption 1 is satisfied. Then a steady-state equilibrium for a sequential offer mechanism with a scoring rule exists.*

*Proof.* The proof proceeds by applying the Brower-Schauder-Tychonoff Fixed Point Theorem (Corollary 17.56, [Aliprantis and Border, 2006](#)). The proof proceeds in three parts.

**Part 1, Definition of  $\Omega$ :** The equilibrium objects are defined by five types of functions:

1. The conditional choice probabilities, given  $t$  and the agent and object characteristics  $x$  and  $z$ . We consider these choice probabilities as a function  $p_\sigma : [0, T] \times \chi \times \zeta \rightarrow [0, 1]$ .

2. The value function  $V : \chi \times [0, T] \rightarrow \mathbb{R}_+$ . It is convenient to define this function, although it is somewhat redundant with the choice probabilities above.
3. The offer probabilities  $\pi : [0, T] \times \chi \times \zeta \rightarrow [0, 1]$  where  $\pi(t; x, z) = H_z(s_{xz}(t)) \times \mathbb{P}(c_{ij} = 1|x, z)$ .
4. The distribution of agent types  $m : \chi \times [0, T] \rightarrow \mathbb{R}_+$ .
5. The queue length  $N \in \mathbb{R}$ .

We denote the tuple of these objects by  $\omega = (p_\sigma, V, \pi, m, N)$ . We endow each of the functions in the first four objects with the supremum norm over its domain. The norm for  $\omega$  is denoted  $\|\omega\| = \|p_\sigma\| + \|V\| + \|\pi\| + \|m\| + |N|$ . Therefore,  $\omega$  is an element of a Banach space.

We further restrict  $\omega$  to belong to a subset  $\Omega$  of this Banach space. Specifically, we restrict its components as follows:

1. The functions  $V_x(t)$  are uniformly bounded by  $\lambda T \sup_{\tau, x, z} \int |\Gamma| dF_{\Gamma|\tau, x, z}$  and are Lipschitz continuous with a common constant  $(1 + \rho + \bar{\delta}) \lambda \sup_{\tau, x, z} \int |\Gamma| dF_{\Gamma|\tau, x, z}$ .

Note that the optimal value of  $V_x(t)$  satisfied this property. To see this, observe that  $\frac{d}{dt} V_x(t) = -\lambda \exp(-\rho(\tau - t)) p(\tau|t, x) L(t) + \lambda \int_t^T (-\rho - \delta_x(t)) L(\tau) d\tau$  where  $L(\tau) = \int \pi_{ij}(\tau) \int \max\{0, \Gamma_{ij}(\tau) - V_i(\tau)\} dG dF$ . The result follows since  $L(\tau)$  is bounded by  $\sup_{\tau, x, z} \int |\Gamma| dF_{\Gamma|\tau, x, z}$ .

2. The functions  $p_\sigma(t; x, z)$  are uniformly bounded by 1 and Lipschitz continuous with a common constant  $K$ , where

$$K = (1 + \rho + \bar{\delta}) \lambda \sup_{\tau, x, z} \left( \int |\Gamma| dF_{\Gamma|\tau, x, z} \sup_{\Gamma} f_{\Gamma|\tau, x, z}(\Gamma) \right) + \sup_{\bar{\Gamma}, x, z, t, t'} \left| F_{\Gamma|t, x, z}(\bar{\Gamma}) - F_{\Gamma|t', x, z}(\bar{\Gamma}) \right| / |t - t'|.$$

Note that Assumption 1 implies that  $K$  is finite. That the equilibrium value satisfies this assumption can be seen from part 1.

3. The functions  $\pi_{x,z}(t)$  such that  $\pi_{x,z}(t) = \pi_{x,z}(t')$  if  $s_{xz}(t) = s_{xz}(t')$  with range  $[0, 1]$ .
4. The term  $N \in [\underline{N}, \bar{N}]$ , where  $\underline{N} = (\sum_x \gamma_x - \lambda) / \bar{\delta}$  and  $\bar{N} = \frac{\sum_x \gamma_x}{\bar{\delta}}$ . These bounds are obtained by considering the extremal cases in which no agent is assigned and when every kidney is assigned. Note that  $\underline{N} > 0$  because Assumption 1 requires that  $\sum_x \gamma_x > \lambda$  and  $\delta(\tau; x)$  is uniformly bounded above.

5. The functions  $m_x(t)$  are uniformly bounded by  $\frac{\sup_x \gamma_x}{N}$  and are Lipschitz continuous with a common constant  $\frac{\sup_x \gamma_x}{N}(\bar{\delta} + \lambda)$ . The steady-state value satisfies this requirement since  $T\gamma_x$  is the maximum mass of agents of type  $x$ , and because

$$|\dot{m}_x(t)| = m_x(t)\kappa_x(t) \leq m_x(t)(\bar{\delta} + \lambda).$$

**Part 2, definition of  $A : \Omega \rightarrow \Omega$ :** Denote  $A_V[\omega]$  as the  $V$  component of  $A[\omega]$ , where  $\omega \in \Omega$ . Likewise, define  $A_\pi$ ,  $A_{p_\sigma}$ ,  $A_m$  and  $A_N$ . This map is defined as follows:

$$\begin{aligned} A_V[\omega](x, t) &= \int_t^T \exp(-\rho(\tau - t)) p(\tau|t; x) \left( \lambda \int \pi(\tau; x, Z) \int \max\{0, \Gamma - V(\tau; x)\} dF_{\Gamma|\tau, x, Z} dF_Z \right) d\tau \\ A_{p_\sigma}[\omega](x, z, t) &= \int 1\{\Gamma \geq A_V[\omega](x, t)\} dF_{\Gamma|x, z, t} \\ A_m[\omega](x, t) &= \gamma_x \exp\left(-\int_0^t \delta(\tau; x) + \lambda \int \pi(\tau; x, Z) p_\sigma(\tau; x, z) dF_Z d\tau\right) / N \\ A_N[\omega] &= \max\left\{N, \min\left\{\frac{\sum_x \gamma_x}{\sum_x \int_0^T m_x(t) \kappa_x(t) dt}, \bar{N}\right\}\right\} \\ A_\pi[\omega](x, z, t) &= H_z(s_{xz}(t); A_{p_\sigma}[\omega], A_m[\omega], A_N[\omega]) \times \mathbb{P}(c_{ij} = 1|x, z), \end{aligned}$$

where

$$p(\tau|t; x) = \exp\left(-\int_t^\tau \delta(\tau'; x) d\tau'\right)$$

is the probability that an agent of type  $x$  departs the list prior to  $\tau$  conditional on being on the list at  $t$ . To ensure that the image is a subset of  $\Omega$ , we need to show that  $A[\omega] \in \Omega$  for all  $\omega \in \Omega$ . We do this for each of the components separately:

1.  $A_V$ : Since  $\exp(-\rho(\tau - t))$ ,  $p(\tau|t; x)$  and  $\pi(\tau; x, Z)$  are in  $[0, 1]$ , and

$$\int \max\{0, \Gamma - V(\tau; x)\} dF_{\Gamma|\tau, x, Z} \leq \int |\Gamma| dF_{\Gamma|\tau, x, Z},$$

we have that  $A_V[\omega]$  is uniformly bounded by  $\lambda T \sup_{\tau, x, z} \int |\Gamma| dF_{\Gamma|\tau, x, z}$ . Further, for any  $t, t' \in [0, T]$ , with  $t < t'$ , we have that

$$\begin{aligned} &|A_V[\omega](t) - A_V[\omega](t')| \\ &= \left| \int_t^{t'} \exp(-\rho(\tau - t)) p(\tau|t; x) \left( \lambda \int \pi(\tau; x, Z) \int \max\{0, \Gamma - V(\tau; x)\} dF_{\Gamma|\tau, x, Z} dF_Z \right) d\tau \right| \\ &\leq \lambda |t' - t| (1 + \rho + \bar{\delta}) \sup_{\tau, x, z} \int |\Gamma| dF_{\Gamma|\tau, x, z}. \end{aligned}$$

Therefore,  $A_V[\omega]$  satisfies the necessary restrictions.

2.  $A_{p_\sigma}$  : Note that  $A_{p_\sigma}[\omega]$  is uniformly bounded by 1. Moreover, for any  $x$  and  $z$ , and  $t, t' \in [0, T]$ , we have that

$$\begin{aligned}
& |A_{p_\sigma}[\omega](t, x, z) - A_{p_\sigma}[\omega](t', x, z)| \\
&= \left| \int 1\{\Gamma \geq A_V[\omega](x, t)\} dF_{\Gamma|x,z,t} - \int 1\{\Gamma \geq A_V[\omega](x, t')\} dF_{\Gamma|x,z,t'} \right| \\
&= \left| \int (1\{\Gamma \geq A_V[\omega](x, t)\} - 1\{\Gamma \geq A_V[\omega](x, t')\}) dF_{\Gamma|x,z,t} \right| \\
&\quad + \left| \int 1\{\Gamma \geq A_V[\omega](x, t')\} d(F_{\Gamma|x,z,t} - F_{\Gamma|x,z,t'}) \right| \\
&\leq \left| \int_{\min\{A_V[\omega](x,t), A_V[\omega](x,t')\}}^{\max\{A_V[\omega](x,t), A_V[\omega](x,t')\}} 1 dF_{\Gamma|x,z,t} \right| \\
&\quad + \left| F_{\Gamma|x,z,t'}(A_V[\omega](x, t')) - F_{\Gamma|x,z,t}(A_V[\omega](x, t')) \right| \\
&\leq \lambda(1 + \rho + \bar{\delta}) |t' - t| \sup_{\tau, x, z} \left( \int |\Gamma| dF_{\Gamma|\tau, x, z} \sup_{\Gamma} f_{\Gamma|\tau, x, z}(\Gamma) \right) + \sup_{\bar{\Gamma}, x, z} \left( |F_{\Gamma|t, x, z}(\bar{\Gamma}) - F_{\Gamma|t', x, z}(\bar{\Gamma})| / |t - t'| \right) \\
&\leq \left[ \lambda(1 + \rho + \bar{\delta}) \sup_{\tau, x, z} \left( \int |\Gamma| dF_{\Gamma|\tau, x, z} \sup_{\Gamma} f_{\Gamma|\tau, x, z}(\Gamma) \right) + \sup_{\bar{\Gamma}, x, z, t, t'} \left( |F_{\Gamma|t, x, z}(\bar{\Gamma}) - F_{\Gamma|t', x, z}(\bar{\Gamma})| / |t - t'| \right) \right] |t - t'|
\end{aligned}$$

Therefore,  $A_{p_\sigma}[\omega]$  satisfies the necessary restrictions.

3.  $A_\pi$  : Observe that  $A_\pi[\omega](x, z, t) \in [0, 1]$  and  $A_\pi[\omega](x, z, t) = A_\pi[\omega](x, z, t')$  if  $s_{xz}(t) = s_{xz}(t')$  by construction.
4.  $A_m$  : Since  $\exp\left(-\int_0^t \delta(\tau; x) + \lambda \int \pi(\tau; x, Z) p_\sigma(\tau; x, z) dF_{\Gamma|\tau, x, Z} d\tau\right) \leq 1$  and  $\underline{N} > 0$ , we have that  $A_m[\omega]$  is uniformly bounded by  $\frac{\sup_x \gamma_x}{\underline{N}}$ . Further, the derivative at  $t$  of  $A_m[\omega](t)$  is equal to

$$\left( -\delta(t; x) - \lambda \int \pi(t; x, Z) p_\sigma(t; x, z) dF_Z \right) A_m[\omega](t).$$

This derivative is bounded in absolute value by  $(\bar{\delta} + \lambda) \frac{\sup_x \gamma_x}{\underline{N}}$ .

5.  $A_N$ : By construction,  $A_N[\omega]$  belongs to  $[\underline{N}, \bar{N}]$ , satisfying the necessary restrictions.

**Part 3, existence of equilibria:** It is straightforward to verify that  $\Omega$  is convex. Lemma 1 implies that the components  $\Omega_V$ ,  $\Omega_m$ , and  $\Omega_{p_\sigma}$  are compact sets. Lemma 2 shows that  $\Omega_\pi$  is compact. Assumption 1 (i), (ii), and (vi) imply that  $\underline{N} > 0$  and  $\bar{N}$  is finite, implying that  $\Omega_N$  is compact. Therefore,  $\Omega$  is compact. Lemma 3 shows that  $A$  is a continuous map. Therefore,

the Brouwer-Schauder-Tychonoff Theorem (Corollary 17.56, [Aliprantis and Border, 2006](#)) implies that there exists  $\omega^* \in \Omega$  such that  $A[\omega^*] = \omega^*$ .

To complete the proof, we show that any fixed point  $\omega^* = (p_\sigma^*, V^*, \pi^*, m^*, N^*)$  corresponds to a steady state equilibrium. Observe that for each  $x$ ,

$$V^*(t; x) = \int_t^T \exp(-\rho(\tau - t)) p(\tau|t; x) \left( \lambda \int \pi^*(\tau; x, Z) \int \max\{0, \Gamma - V^*(\tau; x)\} dF_{\Gamma|\tau, x, Z} dF_Z \right) d\tau.$$

Therefore,  $V^*(t; x)$  is the value of declining an offer and following the optimal strategy given the offer rate  $\pi^*$ . Therefore,

$$p_\sigma^*(x, z, t) = A_{p_\sigma}[\omega^*](x, z, t) = \int 1\{\Gamma \geq V^*(t; x)\} dF_{\Gamma|x, z, t}.$$

For each  $(x, z, t)$ ,  $F_{\Gamma|x, z, t}^{-1}(p_\sigma^*(x, z, t)) = V^*(t; x)$ . Therefore,  $\sigma^*(\Gamma, t) = 1\{\Gamma \geq F_{\Gamma|x, z, t}^{-1}(p_\sigma^*(x, z, t))\}$  is an optimal strategy, satisfying requirement 1 in Definition 1.

By construction,  $\pi^*(x, z, t) = A_\pi[\omega^*](x, z, t) = H_z(s_{xz}(t); p_\sigma^*, m^*, N^*) \times \mathbb{P}(c_{ij} = 1|x, z)$  satisfies requirement 2 of Definition 1 because  $p_\sigma^*$  equals the acceptance probability of a type  $z$  object by an agent of type  $x$  at time  $t$ .

Finally,  $m^* = A_m[\omega^*]$  and  $N^* = A_N[\omega^*]$  together satisfy requirement 3 in Definition 1. The restriction of  $A_N[\omega^*]$  to  $[\underline{N}, \bar{N}]$  cannot strictly bind because  $\underline{N}$  and  $\bar{N}$  denote the smallest and largest possible queue lengths given the exogenous arrival and departure rates.

□

## D.2 Lemmata

**Lemma 1.** *Suppose  $X \subset C([a, b])$  is the set of all functions on the bounded interval  $[a, b]$  that are uniformly bounded by  $K_1$  and have a common Lipschitz constant  $K_2$ . Then  $X$  is compact.*

*Proof.* Note that the set of functions  $X$  is uniformly equicontinuous. By the Arzela-Ascoli theorem, any sequence of functions  $x_n \in X$  has a uniformly convergent subsequence  $x_{n_k}$ . Denote the limit of this sequence by  $x^*$ , i.e. for each  $t$ ,  $x^*(t) = \lim_{k \rightarrow \infty} x_{n_k}(t)$ . Therefore,  $\sup_t |x^*(t)| \leq \lim_{k \rightarrow \infty} \sup_t |x_{n_k}(t)| \leq K_1$ . Similarly,  $|x^*(t) - x^*(t')| = \lim_{k \rightarrow \infty} |x_{n_k}(t) - x_{n_k}(t')| \leq K_2 |t - t'|$ . Hence,  $x^* \in X$ . Consequently, we have that  $X$  is sequentially compact, which is equivalent to  $X$  being compact. □

**Lemma 2.** *Assumption 1(vii) implies that the set  $\Omega_\pi$  consisting of functions  $\pi : [0, T] \times \chi \times \zeta \rightarrow [0, 1]$  endowed with the supremum norm such that  $\pi_{xz}(t) = \pi_{xz}(t')$  if  $s_{xz}(t) = s_{xz}(t')$  is compact.*

*Proof.* Assumption 1(vii) and finiteness of  $\chi$  and  $\zeta$  imply that the set of scores  $s_{xz}(t)$  over all  $\chi, \zeta$ , and  $t \in [0, T]$  is finite. Therefore,  $\pi$  is an element of a finite dimensional Euclidean space. Further,  $\Omega_\pi$  is closed and bounded by definition. By the Heine-Borel theorem,  $\Omega_\pi$  is compact.  $\square$

**Lemma 3.** *Suppose Assumption 1 is satisfied. Then the map  $A : \Omega \rightarrow \Omega$  is continuous.*

*Proof.* We do this for each component of  $A$  separately.

$A_V$ : Let  $\Omega_0$  be an arbitrary subset of  $\Omega$ . Consider  $\omega \in \bar{\Omega}_0$ , where  $\bar{\Omega}_0$  is the closure of  $\Omega_0$ . Since  $\omega \in \bar{\Omega}_0$ , there exists a sequence  $\omega_n \in \Omega_0$  such that  $\|\omega_n - \omega\| = \varepsilon_n \rightarrow 0$ . Denote  $\tilde{V}_n = A_V[\omega_n]$  and drop  $x$  from the notation as it belongs to a finite set. Now, consider

$$\begin{aligned}
& \left| \tilde{V}_n(t) - \tilde{V}(t) \right| \\
&= \left| \int_t^T \exp(-\rho(\tau - t)) p(\tau|t) \lambda \left( \int \pi_n(\tau; Z) \int \max\{0, \Gamma - V_n(\tau)\} dF_{\Gamma|\tau, Z} dF_Z \right) d\tau \right. \\
&\quad \left. - \int_t^T \exp(-\rho(\tau - t)) p(\tau|t) \lambda \left( \int \pi(\tau; Z) \int \max\{0, \Gamma - V(\tau)\} dF_{\Gamma|\tau, Z} dF_Z \right) d\tau \right| \\
&\leq T\lambda \sup_{t, z} \left| \pi_n(t; z) \int \max\{0, \Gamma - V_n(t)\} dF_{\Gamma|t, z} - \pi(t; z) \int \max\{0, \Gamma - V(t)\} dF_{\Gamma|t, z} \right| \\
&\leq T\lambda \sup_{t, z} \left| \pi_n(t; z) \int |\max\{0, \Gamma - V_n(t)\} - \max\{0, \Gamma - V(t)\}| dF_{\Gamma|t, z} \right| \\
&\quad + T\lambda \sup_{t, z} \left| \pi_n(t; z) - \pi(t; z) \right| \int \max\{0, \Gamma - V(t)\} dF_{\Gamma|t, z} \\
&\leq T\lambda \sup_{t, z} |V_n(t) - V(t)| + T\lambda \sup_{t, z} \int |\Gamma| dF_{\Gamma|t, z} \sup_{t, z} |\pi_n(t; z) - \pi(t; z)| \\
&\leq T\lambda \left( 1 + \sup_{t, z} \int |\Gamma| dF_{\Gamma|t, z} \right) \varepsilon_n.
\end{aligned}$$

Since  $\varepsilon_n \rightarrow 0$ , Assumption 1(i) and (iv) imply that the right hand side converges to zero.

$A_{p_\sigma}$ : Continuity follows by noting that  $A_V$  is continuous in the sup-norm and  $F_{\Gamma|t, x, z}$  is absolutely continuous with respect to Lebesgue measure for each  $t, x, z$  (Assumption 1(iii)).

$A_m$ : It is sufficient to fix  $x$  because  $\chi$  is a finite set. Lemma 4 implies that the map defined by  $A_\kappa[\omega](t) = \delta(t; x) + \lambda \int \pi(t; x, Z) p_\sigma(t; x) dF_Z$  is continuous. Moreover,  $\sup_t A_\kappa[\omega](t)$  is

bounded above (Assumption 1(i)). Therefore,  $A_{\kappa^*}[\omega](t) = -\int_0^t \delta(\tau; x) + \lambda \int \pi(\tau; x, Z) p_\sigma(t; x) dF_Z d\tau$  defines a continuous map from  $\Omega$  to  $C([0, T])$ . Since a composition of continuous functions is continuous, and  $g(a) = \gamma_x \exp(a)/N$  is continuous for all  $N > 0$ ,  $A_m$  is continuous.

$A_N$ : First we show that  $A_N[\omega_n]$  is continuous. Lemma 4 implies that the map  $A_\kappa[\omega](t) = \delta(t; x) + \lambda \int \pi(t; x, Z) p_{\sigma_n}(t; x, Z) dF_Z$  is continuous for each  $x$ . A similar argument implies that  $A_{\bar{\kappa}}[\omega] = \sum_x \int_0^T m_x(t) \kappa_x(t) dt$  is continuous because  $m_x(t)$  is bounded by  $\gamma_x$ . Further,  $A_{\bar{\kappa}}[\omega] \in [\underline{\delta}, \infty]$  since  $\delta(t; x)$  is uniformly bounded below by  $\underline{\delta}$  (Assumption 1(ii)). Since a composition of real-valued continuous functions is continuous, and the reciprocal function is continuous for all arguments other than 0,  $A_N$  is a continuous map.

$A_\pi$ : Denote  $\tilde{A}[\omega] = (A_{p_\sigma}[\omega], A_m[\omega], A_N[\omega])$ . We have shown that  $\tilde{A}$  is continuous and compact. Note that for any sequence  $\omega_n$ ,

$$\sup_{x,z,t} |A_\pi[\omega_n](x, z, t)| \leq \sup_{x,z,t} \left| H_z(s_{xz}(t); \tilde{A}[\omega_n]) \right| \leq \sup_{z,s} \left| H_z(s; \tilde{A}[\omega_n]) \right|,$$

where the first inequality follows from the fact that  $\mathbb{P}(c_{ij} = 1 | x, z) \in [0, 1]$  and the second inequality follows from set inclusion. Therefore, Lemma 5 and continuity of  $\tilde{A}$  imply that for each  $z$ ,  $\sup_s |H_z(s; \tilde{A}[\omega_n]) - H_z(s; \tilde{A}[\omega])| \rightarrow 0$  if  $\omega_n$  converges to  $\omega$ . Since  $z$  belongs to a finite set, we therefore have that  $\sup_{x,z,t} |A_\pi[\omega_n](x, z, t) - A_\pi[\omega](x, z, t)| \rightarrow 0$ . Hence,  $A_\pi$  is a continuous map.  $\square$

**Lemma 4.** *Fix  $x$ . The map  $A_\kappa : \Omega \rightarrow L_\infty([0, T])$ , where  $A_\kappa[\omega](t) = \delta(t; x) + \lambda \int \pi(t; x, Z) p_\sigma(\tau; x, Z) dF_Z$  is continuous if  $\lambda$  is finite, and  $\pi$  and  $p_\sigma$  are uniformly bounded by 1.*

*Proof.* Let  $\Omega_0$  be an arbitrary subset of  $\Omega$ . Consider  $\omega \in \bar{\Omega}_0$ . Since  $\omega \in \bar{\Omega}_0$ , there exists a sequence  $\omega_n \in \Omega_0$  such that  $\|\omega_n - \omega\| = \varepsilon_n \rightarrow 0$ . Now, consider  $A_\kappa[\omega_n](t) = \lambda \int \pi_n(t; x, Z) p_{n,\sigma}(\tau; x, Z) dF_{\Gamma|\tau,x,Z}$ .

$$\begin{aligned} \|A_\kappa[\omega_n] - A_\kappa[\omega]\| &= \lambda \left\| \int \pi_n(t; x, Z) p_{n,\sigma}(t; x, Z) dF_Z - \int \pi(t; x, Z) p_\sigma(t; x, Z) dF_Z \right\| \\ &\leq \lambda \sup_{z,t} |\pi_n(t; x, z) p_{n,\sigma}(t; x, z) - \pi(t; x, z) p_\sigma(t; x, z)| \\ &\leq \lambda \sup_{z,t} |\pi_n(t; x, z) (p_{n,\sigma}(t; x, z) - p_\sigma(t; x, z))| \\ &\quad + \lambda \sup_{z,t} |(\pi_n(t; x, z) - \pi(t; x, z)) p_\sigma(t; x, z)| \\ &\leq \lambda \sup_{z,t} |p_{n,\sigma}(t; x, z) - p_\sigma(t; x, z)| + \lambda \sup_{z,t} |\pi_n(t; x, z) - \pi(t; x, z)| \leq 2\lambda\varepsilon_n. \end{aligned}$$

Therefore,  $A_\kappa [\bar{\Omega}_0] \subset \overline{A_\kappa [\Omega_0]}$ , implying that  $A_\kappa$  is continuous (Theorem 2.27, [Aliprantis and Border, 2006](#)).  $\square$

**Lemma 5.** *Fix  $z$ . The map  $A_H : \Omega \rightarrow L_\infty(\mathbb{R})$  defined by  $A_H[\omega](s) = H_z(s; p_\sigma, m, N)$  is continuous.*

*Proof.* We omit  $z$  from the notation for simplicity as it is fixed. Equation (B.5) derives the following expression for  $A_H$ :

$$A_H[\omega](t, x, z) = \int_0^1 \sum_{q' < q} \frac{e^{-Np(s, \alpha)} (Np(s, \alpha))^{q'}}{q'!} d\alpha,$$

where  $p(s, \alpha)$ ,  $p_H(s)$ , and  $p_E(s)$  are defined in Section B.1.2. We have  $\mathbb{P}(\Gamma(t; x, z) + \varepsilon > V_x(t))$  with the acceptance probabilities  $p_\sigma(t; x, z)$ . Recall that  $m_H(s) = \sum_{t,x} m(t; x) 1\{s(t; x) > s\}$  and  $m_E(s) = \sum_{t,x} m(t; x) 1\{s(t; x) = s\}$ . We prove continuity of  $A_H$  by first proving continuity of the components  $m_H$ ,  $m_E$ ,  $p_H$  and  $p_E$ .

Continuity of  $m_H$  and  $m_E$ : Consider a sequence  $m_n$  that converges in sup norm on  $x, t$  to  $m$ :

$$\begin{aligned} |m_{n,H}(s) - m_H(s)| &\leq \sum_x \int_0^T |m_n(t; x) - m(t; x)| 1\{s(t; x) > s\} dt \\ &\leq |\chi| T \sup_{x,t} |m_n(t; x) - m(t; x)|. \end{aligned}$$

Because this bound is independent of  $s$ ,  $\sup_s |m_{n,H}(s) - m_H(s)|$  converges to zero. Therefore,  $A_{m_H} : \Omega \rightarrow L_\infty(\mathbb{R})$  defined by  $A_{m_H}[\omega](s) = m_H(s)$  is a continuous map because  $A_{m_H}(\bar{\Omega}_0) = \overline{A_{m_H}(\Omega_0)}$  for any  $\Omega_0 \subseteq \Omega$  (Theorem 2.27, [Aliprantis and Border, 2006](#)). An identical argument shows that  $A_{m_E} : \Omega \rightarrow L_\infty(\mathbb{R})$  defined by  $A_{m_E}[\omega](s) = m_E(s)$  is continuous.

Continuity of  $p_H$  and  $p_E$ : We show the argument only for  $p_H$  because the argument for  $p_E$  is identical. Consider a sequence of  $\omega_n$  that converges to  $\omega$ , and the map  $A_{p_H} : \Omega \rightarrow L_\infty(\mathbb{R})$  defined by  $A_{p_H}[\omega](s) = p_0 + (1 - p_0) \frac{1}{m_H(s)} \sum_{t,x} m(t; x) 1\{s(t; x) > s\} p_\sigma(t; x)$ . Since  $p_0$  is fixed, we need to show continuity of the map from  $\omega$  to  $\frac{1}{m_H(s)} \sum_{t,x} m(t; x) 1\{s(t; x) > s\} p_\sigma(t; x)$ .



For each  $s$ ,

$$\begin{aligned}
& \left| \frac{1}{m_{n,H}(s)} \sum_{t,x} m_n(t;x) \mathbf{1}\{s(t;x) > s\} p_{n,\sigma}(t;x,z) - \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} p_\sigma(t;x) \right| \\
& \leq \left| \frac{1}{m_{n,H}(s)} \sum_{t,x} m_n(t;x) \mathbf{1}\{s(t;x) > s\} p_{n,\sigma}(t;x) - \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} p_{n,\sigma}(t;x) \right| \\
& + \left| \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} p_{n,\sigma}(t;x) - \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} p_\sigma(t;x) \right| \\
& \leq \left| \frac{1}{m_{n,H}(s)} \sum_{t,x} m_n(t;x) \mathbf{1}\{s(t;x) > s\} - \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} \right| |p_{n,\sigma}(t;x)| \\
& + \left| \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} \right| |p_{n,\sigma}(t;x) - p_\sigma(t;x)| \\
& \leq \left| \frac{1}{m_{n,H}(s)} \sum_{t,x} m_n(t;x) \mathbf{1}\{s(t;x) > s\} - \frac{1}{m_H(s)} \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\} \right| + |p_{n,\sigma}(t;x) - p_\sigma(t;x)| \\
& = |p_{n,\sigma}(t;x) - p_\sigma(t;x)|
\end{aligned}$$

The first inequality follows from the triangle inequality. The second follows from the fact that  $|p_{n,\sigma}(t;x) - p_\sigma(t;x)|$  is bounded by 1 and  $m_H(s) = \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\}$  by definition. The third follows from  $m_{n,H}(s) = \sum_{t,x} m_n(t;x) \mathbf{1}\{s(t;x) > s\}$  and  $m_H(s) = \sum_{t,x} m(t;x) \mathbf{1}\{s(t;x) > s\}$  for all  $s$ . If  $\omega_n$  converges to  $\omega$ , then  $\sup_{t,x} |p_{n,\sigma}(t;x) - p_\sigma(t;x)|$  converges to zero. Therefore,  $\sup_s |A_{p_H}[\omega_n](s) - A_{p_H}[\omega](s)|$  converges to zero. Hence,  $A_{p_H}$  is continuous because  $A_{p_H}(\bar{\Omega}_0) = \overline{A_{p_H}(\Omega_0)}$  for any  $\Omega_0 \subseteq \Omega$  (Theorem 2.27, [Aliprantis and Border, 2006](#)).

Continuity of  $p(s, \alpha)$ : The map  $A_{p_H} : \Omega \rightarrow L_\infty(\mathbb{R} \times [0, 1])$  defined by  $A_p[\omega](s, \alpha) = m_H(s) p_H(s) + m_E(s) \alpha p_E(s)$  is continuous because  $\alpha$  is bounded by 1, the maps from  $\omega$  to  $m_H(s)$ ,  $p_H(s)$ ,  $m_E(s)$ ,  $p_E(s)$  are continuous.

Continuity of  $A_H$ : The map from  $\Omega$  to  $\sum_{q' < q} \frac{e^{-Np(s,\alpha)} (Np(s,\alpha))^{q'}}{q!'}$  is continuous because the components are continuous. This term is bounded by 1. Therefore,  $\int_0^1 \sum_{q' < q} \frac{e^{-Np(s,\alpha)} (Np(s,\alpha))^{q'}}{q!' } d\alpha$  defines a continuous map from  $\Omega$  to the  $L_\infty([0, T])$  for each  $x$ .  $\square$

## E Additional Empirical Results

Table E.VII: Acceptance Rates and Recent Offers

	Dependent Variable: Current Offer Accepted				
	(1)	(2)	(3)	(4)	(5)
Time Since Last Offer (Years)	0.0765 (0.0127)	0.0129 (0.0119)			0.0127 (0.0119)
Time Since Last Two Offers (Years)			0.0229 (0.0131)		
Time Since Last Five Offers (Years)				0.0590 (0.0205)	
Variables Affecting Priority		X	X	X	X
Patient Characteristics		X	X	X	X
Donor and Match Characteristics		X	X	X	X
Listing Center Fixed Effects		X	X	X	X
Donor Characteristics of Previous Offer					X
Observations	2703549	2703549	2694132	2666270	2703549
R-squared	0.001	0.103	0.103	0.102	0.103
Mean Acceptance Rate	0.14%	0.14%	0.14%	0.14%	0.14%
#REF!	0.005	0.005	0.005	0.005	0.005
#REF!	0.011	0.011	0.008	0.006	0.011

Notes: Estimates from a linear probability model of offer acceptance as a function of the patient's recent offer history. Time Since Last N Offers measures the average number of years since the patient's previous offers, averaged over their last N offers. Column (1) considers all offers and includes no controls for current offer characteristics. Columns (2) – (5) control for current patient, donor, and match characteristics. Column (5) includes controls for donor characteristics of the patient's previous offer. Controls are as described in the notes for Appendix Table B.1. An ideal donor has no history of diabetes; is non-DCD; has creatinine below 3; and is Hepatitis C negative.

## F Convergence Results

### F.1 Consistency of $\hat{W}(x_i, t; \hat{\theta})$

We now show that  $\hat{W}(x_i, t; \hat{\theta})$  consistently estimates the quantity

$$\begin{aligned} W(x_i, t; \theta) &= \int \pi_{ij}(t) \psi(P_{ijt}) dF \\ &= \int H_{z_j, \eta_j}(s(t; x_i, Z)) \mathbb{P}(c_{ij} = 1 | Z, x_i; \theta) \psi(x_i, Z, \eta, t; \theta) dF_{Z, \eta}. \end{aligned}$$

To do this, we need to introduce some notation. Define

$$g_j(\theta) = p_c(z_j, x_i; \theta) \psi(x_i, z_j, \eta_j, t; \theta) 1\{s(t; x_i, z_j) > s_j^*\},$$

Table E.VIII: Tests for Rank Autocorrelation in Cutoff Scores

Donor Type	N	Autocorrelation Statistic	p-value
Panel A: Donor Category			
Healthy, Age < 35	991	2.08	0.90
Healthy, Age >= 35	1525	2.01	0.60
Cardiac Death, Age < 35	166	2.03	0.58
Cardiac Death, Age >= 35	390	2.22	0.99
Low-Quality	2415	1.94	0.06
Low-Quality or Cardiac Death	155	2.16	0.84
Panel B: Donor Category and Origin			
Healthy, Age < 35; from NYRT	212	2.03	0.57
Healthy, Age < 35; not from NYRT	779	2.00	0.48
Healthy, Age >= 35; from NYRT	229	1.99	0.47
Healthy, Age >= 35; not from NYRT	1296	2.03	0.70
Cardiac Death, Age < 35; from NYRT	15	1.91	0.41
Cardiac Death, Age < 35; not from NYRT	151	2.09	0.72
Cardiac Death, Age >= 35; from NYRT	50	2.51	0.97
Cardiac Death, Age >= 35; not from NYRT	340	2.09	0.80
Low-Quality; from NYRT	222	2.03	0.60
Low-Quality; not from NYRT	2193	1.95	0.11
Low-Quality or Cardiac Death; from NYRT	6	2.00	0.54
Low-Quality or Cardiac Death; not from NYRT	149	2.22	0.92
Panel C: Standard Criteria NYRT Donors, by Age and Blood Type			
Healthy, Age < 35; from Type O	111	2.04	0.59
Healthy, Age < 35; from Type B	35	1.40	0.03
Healthy, Age < 35; from Type AB	5	1.50	0.26
Healthy, Age < 35; from Type A	61	1.70	0.12
Healthy, Age >= 35; from Type O	113	1.81	0.15
Healthy, Age >= 35; from Type B	32	2.00	0.48
Healthy, Age >= 35; from Type AB	7	1.61	0.27
Healthy, Age >= 35; from Type A	77	2.00	0.52

Notes: results from tests for autocorrelation of donor cutoffs in the NYRT sample based on the rank version of von Neumann's ratio statistic (Bartels, 1982). Each donor's cutoff is the priority score above which a patient would have received an offer from that donor, which is determined by the last patient in the donor's offer sequence. The rank of each donor's cutoff is its order statistic among the cutoffs of donors of the same type, with ties broken by a random number. The autocorrelation statistic for each donor type is computed for the observed sequence of donor cutoff ranks. Each p-value is the fraction of 1,000 randomly sampled permutations of donor arrival sequences for which the rank autocorrelation statistic is below that observed in sample.

Table E.IX: Patient Ordering Statistics

Re-Ordering of Patients	
% Cases Ever Ahead given Ahead Today	81.6%
% Cases Ahead in Future given Ahead Today	81.5%
# Cases Ahead in Future given Ahead Today	136.7

where  $p_c(z_j, x_i; \theta) = \mathbb{P}(c_{ij} = 1 | z_j, x_i; \theta)$ . For a vector  $x = (x_1, \dots, x_K)$ , we write  $|x| = (|x_1|, \dots, |x_K|)$ .

Finally, we index objects according to the order in which they arrive in our sample. Therefore,  $(z_j, \eta_j)$  denotes the observed and unobserved characteristics of the  $j$ -th donor that arrived. Therefore, the data on  $(z_j, \eta_j, s_j^*)$  generate a sequence of object arrivals.

We make the following assumptions on  $g_j(\theta)$ :

**Assumption 2.** (i)  $(z_j, \eta_j)$  is drawn i.i.d. with CDF  $F$

(ii)  $g_j(\theta_0)$  is weakly stationary<sup>1</sup> with  $\sum_{k=-\infty}^{\infty} \gamma_k < \infty$ , where  $Cov(g_j(\theta_0), g_{j-k}(\theta_0)) = \gamma_k$ .

(iii)  $\hat{\theta}_J$  is  $\sqrt{J}$ -consistent, i.e.  $(\hat{\theta}_J - \theta_0) = O_p(J^{-1/2})$

(iv) There exists a function  $m(z)$ , such that for each  $z_j$   $|g(z_j; \theta) - g(z_j; \theta')| \leq m(z_j) \cdot |\theta - \theta'|$ , and  $m(Z)$  has finite second moments.

Part (i), in our empirical context, assumes that the characteristics of the donor are drawn independently each time a donor arrives. Part (ii) assumes that, at  $\theta_0$ , the offers and their values for any given patient type  $x_i$ , at any given time  $t$ , follows a weakly stationary process. That is, the covariance in these values across any two donors falls as they are further apart in the sequence. Given part (i), the only potential source of dependence between  $g_j(\theta_0)$  and  $g_k(\theta_0)$  is that the characteristics of donor  $j$  may affect the state of the waitlist for donor  $k$  because of patient decisions. However, we expect that this dependence to fall as these donors become further apart in their arrival sequence. Part (iii) assumes that  $\hat{\theta}_J$  is consistently estimated at a rate that is at least as fast as the square-root of the number of donors. These parameters govern the conditional choice probabilities and the probability of a crossmatch failure at biological testing. Part (iv) is a regularity condition, assuming that  $g(z_j; \theta)$  is Lipschitz continuous at each  $z$ , and imposes a bound on the second moment of the distribution of Lipschitz constants. Proposition 1 shows that this property is satisfied under more primitive conditions stated in Assumption 3.

We now show that for each  $x_i, t$ ,  $\hat{W} = \frac{1}{J} \sum_{j=1}^J g_j(\hat{\theta})$  is a  $\sqrt{J}$ -consistent estimator of  $W(x_i, t; \theta_0)$  under this assumption.

**Theorem 2.** Fix  $x_i, t$ . If Assumption 2 is satisfied, then

$$\left| W(x_i, t; \theta_0) - \frac{1}{J} \sum_{j=1}^J g_j(\hat{\theta}) \right| = O_p(J^{-1/2}).$$

---

<sup>1</sup>The process  $\{g_j\}$  is weakly stationary if (i)  $\mathbb{E}[g_j]$  does not depend on  $j$ , and (ii)  $Cov(g_j, g_{j-k})$  exists, is finite and depends only on  $k$ , and not  $j$ .

*Proof.* For each  $x_i, t$ , Assumption 2(i) implies that

$$\begin{aligned}\mathbb{E}[g_j(\theta_0)] &= \mathbb{E}\left[p_c(z_j, x_i; \theta_0) \psi(x_i, z_j, \eta_j, t; \theta_0) \mathbb{1}\{s(t; x_i, z_j) > s_j^*\}\right] \\ &= \mathbb{E}\left[p_c(z_j, x_i; \theta_0) \psi(x_i, z_j, \eta_j, t; \theta_0) \mathbb{E}\left[\mathbb{1}\{s(t; x_i, z_j) > s_j^*\} \mid z_j, \eta_j\right]\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[p_c(z_j, x_i; \theta_0) \psi(x_i, z_j, \eta_j, t; \theta_0) H_{z_j, \eta_j}(s(t; x_i, z_j)) \mid z_j, \eta_j\right]\right] \\ &= W(x_i, t; \theta_0),\end{aligned}$$

where the equalities are a result of the law of iterated expectations, and the definitions of  $H_{z_j, \eta_j}(s)$  and  $W(x_i, t; \theta_0)$ .

Because  $W(x_i, t; \theta_0) = \mathbb{E}[g_j(\theta_0)]$ , Chebychev's inequality implies that,

$$\mathbb{P}\left(J^{1/2} \left|W(x_i, t; \theta_0) - \frac{1}{J} \sum_{j=1}^J g_j(\theta_0)\right| > \varepsilon\right) \leq \frac{\text{Var}\left(\frac{1}{\sqrt{J}} \sum_{j=1}^J g_j(\theta_0)\right)}{\varepsilon^2}.$$

Assumption 2(i) and Proposition 6.8 in Hayashi (2001) imply that

$$\lim_{J \rightarrow \infty} \text{Var}\left(\frac{1}{\sqrt{J}} \sum_{j=1}^J g_j(\theta_0)\right) = K.$$

Therefore, we have that

$$W(x_i, t; \theta_0) - \frac{1}{J} \sum_{j=1}^J g_j(\theta_0) = O_p(J^{-1/2}). \quad (\text{F.6})$$

Lemma 6 implies that

$$W(x_i, t; \theta_0) - \frac{1}{J} \sum_{j=1}^J g_j(\hat{\theta}) = O_p(J^{-1/2}),$$

as requirements (i) and (ii) of Lemma 6 are part of Assumption 2(iv), requirement (iii) is equivalent to Assumption 2(iii), and requirement (iv) is proved in equation (F.6).  $\square$

**Lemma 6.** Fix  $x_i, t$ . Suppose that (i)  $g(z_j; \theta)$  is Lipschitz continuous for each  $z_j$  with the Lipschitz constant  $m(z_j) \in \mathbb{R}^K$  i.e.  $|g(z_j; \theta) - g(z_j; \theta_0)| \leq m(z_j) \cdot |\theta - \theta_0|$ , (ii)  $m(z_j)$  has finite second moments, (iii)  $|\hat{\theta}_J - \theta_0| = O_p(J^{-1/2})$ , and (iv)  $\frac{1}{J} \sum_j g(z_j; \theta_0) - E[g(z_j; \theta_0)] = O_p(J^{-1/2})$ . Then

$$\frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E[g(z_j; \theta_0)] = O_p(J^{-1/2}).$$

*Proof.* Because  $g(z_j; \theta)$  is Lipschitz continuous in  $\theta$ , we have that for any  $\theta \in \Theta$ ,

$$\text{Var}(|g(z_j; \theta) - g(z_j; \theta_0)|) \leq |\theta - \theta_0|^T \mathbb{E} \left( m(z_j) m^T(z_j) \right) |\theta - \theta_0|,$$

where  $x^T$  is the transpose of the vector  $x$ . Therefore, because  $|\hat{\theta}_J - \theta_0| = O_p(J^{-1/2})$ , with probability approaching 1,

$$\text{Var} \left( g(z_j; \hat{\theta}_J) - g(z_j; \theta_0) \right) \leq V_M K J^{-1},$$

for some finite constant  $K > 0$ , where  $V_M = \mathbb{E} \left( \sum_{k,k'} m_k(z_j) m_{k'}(z_j) \right)$  and  $m_k(z_j)$  is the  $k$ -th component of  $m(z_j)$ . By the covariance inequality, with probability approaching 1,

$$\text{Cov} \left( g(z_j; \hat{\theta}_J) - g(z_j; \theta_0), g(z_k; \hat{\theta}_J) - g(z_k; \theta_0) \right) \leq V_M K J^{-1}.$$

Therefore, with probability approaching 1, for all  $j, k \in \{1, \dots, J\}$ ,

$$\begin{aligned} & \text{Var} \left( \frac{1}{\sqrt{J}} \sum_j g(z_j; \hat{\theta}_J) - \frac{1}{\sqrt{J}} \sum_j g(z_j; \theta_0) \right) \\ &= \frac{1}{J} \sum_{k=1}^J \sum_{j=1}^J \text{Cov} \left( g(z_j; \hat{\theta}_J) - g(z_j; \theta_0), g(z_k; \hat{\theta}_J) - g(z_k; \theta_0) \right) \leq V_M K. \end{aligned}$$

By Chebychev's inequality,

$$\mathbb{P} \left( \sqrt{J} \left| \frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E[g(z_j; \hat{\theta}_J)] - \frac{1}{J} \sum_j g(z_j; \theta_0) + E[g(z_j; \theta_0)] \right| > \varepsilon \right) \leq \frac{V_M K}{\varepsilon^2}.$$

Therefore,

$$A_J = \frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E[g(z_j; \hat{\theta}_J)] - \frac{1}{J} \sum_j g(z_j; \theta_0) + E[g(z_j; \theta_0)] = O_p(J^{-1/2}).$$

But, we know that  $\frac{1}{J} \sum_j g(z_j; \theta_0) - E[g(z_j; \theta_0)] = O_p(J^{-1/2})$ . So, it must be that

$$\frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E[g(z_j; \hat{\theta}_J)] = O_p(J^{-1/2}). \quad (\text{F.7})$$

Lipschitz continuity of  $g(z_j; \theta)$  and the Cauchy-Schwarz inequality imply that.

$$\begin{aligned} E \left[ \left| g(z_j; \hat{\theta}_J) - g(z_j; \theta_0) \right| \right] &\leq E \left[ m(z_j) \cdot \left| \hat{\theta}_J - \theta_0 \right| \right] \\ &\leq \sqrt{V_M} \sqrt{E \left[ \left| \hat{\theta}_J - \theta_0 \right| \cdot \left| \hat{\theta}_J - \theta_0 \right| \right]}. \end{aligned}$$

Because  $\hat{\theta}_J$  belongs to the compact set  $\Theta$ , and  $\hat{\theta}_J - \theta_0 = O_p(J^{-1/2})$ , we have that  $\sqrt{E \left[ \left| \hat{\theta}_J - \theta_0 \right| \cdot \left| \hat{\theta}_J - \theta_0 \right| \right]} = O_p(J^{-1/2})$ . Together with the assumption that  $V_M = \mathbb{E} \left( \sum_{k,k'} m_k(z_j) m_{k'}(z_j) \right)$  is finite, we have that

$$E \left[ \left| g(z_j; \hat{\theta}_J) - g(z_j; \theta_0) \right| \right] = O_p(J^{-1/2}). \quad (\text{F.8})$$

Finally, equation (F.7) and (F.8) together imply that

$$\begin{aligned} &\frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E \left[ g(z_j; \hat{\theta}_J) \right] \\ &= \frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E \left[ g(z_j; \theta_0) \right] + E \left[ g(z_j; \theta_0) \right] - E \left[ g(z_j; \hat{\theta}_J) \right] \\ &\leq \frac{1}{J} \sum_j g(z_j; \hat{\theta}_J) - E \left[ g(z_j; \theta_0) \right] + E \left[ \left| g(z_j; \theta_0) - g(z_j; \hat{\theta}_J) \right| \right] \\ &= O_p(J^{-1/2}). \end{aligned}$$

□

*Lipschitz continuity of  $g(z; \theta)$*

We now show primitive regularity conditions under which Assumption 2(iv) is satisfied. Recall that  $g_j(\theta) = \mathbb{P}(c_{ij} = 1 | z_j, x_i; \theta) \psi(x_i, z_j, \eta_j, t; \theta) \mathbb{1} \left\{ s(t; x_i, z_j) > s_j^* \right\}$ . Fix  $x_i, t$  and omit it from the notation for simplicity.

**Assumption 3.** (i)  $\psi(z_j, \eta_j; \theta) = \mathbb{E} \left[ \max \{0, \chi(z_j) \cdot \theta + \eta_j + \varepsilon\} \right]$  where  $\varepsilon$  has cdf  $F_\varepsilon$

(ii) There exists a function  $m(z)$ , such that  $m(Z)$  has finite fourth moments and for all  $\theta, \theta'$ ,

a.  $|\mathbb{P}(c_{ij} = 1 | z_j; \theta) - \mathbb{P}(c_{ij} = 1 | z_j; \theta')| \leq m(z_j) \cdot |\theta - \theta'|$

b.  $|\chi(z)| \leq m(z)$ .

Part (i) follows from the definition of  $\psi(z_j, \eta_j; \theta)$  and the parametrization of the conditional choice probabilities in our model. It is repeated simply to keep this exercise self-contained.



Part (ii) is a regularity condition that assumes lipschitz continuity of primitives, with sufficiently small lipschitz constants.

**Proposition 1.** *If Assumption 3 is satisfied, then for all  $\theta, \theta' \in \Theta$ ,  $|g_j(\theta) - g_j(\theta')| \leq \tilde{m}(z_j) \cdot |\theta - \theta'|$ , where  $\tilde{m}(z_j)$  has finite second moments.*

*Proof.* First, we show that for all  $\theta, \theta' \in \Theta$   $|\psi(z_j, \eta_j; \theta) - \psi(z_j, \eta_j; \theta')| \leq m(z_j) \cdot |\theta - \theta'|$ . Define  $\gamma(x) = \mathbb{E}[\max\{0, x + \varepsilon\}] = \int_{-x}^{\infty} (x + \varepsilon) dF_\varepsilon$ . Libniz's rule implies that

$$\gamma'(x) = \int_{-x-\eta}^{\infty} 1 dF_\varepsilon = 1 - F(-x - \eta) \leq 1.$$

Therefore,  $\gamma(x)$  is Lipschitz continuous with constant 1. Hence, Assumption 3(i) and (ii)b., and Lemma 8 imply that

$$\begin{aligned} |\psi(z_j, \eta_j; \theta) - \psi(z_j, \eta_j; \theta')| &= |\gamma(\chi(z_j)\theta + \eta_j) - \gamma(\chi(z_j)\theta' + \eta_j)| \\ &\leq |\chi(z_j)(\theta - \theta')| \\ &\leq |\chi(z_j)| \cdot |\theta - \theta'| \leq m(z_j) \cdot |\theta - \theta'|, \end{aligned} \quad (\text{F.9})$$

where the first inequality is due to the fact that  $\gamma(\cdot)$  is Lipschitz continuous with constant 1.

Equation (F.9), Assumption 3(ii)a and Lemma 7 imply that  $|g_j(\theta) - g_j(\theta')| \leq 2(m(z) * m(z)) \cdot |\theta - \theta'|$ , where  $*$  is the hadamard (or component-wise) product. Assumption 3(ii) implies that  $\tilde{m}(z) = 2(m(z) * m(z))$  has finite second moments.  $\square$

**Lemma 7.** *Suppose that there exists a function  $m(z)$  such that (i)  $\tilde{\chi}(z) \leq m(z)$  and  $|\chi(z, \theta) - \chi(z, \theta')| \leq m(z) \cdot |\theta - \theta'|$ , and (ii)  $\sup_\theta |\tilde{\chi}(z, \theta)| \leq m(z)$  and  $\sup_\theta |\chi(z, \theta)| \leq m(z)$ . Then,*

$$|\tilde{\chi}(z, \theta)\chi(z, \theta) - \tilde{\chi}(z, \theta')\chi(z, \theta')| \leq 2(m(z) * m(z)) \cdot |\theta - \theta'|,$$

where  $*$  is the hadamard (or component-wise) product.

*Proof.* Suppose that  $|\tilde{\chi}(z, \theta) - \tilde{\chi}(z, \theta')| \leq \tilde{m}(z) \cdot |\theta - \theta'|$  and  $|\chi(z, \theta) - \chi(z, \theta')| \leq m(z) \cdot$

$|\theta - \theta'|$ . Consider

$$\begin{aligned}
& |\tilde{\chi}(z, \theta) \chi(z, \theta) - \tilde{\chi}(z, \theta') \chi(z, \theta')| \\
&= |\tilde{\chi}(z, \theta) \chi(z, \theta) - \tilde{\chi}(z, \theta) \chi(z, \theta') + \tilde{\chi}(z, \theta) \chi(z, \theta') - \tilde{\chi}(z, \theta') \chi(z, \theta')| \\
&\leq |\tilde{\chi}(z, \theta) (\chi(z, \theta) - \chi(z, \theta'))| + |(\tilde{\chi}(z, \theta) - \tilde{\chi}(z, \theta')) \chi(z, \theta')| \\
&\leq |\tilde{\chi}(z, \theta)| m(z) \cdot |\theta - \theta'| + |\chi(z, \theta')| m(z) \cdot |\theta - \theta'| \\
&\leq 2(m(z) * m(z)) \cdot |\theta - \theta'|.
\end{aligned}$$

□

**Lemma 8.** *Suppose that (i)  $\psi : \mathbb{R} \rightarrow \mathbb{R}$  is Lipschitz continuous with constant  $K < \infty$ , and (ii) for each  $z$ , there exists  $m(z) \in \mathbb{R}^{K\theta}$  such that  $|\tilde{\chi}(z, \theta) - \tilde{\chi}(z, \theta')| \leq m(z) \cdot |\theta - \theta'|$ , then  $|\psi(\tilde{\chi}(z, \theta)) - \psi(\tilde{\chi}(z, \theta'))| \leq Km(z) \cdot |\theta - \theta'|$ . In particular, if  $\tilde{\chi}(z, \theta) = \chi(z) \cdot \theta$  where  $\chi(z), \theta \in \mathbb{R}^{K\theta}$ , then  $|\psi(\tilde{\chi}(z, \theta)) - \psi(\tilde{\chi}(z, \theta'))| \leq K|\chi(z)| \cdot |\theta - \theta'|$ .*

*Proof.* The first part follows definitionally because Lipschitz continuity of  $\psi$  implies that

$$|\psi(\tilde{\chi}(z, \theta)) - \psi(\tilde{\chi}(z, \theta'))| \leq K|\tilde{\chi}(z, \theta) - \tilde{\chi}(z, \theta')| \leq Km(z) \cdot |\theta - \theta'|.$$

For the second part, note that

$$\chi(z) \cdot (\theta - \theta') \leq |\chi(z)| \cdot |\theta - \theta'|.$$

□

## F.2 Approximation Result for Long Queues

In this section, we show that the stationary distribution of the length of the waitlist concentrates mass around the steady state value,  $N^*$ . This result suggests that using a deterministic queue length is a good approximation for large values of  $N^*$ .

**Theorem 3.** *Consider the continuous time Markov process  $N_\tau$ , where  $\tau$  denotes calendar time. For any given  $(\pi^*, m^*, \sigma^*)$ ,  $N_\tau$  is an ergodic process with stationary distribution  $p_n$ . Moreover, for any  $\varepsilon > 0$ ,*

$$\lim_{\tau \rightarrow \infty} \mathbb{P}(|N_\tau - N^*| > \varepsilon N^*) \leq 2 \exp\left(-\frac{N^* \varepsilon^2}{2(1 + \varepsilon)}\right),$$

where

$$N^* = \frac{\int \gamma_x dF_X}{\int \int m_x^*(t) \kappa_x(t) dt dF_X}.$$

*Proof.* Define  $\kappa = \int \int m_x^*(t) \kappa_x(t) dt dF_X$  and  $\gamma = \int \gamma_x dF_X$ . Observe that  $N_\tau$  follows a birth-death process with birth-rate  $\gamma$  and death rate  $N_\tau \kappa$ . Therefore,  $N_\tau$  is an ergodic Markov process. The stationary distribution,  $p_n$ , satisfies the detail balance conditions:

$$\begin{aligned} \gamma p_0 &= \kappa p_1 \\ (\gamma + n\kappa) p_n &= \gamma p_{n-1} + (n+1) \kappa p_{n+1} \text{ for } n \geq 1, \end{aligned}$$

where the left-hand side denotes the exit rate from state  $n$  and the right hand side is the entry rate. Solving the system recursively and using the fact that  $\sum_n p_n = 1$  yields

$$\begin{aligned} p_0 &= \left[ 1 + \sum \frac{\gamma^n}{n! \kappa^n} \right]^{-1} \\ &= \exp\left(-\frac{\gamma}{\kappa}\right). \end{aligned}$$

and

$$p_n = \frac{\gamma^n}{n! \kappa^n} \exp\left(-\frac{\gamma}{\kappa}\right).$$

Note that  $p_n$  is the probability that a random variable  $N$  following a Poisson distribution with parameter  $N^* = \frac{\gamma}{\kappa}$  takes on the value  $n$ . Theorem 1, part 3 in [Canone \(2017\)](#) implies the result.  $\square$

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