Appendix for Online Publication

A Proofs for sections 2 and 3

A.1 The standard New Keynesian model

This section shows that, in the standard New Keynesian model with sticky Calvo prices, the impulse response to the path for prices P_t, real discount rates q_t, real wages w_t and unearned income are those given by my main experiment in figure 1. I only outline the elements of the model relevant to my argument, the reader is referred to the textbook treatments of Woodford (2003) or Galí (2008) for details.

I consider the model in its 'cashless limit', with no aggregate uncertainty. The model features a representative agent with separable utility trading in one-period nominal bonds and holding a fixed stock of capital k, so equation (1) simplifies to

$$\sum \beta^t \{u(c_t) - v(n_t)\}$$
$$P_t c_t + ({}_tQ_{t+1}) B_{t+1} = P_t \pi_t + W_t n_t + B_t + P_t \rho_t k$$

Here ρ_t denotes the real rental rate of capital, so $\rho_t k$ are total real rents, and π_t are real firm profits. Together, rents and profits make up the unearned income in this economy. Consumption c_t is an aggregate of intermediate goods, with constant elasticity of substitution ϵ. Hence the price index, aggregating the individual goods prices p_{jt}, is $P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$.

Each good j is produced under monopolistic competition with constant returns to scale and unit productivity. The production function is

$$y_{jt} = F(k_{jt}, l_{jt}) = k_{jt}^{\alpha} l_{jt}^{1-\alpha}$$

Firms can only adjust their price with probability θ each period, independent across firms and periods (the Calvo assumption). Nominal wages W_t and nominal rents are flexible. Cost minimization by the firm therefore implies

$$\begin{aligned} \rho_t P_t &= \Lambda_{jt} F_k(k_{jt}, l_{jt}) \\ W_t &= \Lambda_{jt} F_l(k_{jt}, l_{jt}) \end{aligned}$$

for some scalar Λ_{jt} representing the nominal marginal cost of production for firm j. Hence

$$\frac{F_k(k_{jt}, l_{jt})}{F_l(k_{jt}, l_{jt})} = \frac{F_k\left(\frac{k_{jt}}{l_{jt}}, 1\right)}{F_l\left(\frac{k_{jt}}{l_{jt}}, 1\right)} = \frac{\rho_t}{w_t}$$

so all firms have the same capital-labor ratio $\frac{k_{jt}}{l_{jt}} = \frac{k_t}{l_t}$, and hence all firms have the same nominal marginal cost of production Λ_t.

As is well-known, a first-order approximation to the equilibrium equations of this model is given by the system of three equations

$$\log\left(\frac{c_t}{\bar{c}}\right) = \log\left(\frac{c_{t+1}}{\bar{c}}\right) - \sigma\left(i_t - \log\left(\frac{P_{t+1}}{P_t}\right) - \varrho\right) \tag{A.1}$$

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \beta\log\left(\frac{P_{t+1}}{P_t}\right) + \kappa\log\left(\frac{c_t}{\bar{c}}\right) \tag{A.2}$$

$$i_t = \varrho + \phi_\pi \log\left(\frac{P_t}{P_{t-1}}\right) + \epsilon_t \tag{A.3}$$

where $\bar{c}$ is the level of consumption that would prevail under flexible prices, which (normalizing $k = 1$) solves

$$\frac{v'\left((\bar{c})^{\frac{1}{1-\alpha}}\right)}{u'(\bar{c})} = \frac{\epsilon - 1}{\epsilon}\frac{(1-\alpha)}{\bar{c}} \equiv \overline{w}$$

$\varrho = \beta^{-1} - 1$ is the steady-state net real interest rate, $\sigma = -\frac{u'(\bar{c})}{cu''(\bar{c})}$ is the elasticity of substitution around $\bar{c}$, and κ is the slope of the Phillips curve (a function of model parameters). Equation (A.3) is a Taylor rule describing the behavior of monetary policy. We assume that $\phi_\pi > 1$, which guarantees equilibrium uniqueness. We consider the effects of a time-0 monetary policy loosening, $\epsilon_0 < 0$ and $\epsilon_t = 0$ for $t \geq 1$, assuming the system was at steady-state at $t = -1$, with constant price level $\overline{P}$.

It is easy to guess and verify that the equilibrium features $i_t = \rho$, $P_t = P_{t-1}$ and $c_t = \bar{c}$ for $t \geq 1$. Solving backwards, this implies that

$$\begin{aligned} i_0 &= \rho + \frac{1}{1+\kappa\sigma\phi_\pi}\epsilon_0 \\ \log\left(\frac{c_0}{\bar{c}}\right) &= -\frac{\sigma}{1+\kappa\sigma\phi_\pi}\epsilon_0 \\ \log\left(\frac{P_0}{\overline{P}}\right) &= -\frac{\kappa\sigma}{1+\kappa\sigma\phi_\pi}\epsilon_0 \end{aligned}$$

In other words, a monetary loosening raises c_t at $t = 0$ only, and raises P_t immediately and permanently. (Firms that get an opportunity to reset at $t = 0$ all increase their price above $\overline{P}$, pulling up the price level to P_0. Thereafter, all firms that get a chance reset their price to P_0, so there is no inflation.) To a first-order approximation, the real wage satisfies

$$w_t = \frac{v'\left(c_t^{\frac{1}{1-\alpha}}\right)}{u'(c_t)}$$

so w_t increases at $t = 0$ only and then reverts to $\overline{w}$. Moreover, real rents are

$$\rho_t = \frac{\alpha}{1-\alpha} w_t c_t^{\frac{1}{1-\alpha}}$$

so they also increase at $t = 0$ and then revert to $\bar{\rho} = \frac{\alpha}{1-\alpha}\overline{w}(\bar{c})^{\frac{1}{1-\alpha}}$.[46] Date-0 nominal and

[46] Since price dispersion rises as a result of the monetary policy shock, the nonlinear solution features a real wage

real state prices are $Q_0 = q_0 = 1$ and, for $t \geq 1$, given that $P_t = P_0$,

$$q_t = Q_t = \prod_{s=0}^{t-1} \left({}_sQ_t \right) = \frac{1}{1+i_0}\beta^{t-1}$$

Hence, the path of q_t and Q_t for $t \geq 1$ is shifted upwards by $\frac{dq_t}{q_t} = \frac{dQ_t}{Q_t} = -\frac{dR}{R}$ where the proportional real interest rate change is $\frac{dR}{R} = \frac{d\epsilon_0}{(1+\kappa\sigma\phi_\pi)} \frac{1}{(1+\rho)}$. Finally, aggregate profits are, to first-order, given by

$$\pi_t = c_t - w_t n_t - \rho_t k = c_t \left(1 - \frac{1}{1-\alpha} \frac{v'\left((c_t)^{\frac{1}{1-\alpha}} \right)}{u'(c_t)} c_t \right) \tag{A.4}$$

Hence they also deviate only at $t = 0$ from their steady state value of $\overline{d} = \frac{\overline{c}}{\epsilon}$. The first term in (A.4) is volume, which rises with c_0. The second term is the markup, which falls with c_0. In typical calibrations, the markup effect dominates and profits fall in response to an expansionary monetary shock $\epsilon_0 < 0$.

Collecting results, the timing of changes for w_t, P_t and q_t, as well as unearned income $\rho_t k + \pi_t$, is exactly that depicted in figure 1, as claimed in the main text.

A.2 Proof of theorem 1

The proof is greatly simplified by first applying a simple renormalization of discount factors. Instead of the present value normalization $q_0 = 1$, I normalize $q_1 = 1$ and let q_0 vary. Then, setting

$$\frac{dq_0}{q_0} = \frac{dR}{R} \tag{A.5}$$

yields the experiment in figure 1. Intuitively, a rise in the relative price of future goods relative to a current good is the same as a fall in the price of that current good relative to all future goods. This renormalization is innocuous since there is a degree of freedom in choosing discount factors.

Given the experiment, we can hold q_t fixed for $t \geq 1$. Hence, only three parameters y_0, w_0 and q_0 vary, together with the sequence $\{P_t\}$.

With this renormalization, the proof has three steps: first, I apply Slutky's theorem to break down dc and dn into income and substitution effects. Second, I work out explicit expressions for MPC and MPN. Finally, I calculate compensated derivatives, and use my expressions from the second step to simplify their expressions.

that is different from steady state even beyond $t \geq 1$, but the difference is second order in ϵ_0.

Step 1: Slutky's theorem. Recall that the sequences $\{q_t\}$ and $\{w_t\}$ are fixed in the experiment, except for q_0 and w_0. Define the following expenditure function

$$e\left(q_0, w_0, U\right) = \min\left\{\sum_t q_t\left(c_t - w_t n_t\right) \quad \text{s.t.} \quad \sum_t \beta^t\left\{u\left(c_t\right) - v\left(n_t\right)\right\} \geq U\right\} \tag{A.6}$$

and let c_0^h, n_0^h be the resulting compensated (Hicksian) demands for time-0 consumption and hours. Applying the envelope theorem, we obtain a version of Shephard's lemma:

$$e_{q_0} = c_0 - w_0 n_0 \tag{A.7}$$

$$e_{w_0} = -q_0 n_0 \tag{A.8}$$

Define 'unearned' wealth as

$$\widetilde{\omega} \equiv \sum_{t\geq 0} q_t\left(y_t + \left({}_{-1}b_t\right) + \left(\frac{{}_{-1}B_t}{P_t}\right)\right)$$

and note that, given the variation we consider,

$$d\widetilde{\omega} = \left(y_0 + \left({}_{-1}b_0\right) + \left(\frac{{}_{-1}B_0}{P_0}\right)\right)dq_0 + q_0 dy_0 - \sum_{t\geq 0} q_t\left(\frac{{}_{-1}B_t}{P_t}\right)\frac{dP_t}{P_t} \tag{A.9}$$

Using the Fisher equation $\frac{q_t}{P_t} = \frac{Q_t}{P_0}$, and the fact that $\frac{dP_t}{P_t} = \frac{dP}{P}$ is a constant, the last term rewrites

$$\sum_{t\geq 0} q_t\left(\frac{{}_{-1}B_t}{P_t}\right)\frac{dP_t}{P_t} = \sum_{t\geq 0} Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)\frac{dP}{P} = q_0 NNP\frac{dP}{P}$$

where we have defined the household's net nominal position as the present value of his nominal assets

$$q_0 NNP \equiv \sum_{t\geq 0} Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)$$

Moreover, defining

$$URE \equiv w_0 n_0 + y_0 + \left({}_{-1}b_0\right) + \left(\frac{{}_{-1}B_0}{P_0}\right) - c_0$$

we can rewrite (A.9) as

$$d\widetilde{\omega} = \left(URE + c_0 - w_0 n_0\right)dq_0 + q_0 dy_0 - q_0 NNP\frac{dP}{P} \tag{A.10}$$

Next, define the indirect utility function that attains $\widetilde{\omega}$ as

$$V\left(q_0, w_0, \widetilde{\omega}\right) = \max\left\{\sum_t \beta^t\left\{u\left(c_t\right) - v\left(n_t\right)\right\} \quad \text{s.t.} \quad \sum_t q_t\left(c_t - w_t n_t\right) = \widetilde{\omega}\right\} \tag{A.11}$$

Let c_0, n_0 denote the resulting Marshallian demands. Applying the envelope theorem, we find

$$\frac{\partial V}{\partial q_0} = -\frac{u'(c_0)}{q_0}(c_0 - w_0 n_0) \tag{A.12}$$

$$\frac{\partial V}{\partial w_0} = \frac{u'(c_0)}{q_0} q_0 n_0 \tag{A.13}$$

$$\frac{\partial V}{\partial \widetilde{\omega}} = \frac{u'(c_0)}{q_0} \tag{A.14}$$

As in the proof of Slutky's theorem, we next differentiate along the identities

$$\begin{aligned} c_0^h(q_0, w_0, U) &= c_0(q_0, w_0, e(q_0, w_0, U)) \\ n_0^h(q_0, w_0, U) &= n_0(q_0, w_0, e(q_0, w_0, U)) \end{aligned}$$

to find that Marshallian and Hickisan derivatives are related via

$$\frac{\partial c_0^h}{\partial q_0} = \frac{\partial c_0}{\partial q_0} + \frac{\partial c_0}{\partial \widetilde{\omega}} e_{q_0} \qquad \frac{\partial c_0^h}{\partial w_0} = \frac{\partial c_0}{\partial w_0} + \frac{\partial c_0}{\partial \widetilde{\omega}} e_{w_0} \tag{A.15}$$

$$\frac{\partial n_0^h}{\partial q_0} = \frac{\partial n_0}{\partial q_0} + \frac{\partial n_0}{\partial \widetilde{\omega}} e_{q_0} \qquad \frac{\partial n_0^h}{\partial w_0} = \frac{\partial n_0}{\partial w_0} + \frac{\partial n_0}{\partial \widetilde{\omega}} e_{w_0} \tag{A.16}$$

Next, define

$$MPC \equiv q_0 \frac{\partial c_0}{\partial \widetilde{\omega}} \tag{A.17}$$

$$MPN \equiv q_0 \frac{\partial n_0}{\partial \widetilde{\omega}} \tag{A.18}$$

these express the dollar-for-dollar (or hour-for-dollar) marginal propensities to consume and work at date 0: indeed,

$$\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial \widetilde{\omega}} \frac{\partial \widetilde{\omega}}{\partial y_0} = \frac{MPC}{q_0} q_0 = MPC$$

and similarly $\frac{\partial n_0}{\partial y_0} = MPN$.

Totally differentiating the Marshallian consumption function and using (A.10), we find

$$dc_0 = \frac{\partial c_0}{\partial q_0} dq_0 + \frac{\partial c_0}{\partial w_0} dw_0 + \frac{\partial c_0}{\partial \widetilde{\omega}} \left((URE + c_0 - w_0 n_0)\, dq_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \right)$$

Using (A.15)–(A.16),

$$\begin{aligned} dc_0 &= \left(\frac{\partial c_0^h}{\partial q_0} - \frac{\partial c_0}{\partial \widetilde{\omega}} e_{q_0} \right) dq_0 + \left(\frac{\partial c_0^h}{\partial w_0} - \frac{\partial c_0}{\partial \widetilde{\omega}} e_{w_0} \right) dw_0 \\ &\quad + \frac{\partial c_0}{\partial \widetilde{\omega}} \left((URE + c_0 - w_0 n_0)\, dq_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \right) \\ &= \frac{\partial c_0}{\partial \widetilde{\omega}} \left(-e_{w_0} dw_0 + q_0 dy_0 + (-e_{q_0} + URE + c_0 - w_0 n_0)\, dq_0 - NNP \frac{dP}{P} \right) + \frac{\partial c_0^h}{\partial q_0} dq_0 + \frac{\partial c_0^h}{\partial w_0} dw_0 \end{aligned}$$

and using (A.7), (A.8) and (A.17) to replace e_{w_0}, e_{q_0} and $\frac{\partial c_0}{\partial \tilde{\omega}}$, we find

$$\begin{aligned} dc_0 &= \frac{MPC}{q_0}\left(q_0 n_0 dw_0 + q_0 dy_0 + UREdq_0 - q_0 NNP\frac{dP}{P}\right) + \frac{\partial c_0^h}{\partial q_0}dq_0 + \frac{\partial c_0^h}{\partial w_0}dw_0 \\ &= MPC\left(n_0 dw_0 + dy_0 + URE\frac{dq_0}{q_0} - NNP\frac{dP}{P}\right) + c_0\left(\frac{q_0}{c_0}\frac{\partial c_0^h}{\partial q_0}\frac{dq_0}{q_0} + \frac{w_0}{c_0}\frac{\partial c_0^h}{\partial w_0}\frac{dw_0}{w_0}\right) \end{aligned}$$

Finally, dropping time subscripts for ease of notation, using (A.5), and defining compensated elasticities by

$$\begin{aligned} \epsilon_{c,q}^h &\equiv \frac{q_0}{c_0}\frac{\partial c_0^h}{\partial q_0} \\ \epsilon_{c,w}^h &\equiv \frac{w_0}{c_0}\frac{\partial c_0^h}{\partial w_0} \end{aligned}$$

we obtain

$$dc = MPC\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + c\left(\epsilon_{c,q}^h\frac{dR}{R} + \epsilon_{c,w}^h\frac{dw}{w}\right) \tag{A.19}$$

In a completely analogous way, we also find

$$dn = MPN\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + n\left(\epsilon_{n,q}^h\frac{dR}{R} + \epsilon_{n,w}^h\frac{dw}{w}\right) \tag{A.20}$$

The rest of the proof calculates the compensated elasticities and relates them to MPC and MPN, which will yield our expressions for consumption and labor supply. To get my expression for welfare, totally differentiate the indirect utility function and use (A.12)–(A.14) and (A.10) to obtain

$$\begin{aligned} dU &= \frac{\partial V}{\partial q_0}dq_0 + \frac{\partial V}{\partial w_0}dw_0 + \frac{\partial V}{\partial \tilde{\omega}}d\tilde{\omega} \\ &= \frac{u'(c_0)}{q_0}\cdot\left(UREdq_0 + q_0 n_0 dw_0 + q_0 dy_0 - q_0 NNP\frac{dP}{P}\right) \end{aligned}$$

This yields my expression in (5),

$$dU = u'(c)\cdot\left(dy + ndw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right)$$

Step 2: Marginal propensities. I now derive explicit expressions for marginal propensities to consume, that is, the Marshallian derivatives of the consumption and labor supply functions that are solutions to (A.11). Inverting the first-order conditions

$$u'(c_t) = \beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0) \tag{A.21}$$

$$v'(n_t) = \beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0) \tag{A.22}$$

and inserting the resulting values for c_t and n_t into the budget constraint (redefining $W = q_0\widetilde{\omega}$ as present-value wealth for simplicity)

$$\sum_{t\geq 0}\frac{q_t}{q_0}\left(c_t - w_t n_t\right) = W$$

we obtain

$$c_0 + \sum_{t\geq 1}\frac{q_t}{q_0}\left(u'\right)^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)u'\left(c_0\right)\right] - w_0\left(n_0 + \sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\left(v'\right)^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'\left(n_0\right)\right]\right) = W \tag{A.23}$$

Recall that $MPC = \frac{\partial c_0}{\partial W}$ and $MPN = \frac{\partial n_0}{\partial W}$. Differentiating (A.23) with respect to W, we obtain

$$MPC\left(1 + \sum_{t\geq 1}\frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''\left(c_0\right)}{u''\left(c_t\right)}\right) - w_0 MPN\left(1 + \sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''\left(n_0\right)}{v''\left(n_t\right)}\right) = 1 \tag{A.24}$$

moreover, the intratemporal first order condition

$$v'\left(n_0\right) = w_0 u'\left(c_0\right) \tag{A.25}$$

implies

$$\begin{aligned} v''\left(n_0\right) MPN &= w_0 u''\left(c_0\right) MPC \\ \frac{v''\left(n_0\right)}{v'\left(n_0\right)} MPN &= \frac{u''\left(c_0\right)}{u'\left(c_0\right)} MPC \end{aligned}$$

so, using the definition of the local elasticities of substitution,

$$-\sigma\left(c_t\right) c_t u''\left(c_t\right) = u'\left(c_t\right) \tag{A.26}$$

$$\psi\left(n_t\right) n_t v''\left(n_t\right) = v'\left(n_t\right) \tag{A.27}$$

we see that MPC and MPN are related through

$$MPN = -\frac{\psi\left(n_0\right)}{\sigma\left(c_0\right)}\frac{n_0}{c_0} MPC$$

Inserting into (A.24), this gives

$$MPC = \left(1 + \sum_{t\geq 1}\frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''\left(c_0\right)}{u''\left(c_t\right)} + \frac{\psi\left(n_0\right)}{\sigma\left(c_0\right)}\frac{w_0 n_0}{c_0}\sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''\left(n_0\right)}{v''\left(n_t\right)}\right)^{-1} \tag{A.28}$$

as well as

$$\begin{aligned} MPS &= 1 - MPC + w_0 MPN \\ &= MPC\left(\sum_{t\geq 1}\frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''\left(c_0\right)}{u''\left(c_t\right)}\right. \\ &\qquad \left. + \frac{\psi\left(n_0\right)}{\sigma\left(c_0\right)}\frac{w_0 n_0}{c_0}\sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''\left(n_0\right)}{v''\left(n_t\right)}\right) \end{aligned} \tag{A.29}$$

Expressions (A.28) and (A.29) can also be rewritten using the fact that (A.21)-(A.22) together with (A.26)-(A.27) yield

$$\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)} = \frac{\sigma(c_t)\,c_t}{\sigma(c_0)\,c_0} \qquad \beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)} = \frac{\psi(n_t)\,n_t}{\psi(n_0)\,n_0}$$

So, we also have

$$MPC = \left(1+\sum_{t\geq 1}\frac{q_t}{q_0}\frac{\sigma(c_t)\,c_t}{\sigma(c_0)\,c_0} + \frac{\psi(n_0)}{\sigma(c_0)}\frac{w_0 n_0}{c_0}\left(1+\sum_{t\geq 1}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{\psi(n_t)\,n_t}{\psi(n_0)\,n_0}\right)\right)^{-1}$$

Step 3: Hicksian elasticities. The solution to the expenditure minimization problem in (A.6) also involves the first-order conditions (A.21)-(A.22) , from which we obtain

$$u(c_t) = u\left(\left(u'\right)^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0)\right]\right) \qquad v(n_t) = v\left(\left(v'\right)^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0)\right]\right)$$

attaining utility U requires that the initial values c_0, n_0 satisfy

$$\begin{aligned} u(c_0) + \sum_{t\geq 1}\beta^t u\left(\left(u'\right)^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0)\right]\right) - v(n_0) \\ - \sum_{t\geq 1}\beta^t v\left(\left(v'\right)^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0)\right]\right) = U \end{aligned} \tag{A.30}$$

Differentiating with respect to q_0 along the indifference curve (A.30) results in

$$\begin{aligned} &\frac{\partial c_0}{\partial q_0}\left(u'(c_0) + \sum_{t\geq 1}\beta^t u'(c_t)\,\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\right) \\ &-\frac{\partial n_0}{\partial q_0}\left(v'(n_0) + \sum_{t\geq 1}\beta^t v'(n_t)\,\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right) \\ -\sum_{t\geq 1}\beta^t\frac{u'(c_t)}{u''(c_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}u'(c_0)\right) - \sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}&\left(\beta^{-t}\frac{q_t}{q_0^2}\left(\frac{w_t}{w_0}\right)v'(n_0)\right) \quad = \quad 0 \end{aligned}$$

dividing by $u'(c_0)$ and using (A.21), (A.25), (A.26) and (A.27) we find

$$\begin{aligned} \frac{\partial c_0}{\partial q_0}\left(1+\sum_{t}\frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\right) - \frac{\partial n_0}{\partial q_0}w_0\left(1+\sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right) \\ = \frac{1}{u'(c_0)}\left(\sum_{t\geq 1}\beta^t\frac{u'(c_t)}{u''(c_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}u'(c_0)\right) + \sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}\left(\frac{w_t}{w_0}\right)v'(n_0)\right)\right) \end{aligned}$$

moreover, differentiating (A.25) we also find

$$\frac{\partial n_0}{\partial q_0} = -\frac{\psi(n_0)}{\sigma(c_0)}\frac{n_0}{c_0}\frac{\partial c_0}{\partial q_0}$$

Gathering results, we recognize, on the left-hand-side, the MPC expression in (A.28). We then use first-order conditions on the right hand side to obtain

$$\begin{aligned}\frac{\partial c_0}{\partial q_0} MPC^{-1} &= \frac{1}{u'(c_0)}\left\{\sum_{t\geq 1}\beta^t \frac{u'(c_t)}{u''(c_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}u'(c_0)\right) - \sum_{t\geq 1}\beta^t \frac{v'(n_t)}{v''(n_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}\left(\frac{w_t}{w_0}\right)v'(n_0)\right)\right\} \\ &= \frac{1}{q_0}\left(\sum_{t\geq 1}\frac{u'(c_t)}{u''(c_t)}\frac{q_t}{q_0} - w_0\sum_{t\geq 1}\frac{v'(n_t)}{v''(n_t)}\frac{q_t}{q_0}\left(\frac{w_t}{w_0}\right)\right)\end{aligned}$$

Manipulating the right-hand side, we recognize the expression for (A.29) as

$$\begin{aligned}\frac{\partial c_0}{\partial q_0} MPC^{-1} &= -\frac{1}{q_0}\sigma(c_0)\,c_0\left\{\sum_{t\geq 1}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\frac{q_t}{q_0}\right. \\ &\qquad \left. + \frac{w_0 n_0}{c_0}\frac{\psi(n_0)}{\sigma(c_0)}\sum_{t\geq 1}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\frac{q_t}{q_0}\left(\frac{w_t}{w_0}\right)\right\} \\ &= -\frac{1}{q_0}\sigma(c_0)\,c_0\frac{MPS}{MPC}\end{aligned}$$

and therefore, we finally simply have

$$\left.\frac{\partial c_0}{\partial q_0}\right]_U = -\frac{c_0}{q_0}\sigma(c_0)\,MPS$$

which corresponds to a Hicksian elasticity of

$$\epsilon^h_{c_0,q_0} = -\sigma(c_0)\,MPS \tag{A.31}$$

A similar procedure can be used to differentiate with respect to w_0: from (A.25) we obtain

$$\frac{\partial n_0}{\partial w_0} = -\frac{\psi(n_0)}{\sigma(c_0)}\frac{n_0}{c_0}\frac{\partial c_0}{\partial q_s} + \psi(n_0)\frac{n_0}{w_0}$$

and differentiating along (A.28) we therefore obtain

$$\begin{aligned}&\frac{\partial c_0}{\partial w_0}u'(c_0)\,MPC^{-1} + \psi(n_0)\frac{n_0}{w_0}\left(v'(n_0) + \sum_{t\geq 1}\beta^t v'(n_t)\,\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right) \\ &= \sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}\beta^{-t}\frac{q_t}{q_0}\left(\frac{w_t}{w_0^2}\right)v'(n_0)\end{aligned}$$

We conclude by noticing that $v'(n_0) = \psi(n_0)\,n_0 v''(n_0)$, so

$$\left.\frac{\partial c_0}{\partial w_0}\right]_U = MPC\psi(n_0)\,n_0$$

and

$$\epsilon^h_{c_0,w_0} = MPC\left(\psi(n_0)\frac{w_0 n_0}{c_0}\right) \tag{A.32}$$

Finally, elasticities for n_0 result from a final differentiation of (A.25):

$$\epsilon^h_{n_0,q_0} = -\frac{\psi(n_0)}{\sigma(c_0)}\epsilon^h_{c_0,q_0} \tag{A.33}$$

$$\begin{aligned}\epsilon^h_{n_0,w_0} &= \psi(n_0)\left(1-\frac{1}{\sigma(c_0)}\epsilon^h_{c_0,w_0}\right)\\ &= \psi(n_0)\left(1-\frac{\psi(n_0)}{\sigma(c_0)}\frac{w_0 n_0}{c_0}MPC\right)\\ &= \psi(n_0)\left(1+w_0 MPN\right)\end{aligned} \tag{A.34}$$

Step 4: Putting all expressions together. For consumption, equations (A.31)–(A.32) can be inserted into (A.19) to yield

$$dc = MPC\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + c\left(-\sigma MPS\frac{dR}{R} + \psi MPC\frac{wn}{c}\frac{dw}{w}\right)$$

The first term is the wealth effect, and the last two terms the substitution effects with respect to interest rates and wages. We then simplify the expression to

$$dc = MPC\left(dy + n(1+\psi)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c MPS\frac{dR}{R} \tag{A.35}$$

which is our equation (3).

Similarly, equations (A.33)–(A.34) can be inserted into (A.20) to yield

$$dn = MPN\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + n\left(-\psi MPS\frac{dR}{R} + \psi(1+wMPN)\frac{dw}{w}\right)$$

and we again naturally separate the latter piece to obtain

$$dn = MPN\left(dy + n(1+\psi)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \psi n MPS\frac{dR}{R} + \psi n\frac{dw}{w} \tag{A.36}$$

which is equation (4).

A.3 Extension of Theorem 1 to general preferences and persistent changes

Theorem 1 in the main text is a special case of a general decomposition that holds for arbitrary nonsatiable preferences U over $\{c_t\}$ and $\{n_t\}$ and for any change in the price level $\{P_0, P_1 \ldots\}$, the real term structure $\{q_0 = 1, q_1, q_2 \ldots\}$, the agent's unearned income sequence $\{y_0, y_1 \ldots\}$ and the stream of real wages $\{w_0, w_1 \ldots\}$, with the nominal term structure adjusting instantaneously to make the Fisher equation hold at the post-shock sequences of interest rates and prices. The utility maximization problem is then

$$\begin{aligned}\max \quad & U(\{c_t, n_t\})\\ \text{s.t.} \quad & P_t c_t = P_t y_t + W_t n_t + ({}_{t-1}B_t) + \sum_{s\geq 1}({}_tQ_{t+s})({}_{t-1}B_{t+s} - {}_tB_{t+s})\\ & \qquad + P_t({}_{t-1}b_t) + \sum_{s\geq 1}({}_tq_{t+s})P_{t+s}({}_{t-1}b_{t+s} - {}_tb_{t+s})\end{aligned}$$

and the first order date-0 responses of consumption, labor supply and welfare to the considered change are, in this case, given by

$$dc_0 = MPCd\Omega + c_0 \left(\sum_{t\geq 0} \epsilon^h_{c_0,q_t} \frac{dq_t}{q_t} + \sum_{t\geq 0} \epsilon^h_{c_0,w_t} \frac{dw_t}{w_t} \right)$$

$$dn_0 = MPNd\Omega + n_0 \left(\sum_{t\geq 0} \epsilon^h_{n_0,q_t} \frac{dq_t}{q_t} + \sum_{t\geq 0} \epsilon^h_{n_0,w_t} \frac{dw_t}{w_t} \right)$$

$$dU = U_{c_0} d\Omega$$

where $\epsilon^h_{x_0,y_t} = \frac{\partial x^h_0}{\partial y_t}\frac{y_t}{x_0}$ for $x \in \{c, n\}$ and $y \in \{q, w\}$ are Hicksian elasticities and $d\Omega = dW - \sum_{t\geq 0} c_t dq_t$, the net-of-consumption wealth change, is given by

$$d\Omega = \underbrace{\sum_{t\geq 0} (q_t y_t) \frac{dy_t}{y_t}}_{\text{Real unearned income change}} + \underbrace{\sum_{t\geq 0} (q_t w_t n_t) \frac{dw_t}{w_t}}_{\text{Real earned income change}}$$

$$+ \underbrace{\sum_{t\geq 0} q_t \left(y_t + w_t n_t + \left(\frac{{}_{-1}B_t}{P_t} \right) + ({}_{-1}b_t) - c_t \right) \frac{dq_t}{q_t}}_{\text{Revaluation of net savings flows}} - \underbrace{\sum_{t\geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0} \right) \frac{dP_t}{P_t}}_{\text{Revaluation of net nominal position}} \quad (A.37)$$

The proof is a generalization of that in section A.2. I omit it here in the interest of space.

Values of all elasticities with separable preferences in a steady-state with no growth. Following once more the steps of section A.2, it is possible to derive the value of Hicksian elasticities for a change at any horizon. Here I just report the values of these elasticities in the case of an infinite horizon model where $\frac{q_s}{q_0} = \beta^s$ and $w_s = w^*$, $\forall s$. These prices correspond to those prevailing in a steady-state with no growth of any such model, and the resulting elasticities are relevant, for example, to determine the impulse responses in many RBC and DSGE models. The first order conditions imply that consumption and labor supply are constant. Let us call the solutions c^* and n^*, respectively. Writing $\vartheta \equiv \frac{w^* n^*}{c^*}$ for the share of earned income in consumption and $\kappa \equiv \frac{\frac{\psi}{\sigma}\vartheta}{1+\frac{\psi}{\sigma}\vartheta} \in (0, 1)$, obtain values of elasticities summarized in table A.1.

Table A.1: Steady-state moments, separable preferences

ϵ^h	q_0	$q_s, s \geq 1$	w_0	$w_s, s \geq 1$	Marg. propensity	
c_0	$-\sigma\beta$	$\sigma(1-\beta)\beta^s$	$\sigma\kappa(1-\beta)$	$\sigma\kappa(1-\beta)\beta^s$	MPC	$(1-\kappa)(1-\beta)$
n_0	$\psi\beta$	$-\psi(1-\beta)\beta^s$	$\psi(1-\kappa(1-\beta))$	$-\psi\kappa(1-\beta)\beta^s$	MPN	$-\frac{1}{w^*}\kappa(1-\beta)$
					MPS	$(1-\beta)$

A.4 Proof of corollary 1

Rewrite equations (A.35) and (A.36) as

$$\begin{aligned} dc &= MPC\left(dY+\psi dw-wdn+URE\frac{dR}{R}-NNP\frac{dP}{P}\right)-\sigma cMPS\frac{dR}{R} \\ wdn-\psi ndw &= wMPN\left(dY+\psi dw-wdn+URE\frac{dR}{R}-NNP\frac{dP}{P}\right)+\psi wnMPS\frac{dR}{R} \end{aligned}$$

Hence

$$wdn-\psi ndw=\frac{1}{1+wMPN}\left\{wMPN\left(dY+URE\frac{dR}{R}-NNP\frac{dP}{P}\right)+\psi wnMPS\frac{dR}{R}\right\}$$

which, inserted into the expression for dc yields

$$dc=MPC\left(1-\frac{wMPN}{1+wMPN}\right)\left(dY+URE\frac{dR}{R}-NNP\frac{dP}{P}\right)-\sigma cMPS\left(1+MPC\frac{\psi wn}{\sigma c}\frac{1}{1+wMPN}\right)\frac{dR}{R}$$

But $MPC\frac{\psi n}{\sigma c}=-MPN$ so this is

$$dc=\left(\frac{MPC}{1+wMPN}\right)\left(dY+URE\frac{dR}{R}-NNP\frac{dP}{P}\right)-\sigma c\frac{MPS}{1+wMPN}\frac{dR}{R}$$

and noting that

$$1+wMPN=MPC+MPS$$

we can finally rewrite this in terms of $\hat{MPC}=\frac{MPC}{MPC+MPS}$ as

$$dc=\hat{MPC}\left(dY+URE\frac{dR}{R}-NNP\frac{dP}{P}\right)-\sigma c\left(1-\hat{MPC}\right)\frac{dR}{R}$$

as claimed.

A.5 Adding durable goods

This section shows the consequences of adding durable goods to the model.

I consider a standard durable goods problem. For simplicity, I ignore labor supply and nominal assets, neither of which interacts with the conclusions below. A consumer maximizes a separable intertemporal utility function

$$\begin{aligned} \max \quad & \sum\beta^t\left\{u\left(C_t\right)+w\left(D_t\right)\right\} \\ \text{s.t.} \quad & C_t+p_tI_t=Y_t+\left({}_{t-1}b_t\right)+\sum_{s\geq 1}\left({}_tq_{t+s}\right)\left({}_{t-1}b_{t+s}-{}_tb_{t+s}\right) \\ & D_t=I_t+D_{t-1}\left(1-\delta\right) \\ & D_{-1},\left\{{}_{-1}b_t\right\} \qquad \text{given} \end{aligned}$$

where C_t is now *nondurable* consumption, D_t is the consumer's stock of durables, and p_t is the relative price of durable goods in period t.

I am interested in the response of the demand for nondurable goods C_t and durables goods I_t, as well as that of total expenditures

$$X_t \equiv C_t + p_t I_t \tag{A.38}$$

to a change in the time-0 nondurable real interest rate R_0 and (potentially) a simultaneous change in the price of durables p_0. As I argue below, the notion of *aggregate demand* makes most sense when the relative price of durables does not change with R_0, but I start by covering the general case in which p_0 can change.

The intertemporal budget constraint reads

$$\sum_{t\geq 0} q_t \left(C_t + p_t I_t\right) = \sum_{t\geq 0} q_t Y_t + \sum_{t\geq 0} q_t \left({}_{-1}b_t\right)$$

Defining $R_t \equiv \frac{q_t}{q_{t+1}}$, the first-order conditions of this problem are, for all $t \geq 0$

$$u'\left(C_t\right) = \beta R_t u'\left(C_{t+1}\right) \tag{A.39}$$

$$w'\left(D_t\right) = u'\left(C_t\right)\left[p_t - \frac{\left(1-\delta\right) p_{t+1}}{R_t}\right] \tag{A.40}$$

Equation (A.39) is the standard Euler equation for nondurable consumption. Equation (A.40) shows that the consumer equates the marginal rate of substitution between the stock of durables and consumption to the user cost of durables, $p_t - \frac{(1-\delta)p_{t+1}}{R_t}$. A fall in the nondurable real interest rate at date 0, R_0, increases the desired level of nondurable consumption and of the stock of nondurables (an intertemporal substitution effect). Holding p_1 constant, it also reduces the user cost of durables, increasing the desired stock of durables relative to nondurable consumption. A fall in p_0 has the same effect of reducing the durable user cost, but it does not affect intertemporal substitution in consumption.

Suppose that the path for interest rates $\{R_t\}$, relative prices $\{p_t\}$ and income $\{Y_t\}$ delivers the solution $\{C_t, D_t\}$. Consider the solution under the alternative paths $\{\overline{R_0}, R_1, R_2 \ldots\}$, $\{\overline{p_0}, p_1, p_2 \ldots\}$, and $\{\overline{Y_0}, Y_1, Y_2 \ldots\}$. Let $dR = \overline{R_0} - R_0$, $dp = \overline{p_0} - p_0$ and $dY = \overline{Y_0} - Y_0$. I am interested in the response of the paths of nondurable and durable expenditures to these changes. To obtain this, I find the paths for consumption $\{C_t\}$ and durables $\{D_t\}$, and then find the implied path for durable expenditures $\{p_t I_t\}$.

Marshallian demand. In order to determine the Marshallian demands, I could follow the same proof as that of section A.2, but here I follow an alternative and somewhat more intuitive procedure. The procedure is in two steps. First, I determine a variation that respects all the first-order conditions (A.39)–(A.40) at the new prices. This gives dC^* and dD^*, which result in a budgetary cost $d\Omega^*$ at the old prices. Second, I determine the change in net wealth $d\Omega$ that results from the change in prices. The Marshaling demands are then

$$dC = dC^* + MPC\left(d\Omega - d\Omega^*\right) \tag{A.41}$$

$$dD = dD^* + MPD\left(d\Omega - d\Omega^*\right) \tag{A.42}$$

where $MPD = \frac{\partial D}{\partial Y}$ is the increase in the stock of date-0 durables that results from a date-0 increase in income. Note that MPC and MPD are related: differentiating (A.40), we find

$$w''(D_0)\, MPD = u''(C_0)\, MPC \left[p_0 - \frac{(1-\delta)\, p_1}{R_0}\right]$$

so

$$MPD = \frac{\sigma_D}{\sigma_C}\frac{D_0}{C_0} MPC$$

where $\sigma_C \equiv -\frac{u'(C_0)}{u''(C_0)C_0}$ and $\sigma_D \equiv -\frac{w'(D_0)}{w''(D_0)D_0}$ are the elasticities of intertemporal substitution in consumption and in the stock of durables. Since $D_0 = I_0 + D_{-1}(1-\delta)$ and the initial stock D_{-1} is fixed, the total constant-p marginal propensity to spend at date 0 is

$$\begin{aligned} MPX \equiv \frac{\partial (C+pI)}{\partial Y} = \frac{\partial C}{\partial Y} + p\frac{\partial D}{\partial Y} &= MPC + pMPD \\ &= MPC\left(1 + \frac{\sigma_D}{\sigma_C}\frac{pD}{C}\right) \end{aligned}$$

Step 1: variation respecting FOCs. The simplest variation that respects all FOCs holds the paths $\{C_t\}$ and $\{D_t\}$ fixed for all $t \geq 1$ and adjusts C_0 and D_0 by dC (respectively dD) such that (A.39) and (A.40) are satisfied at $t = 0$. Differentiating these equations, I obtain

$$\begin{aligned} -\frac{1}{\sigma_C}\frac{dC}{C} &= \frac{dR}{R} \\ -\frac{1}{\sigma_D}\frac{dD}{D} &= -\frac{1}{\sigma_C}\frac{dC}{C} + \frac{p_1\frac{1-\delta}{R}}{p_0 - p_1\frac{1-\delta}{R}}\frac{dR}{R} + \frac{p_0}{p_0 - p_1\frac{1-\delta}{R}}\frac{dp}{p} \end{aligned}$$

Hence we find

$$dC^* = -\sigma_C C\frac{dR}{R} \tag{A.43}$$

and

$$dD^* = -\sigma_D D\left[\frac{p_0}{p_0 - p_1\frac{1-\delta}{R}}\right]\left(\frac{dR}{R} + \frac{dp}{p}\right) \tag{A.44}$$

These responses are very intuitive: one way to respond to a fall in real interest rates is to raise nondurable consumption and the stock of durables. The relevant elasticity for durables is higher than σ_D because of the additional substitution effect coming from the change in the user cost. A lower current relative price of durables has a symmetric effect on the demand for durables as that of a lower real interest rate (in other words, it is the real interest rate in terms of durables that matters for durables demand).

We are now ready to determine the net cost of this variation. Since

$$\begin{aligned} D_0 &= (1-\delta)\, D_{-1} + I_0 \\ D_1 &= (1-\delta)\, D_0 + I_1 \end{aligned}$$

the sequence of investment that achieves this variation consists naturally in an increase of

dD^* followed by a subsequent decrease:

$$\begin{aligned} dI_0^* &= dD^* \\ dI_1^* &= -(1-\delta)\,dD^* \end{aligned}$$

Hence the total budgetary cost of this 'star' variation at the old prices p and R has the simple form

$$\begin{aligned} d\Omega^* &= dC^* + p_0 dI_0^* + p_1 \frac{dI_1^*}{R} \\ &= dC^* + \left(p_0 - p_1 \frac{1-\delta}{R}\right) dD^* \\ &= -\left(\sigma_C C + p_0 \sigma_D D\right) \frac{dR}{R} - \sigma_D p_0 D \frac{dp}{p} \end{aligned}$$

Step 2: change in net wealth. Let Ω be defined as

$$\Omega \equiv \sum_{t\geq 0} q_t \left\{Y_t + ({}_{-1}b_t) - C_t - p_t I_t\right\}.$$

At the initial prices, the intertemporal budget constraint implies $\Omega = 0$. The exogenous variation dR, dp and dY yields

$$\begin{aligned} d\Omega &= dY - Idp + \sum_{t\geq 0} dq_t \left\{Y_t + ({}_{-1}b_t) - C_t - p_t I_t\right\} \\ &= dY - Idp - \sum_{t\geq 1} q_t \left\{Y_t + ({}_{-1}b_t) - C_t - p_t I_t\right\} \frac{dR}{R} \\ &= dY - pI_0 \frac{dp}{p} + \Bigg(\underbrace{Y_0 + ({}_{-1}b_0) - C_0 - p_0 I_0}_{URE}\Bigg) \frac{dR}{R} \end{aligned} \tag{A.45}$$

The intuition is as follows. Suppose that the nondurable real interest rate falls at date 0. As before, this benefits consumers that have a negative URE, that is, maturing liabilities $C_0 + p_0 I_0$ in excess maturing assets $Y_0 + ({}_{-1}b_0)$. Note that, for this effect, total expenditures *including expenditures on durables* are counted as part of URE. In that sense, URE measures the true balance-sheet exposure to a change in the real interest rate. In particular, ceteris paribus, when investment is higher today the consumer benefits more from a fall in real interest rates.

Suppose however that, in parallel, the relative price of durables rises. In the general equilibrium model of Barsky, House and Kimball (2007), for example, this happens in response to an accommodative monetary policy shock when durable goods prices are more flexible than nondurable goods prices. In that case, equation (A.45) shows that there is an additional capital loss on wealth due to the rise in the durable relative price. While conceptually distinct, these two effects could be consolidated into a single one, if we restrict ourselves to variations that feature a constant elasticity of the durable-good price to the nondurable real interest rate

$$\epsilon_{pR} \equiv -\frac{\partial p}{p}\frac{R}{\partial R} \tag{A.46}$$

The benchmark case where p is constant corresponds to $\epsilon_{pR} = 0$, the case where the durable real interest rate is constant to $\epsilon_{pR} = 1$. Then,

$$d\Omega = dY + \left(\underbrace{Y_0 + ({}_{-1}b_0) - C_0 - p_0 I_0 \left(1-\epsilon_{pR}\right)}_{URE^{\epsilon}}\right)\frac{dR}{R} \tag{A.47}$$

In other words, once we net out the capital revaluation effect, an alternative measure of URE becomes URE^{ϵ}, which subtracts a fraction $\left(1-\epsilon_{pR}\right)$ of durable expenditures.

Step 3: demand for durables and nondurables. Combining (A.41)–(A.42) with (A.43), (A.44) and (A.45), I obtain the Marshallian demands (recall that $dI = dD$ at time 0)

$$\begin{aligned}
dC &= MPC\left(dY + URE\frac{dR}{R} + \left(\sigma_C C + p\sigma_D D\right)\frac{dR}{R} + \left(p\sigma_D D - pI_0\right)\frac{dp}{p}\right) - \sigma_C C\frac{dR}{R} \\
dD &= MPD\left(dY + URE\frac{dR}{R} + \left(\sigma_C C + p\sigma_D D\right)\frac{dR}{R} + \left(p\sigma_D D - pI_0\right)\frac{dp}{p}\right) \\
&\quad -\sigma_D D\left[\frac{p_0}{p_0 - p_1\frac{1-\delta}{R}}\right]\left(\frac{dR}{R} + \frac{dp}{p}\right)
\end{aligned}$$

This separates out the separate effects from changing R and p. Given the elasticity ϵ_{pR} in (A.46), we can also rewrite this as

$$\begin{aligned}
dC &= MPC\left(dY + URE^{\epsilon}\frac{dR}{R}\right) - \sigma_C C\left(1-MPC\right)\frac{dR}{R} \\
&\quad +\sigma_D \cdot pD \cdot MPC \cdot \left(1-\epsilon_{pR}\right) \cdot \frac{dR}{R}
\end{aligned} \tag{A.48}$$

$$\begin{aligned}
dD &= MPD\left(dY + URE^{\epsilon}\frac{dR}{R}\right) + \sigma_C \cdot MPD \cdot C \cdot \frac{dR}{R} \\
&\quad -\sigma_D \cdot pD \cdot \left(1-\epsilon_{pR}\right) \cdot \left(1-MPD\right) \cdot \left[\frac{1}{p_0 - p_1\frac{1-\delta}{R}}\right]\frac{dR}{R}
\end{aligned} \tag{A.49}$$

Where URE^{ϵ} is defined in (A.47).

Special case with constant durable real interest rate ($\epsilon_{pR} = 1$). When $\epsilon_{pR} = 1$, equations (A.48)–(A.49) simplify to

$$\begin{aligned}
dC &= MPC\left(dY + URE^{1}\frac{dR}{R}\right) - \sigma_C C\left(1-MPC\right)\frac{dR}{R} \\
dD &= MPD\left(dY + URE^{1}\frac{dR}{R}\right) + \sigma_C \cdot MPD \cdot C \cdot \frac{dR}{R}
\end{aligned}$$

which are simple extensions of expressions in the main text, with URE^1 (which does not subtract durable expenditures) replacing URE. Note that to the extent that $URE^1 \geq 0$, the expression for dD implies a contraction in durable goods from an increase in real interest rates, as in Barsky, House and Kimball (2007). This is counterfactual, suggesting that $\epsilon_{pR} = 1$ may be too high an elasticity in practice.

Special case with constant relative price ($\epsilon_{pR} = 0$). While the cases where $\epsilon_{pR} \neq 0$ are interesting in principle, they prevent a straightforward definition of *aggregate demand* $X = C + pI$: if the relative price of two goods can change, then the relative demands for these two goods (as well as their relative supplies) will matter for general equilibrium. Therefore, the case where $\epsilon_{pR} = 0$ is the most relevant for my purposes. Assume then that $p_0 = p_1 = p$. In this case, we can combine (A.48) and (A.49) to obtain an expression for the change in aggregate demand $dX = dC + pdD$ as a function of the marginal propensity to spend $MPX = MPC + pMPD$ and other variables

$$dX = MPX\left(dY + URE\frac{dR}{R} + \sigma_C C + \sigma_D pD\right) - \left(\sigma_C C + \frac{\sigma_D pD}{1 - \frac{1-\delta}{R}}\right)\frac{dR}{R}$$

This can further be simplified to yield an expression with the same form as the expression in the main text,

$$dX = MPX\left(dY + URE\frac{dR}{R}\right) - \sigma_X\left(1 - MPX\right)X\frac{dR}{R} \tag{A.50}$$

where σ_X is defined as

$$\sigma_X \equiv \frac{C}{X}\cdot\sigma_C + \left(1 - \frac{C}{X}\right)\cdot\sigma_D\cdot\frac{pD}{pI}\cdot\frac{\frac{1}{1-\frac{1-\delta}{R}} - MPX}{1 - MPX} \tag{A.51}$$

In other words, σ_X is a weighted average of σ_C and the relevant elasticity of substitution in durable expenditures: the product of σ_D by the stock-flow ratio $\frac{pD}{pI}$, multiplied by a term that increases in the elasticity of the user cost to the real interest rate.

Quantitatively, the second term is likely to be much larger than the first. If initially durable expenditures cover replacement costs $I = D\delta$, then the stock-flow ratio is $\frac{1}{\delta}$. Hence, with $\delta = 5\%$ and $R = 1.05$ at annual rates, the second term in (A.51) is at least as large as $\frac{1}{20} \times \frac{1}{10} \times \sigma_D = 200\sigma_D$. This makes aggregate demand very sensitive to given changes in the real interest rate because of the large substitution effect that results from the presence of long-lived durables, a point made by Barsky et al. (2007).

A.6 Proof of theorem 2

After dividing through by P_t, defining the real bond position as $\lambda_t \equiv \frac{\Lambda_t}{P_{t-1}}$ and writing $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ for the inflation rate between $t-1$ and t, the budget constraint (9) becomes

$$c_t + Q_t\left(\lambda_{t+1} - \delta\frac{\lambda_t}{\Pi_t}\right) + (\theta_{t+1} - \theta_t)\cdot \mathbf{S}_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot \mathbf{d}_t$$

In this notation, the consumer's date-t net nominal position is

$$NNP_t = (1 + Q_t\delta)\frac{\lambda_t}{\Pi_t}$$

while his unhedged interest rate exposure is:

$$URE_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot \mathbf{d}_t - c_t = Q_t\left(\lambda_{t+1} - \delta\frac{\lambda_t}{\Pi_t}\right) + (\theta_{t+1} - \theta_t)\cdot \mathbf{S}_t$$

His optimization problem can be represented using the recursive formulation

$$\max_{c,n,\lambda',\theta'} u(c) - v(n) + \underbrace{\beta\mathbb{E}\left[V\left(\lambda',\theta';y',w',Q',\Pi',\mathbf{d}',\mathbf{S}'\right)\right]}_{\equiv W(\lambda',\theta')}$$

$$\text{s.t.} \quad c + Q\left(\lambda' - \delta\frac{\lambda}{\Pi}\right) + \left(\theta' - \theta\right)\mathbf{S} = y + wn + \frac{\lambda}{\Pi} + \theta\mathbf{d} \tag{A.52}$$

$$Q\lambda' + \theta'\mathbf{S} \geq \frac{\overline{D}}{R}$$

The function V corresponds to the value from optimizing given a starting real level of bonds λ' and shares θ', and includes the possibility of hitting future borrowing constraints.

I consider the predicted effects on c and n resulting from a simultaneous unexpected change in unearned income dy, the real wage dw, the price level $\frac{dP}{P} = \frac{d\Pi}{\Pi}$ and the real interest rate dR, which result in a change in asset prices $\frac{dQ}{Q} = \frac{dS_j}{S_j} = -\frac{dR}{R}$ for $j = 1 \ldots N$. By leaving the future unaffected, this purely transitory change does not alter the value from future optimization starting at (λ',θ')— that is, the function W is unchanged. I claim that, provided the consumption and labor supply functions are differentiable, their first order differentials are

$$dc = MPC\left(dy + n(1+\psi)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c MPS\frac{dR}{R} \tag{A.53}$$

$$dn = MPN\left(dy + n(1+\psi)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + \psi n MPS\frac{dR}{R} + \psi n\frac{dw}{w} \tag{A.54}$$

where $\sigma \equiv -\frac{u'(c)}{cu''(c)}$ and $\psi = \frac{v'(n)}{nv''(n)}$ are the local elasticities of intertemporal substitution and labor supply, respectively, $MPC = \frac{\partial c}{\partial y}$, $MPN = \frac{\partial n}{\partial y}$ and $MPS = 1 - MPC + wMPN$.

In order to prove (A.53) and (A.54), there are two cases to consider. In the first case, the consumer is at a binding borrowing limit or lives hand-to-mouth. The problem is then a static choice between c and n. In the second case, the consumer is at an interior optimum. The result then follows from application of the implicit function theorem to the set of $N+2$

first-order conditions which, together with the budget constraint, characterize the solution to the problem in (A.52). Here, to simplify the notation and the proof, I first prove the statement in the case where all variables are changing but $N = 0$, and then consider the case with stocks ($N > 0$) but without bonds and assuming only R is changing.

Case 1. Binding borrowing limit and hand-to-mouth agents.

Proof. The consumption of an agent at the borrowing limit is given by

$$c = wn + Z \tag{A.55}$$

where

$$Z = z + (1 + Q\delta)\frac{\lambda}{\Pi} + \theta \cdot (\mathbf{d} + \mathbf{S}) + \frac{\overline{D}}{R}$$

Similarly, the consumption of an agent that lives hand to mouth is

$$c = wn + z$$

Given that $d\mathbf{S} = -\frac{\mathbf{S}}{R}dR$, $dQ = -\frac{Q}{R}dR$ and $d\left(\frac{1}{\Pi}\right) = -\frac{1}{\Pi^2}d\Pi = -\frac{1}{\Pi}\frac{dP}{P}$, we have, if the agent is at the borrowing limit

$$dZ = dz - \underbrace{(1 + Q\delta)\frac{\lambda}{\Pi}}_{\text{NNP}}\frac{dP}{P} + \underbrace{\left(Q\delta\frac{\lambda}{\Pi} + \theta \cdot \mathbf{S} + \frac{\overline{D}}{R}\right)}_{-\text{URE}}\left(-\frac{dR}{R}\right) \tag{A.56}$$

and, if the agent lives hand to mouth,

$$dZ = dz$$

but since that agent also has

$$NNP = URE = 0$$

equation (A.56) still applies. In both cases, the consumer is making a static choice between c and n given the budget constraint (A.55), and hence has $MPS = 0$. We can then apply the results of section A.2 to find

$$\begin{aligned} dc &= MPC\left(dZ + w\left(1 + \psi\right)\right) \\ dn &= MPN\left(dZ + w\left(1 + \psi\right)\right) + \psi n dw \end{aligned}$$

which yields the desired result. □

Case 2a). $N = 0$**, all variables changing** I first prove the following lemma.

Lemma A.1. *Let $c(z, w, q, b)$ and $n(z, w, q, b)$ be the solution to the following separable consumer choice problem under concave preferences over current consumption $u(c)$ and assets $V(a)$, and convex preferences over hours worked $v(n)$:*

$$\begin{aligned} \max \quad & u(c) - v(n) + V(a) \\ s.t. \quad & c + q(a - b) = wn + z \end{aligned}$$

Assume $c()$ and $n()$ are differentiable. Then the first order differentials are

$$\begin{aligned} dc &= MPC\left(dz + n\left(1+\psi\right)dw - \left(a-b\right)dq + qdb\right) - \sigma c MPS\frac{dq}{q} \\ dn &= MPN\left(dz + n\left(1+\psi\right)dw - \left(a-b\right)dq + qdb\right) + \psi n MPS\frac{dq}{q} + \psi n\frac{dw}{w} \end{aligned}$$

where $MPC = \frac{\partial c}{\partial z}$, $MPN = \frac{\partial n}{\partial z}$ and $MPS = 1 - MPC + wMPN = 1 - MPC\left(1 + \frac{wn}{c}\frac{\psi}{\sigma}\right)$.

Proof. The following first-order conditions are necessary and sufficient for optimality:

$$u'(c) = \frac{1}{w}v'(n) = \frac{1}{q}V'(a) \tag{A.57}$$

I first obtain the expression for MPC by considering an increase in income dz alone. Consider how that increase is divided between current consumption, leisure and assets. (A.57) implies

$$u''(c)\,dc = \frac{1}{w}v''(n)\,dn = \frac{1}{q}V''(a)\,da \tag{A.58}$$

where the changes dc, dn and da are related to dz through the budget constraint

$$dc + qda = wdn + dz \tag{A.59}$$

Define $MPC = \frac{\partial c}{\partial z}$, $MPN = \frac{\partial n}{\partial z}$ and $MPS = q\frac{\partial a}{\partial z}$. Then (A.58) implies

$$\begin{aligned} \frac{MPN}{MPC} &= w\frac{u''(c)}{v''(n)} = \frac{u''(c)}{u'(c)}\frac{v'(n)}{v''(n)} = -\frac{n}{c}\frac{\psi}{\sigma} \\ \frac{MPS}{MPC} &= \frac{q^2u''(c)}{V''(a)} = \frac{q}{c}\frac{V'(a)}{\sigma V''(a)} \end{aligned}$$

where $\sigma \equiv -\frac{u'(c)}{cu''(c)}$ and $\psi \equiv \frac{v'(n)}{nv''(n)}$. Hence the total marginal propensity to spend is

$$1 - MPS = \frac{\partial c}{\partial z} - w\frac{\partial n}{\partial z} = MPC\left(1 + \frac{wn}{c}\frac{\psi(n)}{\sigma(c)}\right) = 1 - \frac{q^2u''(c)}{V''(a)}MPC \tag{A.60}$$

and the marginal propensity to consume is

$$MPC = \frac{1}{1 + q^2\frac{u''(c)}{V''(a)} - w^2\frac{u''(c)}{v''(n)}} = \frac{V''(a)\,v''(n)}{V''(a)\,v''(n) + q^2u''(c)\,v''(n) - w^2u''(c)\,V''(a)}$$

Consider now the overall effect on c, n and a of a change in q, w, z and b. Applying the implicit function theorem to the system of equations

$$\begin{cases} v'(n) - wu'(c) = 0 \\ V'(a) - qu'(c) = 0 \\ c + q\left(a-b\right) - wn - z = 0 \end{cases}$$

results in the following expression for partial derivatives:

$$\begin{bmatrix} \frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \\ \frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \\ \frac{\partial a}{\partial q} & \frac{\partial a}{\partial z} & \frac{\partial a}{\partial w} & \frac{\partial a}{\partial b} \end{bmatrix} = -\underbrace{\begin{bmatrix} -wu''(c) & v''(n) & 0 \\ -qu''(c) & 0 & V''(a) \\ 1 & -w & q \end{bmatrix}^{-1}}_{\equiv A} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix} \tag{A.61}$$

now

$$\det(A) = v''(n)V''(a) - w^2u''(c)V''(a) + q^2u''(c)v''(n) = \frac{V''(a)v''(n)}{MPC}$$

and so

$$A^{-1} = \frac{MPC}{V''(a)v''(n)} \begin{bmatrix} wV''(a) & -v''(n)q & v''(n)V''(a) \\ q^2u''(c) + V''(a) & -wqu''(c) & wu''(c)V''(a) \\ qwu''(c) & w^2u''(c) - v''(n) & qu''(c)v''(n) \end{bmatrix}$$

therefore, the first row of (A.61)

$$\begin{bmatrix} \frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \end{bmatrix} = MPC \begin{bmatrix} -\frac{w}{v''(n)} & \frac{q}{V''(a)} & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix} \tag{A.62}$$

Using (A.60) we find

$$-q\frac{u'(c)}{V''(a)}MPC = \frac{\sigma c}{q}q^2\frac{u''(c)}{V''(a)}MPC = \frac{\sigma c}{q}MPS$$

so that the first column of the matrix equation (A.62) reads

$$\frac{\partial c}{\partial q} = \frac{\sigma c}{q}MPS - (a-b)MPC$$

The second and fourth column of (A.62) yield directly

$$\begin{aligned} \frac{\partial c}{\partial z} &= MPC \\ \frac{\partial c}{\partial b} &= qMPC \end{aligned}$$

Finally, using (A.57) we have

$$w\frac{u'(c)}{v''(n)} = \frac{v'(n)}{v''(n)} = \psi n$$

so that the third column of (A.62) reads

$$\begin{aligned} \frac{\partial c}{\partial w} &= MPC\psi n + MPCn \\ &= MPC(1+\psi)n \end{aligned}$$

The first-order total differential dc is then

$$\begin{aligned} dc &= \frac{\partial c}{\partial z}dz + \frac{\partial c}{\partial b}db + \frac{\partial c}{\partial q}dq + \frac{\partial c}{\partial w}dw \\ &= MPC\left(dz + qdb - (a-b)\,dq + (1+\psi)\,ndw\right) + \sigma c MPS\frac{dq}{q} \end{aligned} \tag{A.63}$$

as claimed. Similarly, after using $MPN = MPCw\frac{u''(c)}{v''(n)}$, the second row of (A.61) is

$$\begin{bmatrix} \frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \end{bmatrix} = MPN \begin{bmatrix} -\frac{q^2+V''(a)/u''(c)}{wV''(a)} & \frac{q}{V''(a)} & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix} \tag{A.64}$$

Using (A.60) we find

$$-q\frac{u'(c)}{V''(a)}MPN = \frac{\sigma c}{q}q^2\frac{u''(c)}{V''(a)}MPC\left(\frac{-n\psi}{\sigma c}\right) = -\frac{n\psi}{q}MPS$$

Again the first column yields

$$\frac{\partial c}{\partial q} = -\frac{n\psi}{q}MPS - (a-b)\,MPN$$

The second and fourth column of (A.62) yield directly

$$\begin{aligned} \frac{\partial c}{\partial z} &= MPN \\ \frac{\partial c}{\partial b} &= qMPN \end{aligned}$$

Finally, since

$$\begin{aligned} \left(q^2 + \frac{V''(a)}{u''(c)}\right)\frac{u'(c)}{V''(a)} &= -\sigma c\left(q^2\frac{u''(c)}{V''(a)} + 1\right) \\ &= \psi n\frac{MPC}{MPN}\left(\frac{MPS}{MPC} + 1\right) \end{aligned}$$

the third column yields

$$\frac{\partial n}{\partial w} = \frac{1}{w}\psi n\left(MPS + MPC\right) + MPNn = \frac{1}{w}\psi n\left(1 + wMPN\right) + MPNn = \psi n\frac{1}{w} + MPN\left(n + \psi n\right)$$

The first-order total differential dn is then

$$\begin{aligned} dn &= \frac{\partial n}{\partial z}dz + \frac{\partial n}{\partial b}db + \frac{\partial n}{\partial q}dq + \frac{\partial n}{\partial w}dw \\ &= MPN\left(dz + qdb - (a-b)\,dq + (1+\psi)\,ndw\right) - \psi nMPS\frac{dq}{q} + \psi n\frac{dw}{w} \end{aligned} \tag{A.65}$$

□

Proof of theorem 2 in case 2a). If the policy functions are differentiable and the consumer is at an interior optimum, then the conditions of lemma A.1 are satisfied: the borrowing constraint is not binding so can be ignored, and the value function is concave per standard dynamic programming arguments. The notation of theorem 2 can be cast using that of the lemma by using the mapping

$$q \equiv Q \quad z \equiv y + \frac{\lambda}{\Pi} \quad a \equiv \lambda' \quad b \equiv \delta \frac{\lambda}{\Pi}$$

with $\frac{dP}{P} = \frac{d\Pi}{\Pi}$ and $\frac{dQ}{Q} = -\frac{dR}{R}$. Hence $dz = dy - \frac{\lambda}{\Pi}\frac{dP}{P}$, $db = -\delta \frac{\lambda}{\Pi}\frac{dP}{P}$ and $\frac{dq}{q} = -\frac{dR}{R}$; so

$$dz + qdb - (a-b)\, dq = dy - \underbrace{(1+Q\delta)\frac{\lambda}{\Pi}}_{\text{NNP}}\frac{dP}{P} + \underbrace{\left(\lambda' - \delta\frac{\lambda}{\Pi}\right) Q}_{\text{URE}}\frac{dR}{R}$$

Inserting this equation into (A.63) and (A.65) yields the desired result. □

Case 2b) $N > 0$, no bonds, only R changing. Since we are not considering changes in wages, it is sufficient to restrict the analysis to a choice between consumption and assets. The following lemma then proves the result for dc. The result for dn follows as a straightforward extension.

Lemma A.2. *Let $c(\theta, Y, R)$ be the solution to the following consumer choice problem under concave preferences over current consumption $u(c)$ and assets $W(\theta')$*

$$\begin{array}{ll} \max_{c,\theta'} & u(c) + W(\theta') \\ s.t. & c + (\theta' - \theta)\,\mathbf{S} = Y + \theta \mathbf{d} \end{array}$$

where $\frac{d\mathbf{S}}{dR} = -\frac{\mathbf{S}}{R}$. Then, to first order

$$dc = MPC\left(dY + URE\frac{dR}{R}\right) - \sigma(c)\, c\,(1 - MPC)\frac{dR}{R}$$

where $\sigma(c) \equiv -\frac{u'(c)}{cu''(c)}$ is the local elasticity of intertemporal substitution, $MPC = \frac{\partial c}{\partial Y}$, and $URE = Y + \theta \mathbf{d} - c$

Proof. The following first-order conditions characterize the solution

$$S^i u'\left(Y + \theta\mathbf{d} - (\theta' - \theta)\,\mathbf{S}\right) = W_{\theta^i}(\theta') \quad \forall i = 1 \ldots N \tag{A.66}$$

Consider first an increase in income dY alone. Differentiating along (A.66) we find

$$S^i u''(c)\left(1 - \sum_j S^j \frac{d\theta'^j}{dY}\right) = \sum_j W_{\theta^i\theta^j}(\theta')\frac{d\theta'^j}{dY} \quad \forall i \tag{A.67}$$

Define $\eta^j \equiv S^j \frac{d\theta'^j}{dY}$. Then (A.67) rewrites

$$\sum_j \left(\frac{1}{S^i S^j} W_{\theta^i\theta^j}(\theta') + u''(c)\right)\eta^j = u''(c) \quad \forall i$$

Defining the matrix M with elements

$$m_{ij} \equiv \frac{1}{S^i S^j} W_{\theta^i\theta^j}(\theta') + u''(c)$$

this system can also be written in matrix form as

$$M\eta = u''(c)\,\mathbf{1}$$

or

$$\eta = u''(c)\,M^{-1}\mathbf{1}$$

The budget constraint then implies that

$$MPC = \frac{dc}{dY} = 1 - \sum_j \eta^j = 1 - u''(c)\,m \tag{A.68}$$

where m is defined as

$$m \equiv \mathbf{1}M^{-1}\mathbf{1} \tag{A.69}$$

Next, consider an increase in the real interest rate dR. Differentiating along (A.66) we now have

$$\frac{dS^i}{dR}u'(c) + S^i u''(c)\left(-\sum_j S^j \frac{d\theta'^j}{dR} - \sum_j \frac{dS^j}{dR}\left(\theta'^j - \theta^j\right)\right) = \sum_j W_{\theta^i\theta^j}(\theta')\frac{d\theta'^j}{dR} \quad \forall i$$

Using $\frac{dS^i}{S^i} = -\frac{dR}{R}$ this rewrites

$$-\frac{S^i}{R}u'(c) + S^i u''(c)\left(-\sum_j S^j \frac{d\theta'^j}{dR} + \sum_j \frac{S^j}{R}\left(\theta'^j - \theta^j\right)\right) = \sum_j W_{\theta^i\theta^j}(\theta')\frac{d\theta'^j}{dR} \quad \forall i \tag{A.70}$$

Defining now $\gamma^j \equiv S^j\frac{d\theta^{j\prime}}{dR}$, (A.70) shows that γ^j solves

$$\sum_j m_{ij}\gamma^j = -\frac{1}{R}u'(c) + u''(c)\sum_j \frac{S^j}{R}\left(\theta'^j - \theta^j\right) \quad \forall i$$

which rewrites in matrix form

$$M\gamma = \left(-\frac{1}{R}u'(c) + u''(c)\sum_j \frac{S^j}{R}\left(\theta'^j - \theta^j\right)\right)\mathbf{1}$$

or

$$\gamma = \left(-\frac{1}{R}u'(c) + u''(c)\sum_j \frac{S^j}{R}\left(\theta'^j - \theta^j\right)\right)M^{-1}\mathbf{1} \tag{A.71}$$

Differentiating with respect to R along the budget constraint $c = Y + \theta\mathbf{d} - (\theta' - \theta)\,\mathbf{S}$, we next see that

$$\frac{dc}{dR} = -\sum_j S^j\frac{\theta'^j}{dR} + \sum_j \frac{S^j}{R}\left(\theta^j - \theta'^j\right) = -\sum_j \gamma^j + \sum_j \frac{S^j}{R}\left(\theta^j - \theta'^j\right)$$

inserting (A.71) and using the definition of m,

$$\frac{dc}{dR} = -\left(-\frac{1}{R}u'(c) + u''(c)\sum_j \frac{S^j}{R}\left(\theta'^j - \theta^j\right)\right)m + \sum_j \frac{S^j}{R}\left(\theta^j - \theta'^j\right) \tag{A.72}$$

rearranging terms and using $u'(c) \equiv -c\sigma(c)\,u''(c)$ we find

$$\frac{dc}{dR} = -\sigma(c)\frac{c}{R}u''(c)\,m + \sum_j \frac{S^j}{R}\left(\theta'^j - \theta^j\right)\left(1 - u''(c)\,m\right)$$

But using the expression for MPC in (A.68), this is simply

$$\frac{dc}{dR} = -\sigma(c)\frac{c}{R}(1 - MPC) + \sum_j \frac{S^j}{R}\left(\theta^{j\prime} - \theta^j\right) MPC$$

and using the budget constraint $\sum_j S^j\left(\theta^{j\prime} - \theta^j\right) = (\theta' - \theta)\cdot \mathbf{S}_t = URE$ we obtain

$$\frac{dc}{dR} = -\sigma(c)\frac{c}{R}(1 - MPC) + \frac{1}{R}URE \cdot MPC \tag{A.73}$$

Finally, considering a simultaneous change in income and the real interest rate, combining (A.68) and (A.73) we obtain the first order differential

$$dc = MPC\left(dY + URE\frac{dR}{R}\right) - \sigma(c)\,c\,(1 - MPC)\frac{dR}{R}$$

as was to be shown. □

A.7 Proof of theorem 3

Given the assumption of fixed balance sheets and purely transitory shocks, Theorem 2 shows that

$$dc_i = M\hat{P}C_i\left(dY_i - dt_i + URE_i\frac{dR}{R} - NNP_i\frac{dP}{P}\right) - \sigma_i c_i\left(1 - M\hat{P}C_i\right)\frac{dR}{R}$$

where, where $dY_i = n_i e_i dw + we_i dn_i + d(d_i)$ is the change in gross income at the individual level and dt_i the change in taxes. We can further decompose the change in gross income as

$$dY_i = \frac{Y_i}{Y}dY + dY_i - \frac{Y_i}{Y}dY$$

and note that, since $\mathbb{E}_I[Y_i] = Y$,

$$\mathbb{E}_I\left[dY_i - \frac{Y_i}{Y}dY\right] = dY - \frac{\mathbb{E}_I[Y_i]}{Y}dY = 0 \tag{A.74}$$

Hence,

$$dc_i = M\hat{P}C_i\left(\frac{Y_i}{Y}dY + dY_i - \frac{Y_i}{Y}dY - dt_i + URE_i\frac{dR}{R} - NNP_i\frac{dP}{P}\right) - \sigma_i c_i\left(1 - M\hat{P}C_i\right)\frac{dR}{R}$$

and taking a cross-sectional average

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathbb{E}_I\left[M\hat{P}C_i\left(dY_i - \frac{Y_i}{Y}dY\right)\right] - \mathbb{E}_I\left[M\hat{P}C_i\,(dt_i)\right] - \mathbb{E}_I\left[M\hat{P}C_i NNP_i\right]\frac{dP}{P} \\ &\quad + \left(\mathbb{E}_I\left[M\hat{P}C_i URE_i\right] - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} \end{aligned} \tag{A.75}$$

Now, the government budget (13) with the fiscal rule $G_t = \overline{G}$ and target $\frac{B_t}{P_t} = \overline{b}$ reads

$$\mathbb{E}_I[t_{it}] = \overline{G} + \frac{B_t}{P_t} - \frac{\overline{b}}{R_t}$$

Using the fact that at the margin, taxes are adjusted lump-sum, and the fact that $NNP_g = -\overline{b}$ as well as $URE_g = -\frac{\overline{b}}{R}$, this implies

$$dt_i = dt = NNP_g \frac{dP}{P} - URE_g \frac{dR}{R}$$

In other words, taxes fall with unexpected increases in prices which reduce the government debt burden, and they fall with reductions in real interest rates which reduces the government's debt servicing costs. But the market clearing conditions (17) and (18) imply that these gains and losses have counterparts at the household level:

$$dt_i = dt = -\mathbb{E}_I[NNP_i] \frac{dP}{P} + \mathbb{E}_I[URE_i] \frac{dR}{R} \tag{A.76}$$

Hence, (A.75) rewrites

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y} M\hat{P}C_i\right] dY + \mathbb{E}_I\left[M\hat{P}C_i\left(dY_i - \frac{Y_i}{Y} dY\right)\right] - \mathbb{E}_I\left[M\hat{P}C_i\right](dt) - \mathbb{E}_I\left[M\hat{P}C_i NNP_i\right] \frac{dP}{P} \\ &\quad + \left(\mathbb{E}_I\left[M\hat{P}C_i URE_i\right] - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right) c_i\right]\right) \frac{dR}{R} \end{aligned}$$

so

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y} M\hat{P}C_i\right] dY + \mathbb{E}_I\left[M\hat{P}C_i\left(dY_i - \frac{Y_i}{Y} dY\right)\right] + \mathbb{E}_I\left[M\hat{P}C_i\right]\mathbb{E}_I[NNP_i] \frac{dP}{P} - \mathbb{E}_I\left[M\hat{P}C_i NNP_i\right] \frac{dP}{P} \\ &\quad + \left(\mathbb{E}_I\left[M\hat{P}C_i URE_i\right] - \mathbb{E}_I\left[M\hat{P}C_i\right]\mathbb{E}_I[URE_i] - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right) c_i\right]\right) \frac{dR}{R} \end{aligned}$$

and finally, using (A.74)

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y} M\hat{P}C_i\right] dY + \mathrm{Cov}_I\left(M\hat{P}C_i, dY_i - Y_i \frac{dY}{Y}\right) - \mathrm{Cov}_I\left(M\hat{P}C_i, NNP_i\right) \frac{dP}{P} \\ &\quad + \left(\mathrm{Cov}_I\left(M\hat{P}C_i, URE_i\right) - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right) c_i\right]\right) \frac{dR}{R} \end{aligned}$$

as claimed.

Case with heterogeneous taxes. If the taxes were not lump-sum, equation (A.76) would be replaced by

$$\mathbb{E}_I[dt_i] = -\mathbb{E}_I[NNP_i] \frac{dP}{P} + \mathbb{E}_I[URE_i] \frac{dR}{R}$$

we would therefore use the fact that

$$\mathbb{E}_I\left[M\hat{P}C_i(dt_i)\right] = \mathbb{E}_I\left[M\hat{P}C_i\right]\mathbb{E}_I[dt_i] + \mathrm{Cov}_I\left(M\hat{P}C_i, dt_i\right)$$

to finally obtain

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathrm{Cov}_I\left(M\hat{P}C_i, dY_i - Y_i\frac{dY}{Y}\right) - \mathrm{Cov}_I\left(M\hat{P}C_i, NNP_i\right)\frac{dP}{P} \\ &\quad + \left(\mathrm{Cov}_I\left(M\hat{P}C_i, URE_i\right) - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} - \mathrm{Cov}_I\left(M\hat{P}C_i, dt_i\right) \end{aligned}$$

The additional heterogeneous-taxation term is very natural. Suppose for example that, at the margin, gains from the government budget ($\mathbb{E}_I\left[dt_i\right] < 0$) lead to disproportionate reductions of taxes on high-MPC agents. Then $\mathrm{Cov}_I\left(M\hat{P}C_i, dt_i\right) < 0$, so aggregate consumption increases by more than the benchmark from Theorem 1. The opposite happens when tax reductions fall disproportionately on low-MPC agents.

A.8 Proof of corollary 2

From the definition of γ_i in (24), we have

$$d\left(\frac{Y_i}{Y}\right) = \gamma_i\left(\frac{Y_i}{Y} - 1\right)\frac{dY}{Y}$$

Moreover,

$$dY_i - Y_i\frac{dY}{Y} = Yd\left(\frac{Y_i}{Y}\right) = \gamma_i\left(\frac{Y_i}{Y} - 1\right)dY \tag{A.77}$$

Next, rewrite equation (19) in elasticity terms by dividing by per-capita consumption $C = \mathbb{E}_I\left[c_i\right]$ and using (A.77). We find

$$\begin{aligned} \frac{dC}{C} &= \underbrace{\mathbb{E}_I\left[\frac{Y_i}{\mathbb{E}_I\left[c_i\right]}M\hat{P}C_i\right]\frac{dY}{Y}}_{\text{Aggregate income channel}} + \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \gamma_i\frac{Y_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\text{Earnings heterogeneity channel}}\frac{dY}{Y} - \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{NNP_i}{\mathbb{E}_I\left[c_i\right]}\right)\frac{dP}{P}}_{\text{Fisher channel}} \\ &\quad + \left(\underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{URE_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\text{Interest rate exposure channel}} - \underbrace{\mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)\frac{c_i}{\mathbb{E}_I\left[c_i\right]}\right]}_{\text{Substitution channel}}\right)\frac{dR}{R} \end{aligned}$$

Imposing $\gamma_i = \gamma$ and $\sigma_i = \sigma$ for all i, this equation writes

$$\begin{aligned} \frac{dC}{C} &= \underbrace{\mathbb{E}_I\left[\frac{Y_i}{\mathbb{E}_I\left[c_i\right]}M\hat{P}C_i\right]}_{\mathcal{M}}\frac{dY}{Y} + \gamma \times \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{Y_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\mathcal{E}_Y}\frac{dY}{Y} - \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{NNP_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\mathcal{E}_P}\frac{dP}{P} \\ &\quad + \left(\underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{URE_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\mathcal{E}_R} - \sigma \times \underbrace{\mathbb{E}_I\left[\left(1 - M\hat{P}C_i\right)\frac{c_i}{\mathbb{E}_I\left[c_i\right]}\right]}_{\mathcal{S}}\right)\frac{dR}{R} \end{aligned}$$

which is equation (25).

B Data appendix

This section starts out by providing more details about the data and the MPC identification strategies for the SHIW (section B.1), the PSID (section B.2), and the CE (section B.3). Section B.4 contrasts the financial asset and liability information available in the PSID and the CE, and compares it to that available in the Survey of Consumer Finance (SCF).

Section B.5 performs a sensitivity analysis along several dimensions. Section B.5.1 considers the consequence of using total consumption expenditure to estimate MPC in the PSID and CE. Section B.5.2 considers the effect of excluding durable expenditures from URE. Section B.5.3 considers alternative maturity assumptions for assets and liabilities. Section B.5.4 considers robustness to the number of bins used to stratify the population in the PSID and in the CE. Finally, section B.5.5 considers alternative sample selections with respect to age in the PSID.

Section B.6 then cuts the data in various ways to examine the empirical drivers of the correlations in the data. Section B.6.1 looks at the influence of age, and section B.6.2 examines the role of income. Finally, section B.6.3 generalizes my covariance decomposition procedure from section 4.4 to multiple covariates and reports the decomposition when all of Jappelli and Pistaferri (2014)'s covariates are included.

B.1 SHIW

My first dataset comes from the 2010 wave of the Italian Survey of Household Income and Wealth, which is publicly available from the Bank of Italy's website. This is the data source employed by Jappelli and Pistaferri (2014), and it is very useful for my purposes because it contains a direct household-level measure of MPC, reported as part of a survey question.[47] An additional benefit of this dataset is that it presents detailed information on financial assets and liabilities, allowing a fairly precise measurement of URE and NNP for each household.

B.1.1 Exposure measures

Following the structure of the survey, I measure all my statistics at an annual frequency.[48] Table B.1 presents summary statistics in euros.

[47] "Imagine you unexpectedly receive a reimbursement equal to the amount your household earns in a month. How much of it would you save and how much would you spend? Please give the percentage you would save and the percentage you would spend."

[48] Note that the time frame for *MPC* is not specified in the question, as issue that is left unresolved in Jappelli and Pistaferri (2014). A follow-up question in the 2012 SHIW separates durable and nondurable consumption, and specifies the time frame as a full year. The equivalent "MPC" out of both durable and nondurable consumption has close to the same distribution as that of *MPC* in the 2010 SHIW (respective means are 47 in 2010 and 45 in 2010) which suggests that households tended to assume that the question referred to the full year.

Table B.1: Summary statistics in the SHIW

	N	mean	p5	p25	p50	p75	p95
Income	7,951	36,114	9,565	19,857	30,719	45,340	81,320
Consumption	7,951	27,541	10,600	16,800	24,000	32,900	56,500
Maturing assets	7,951	27,073	0	2,000	10,000	30,000	97,929
Maturing liabilities	7,951	9,440	0	0	0	305	49,000
URE	7,951	26,207	-24,787	2,490	16,214	39,063	113,834
Nominal assets	7,951	22,499	0	1,274	6,796	22,000	77,272
Nominal liabilities	7,951	15,133	0	0	0	4,285	99,000
Net Nominal Position	7,951	7,366	-81,712	-1	3,830	17,115	71,218
MPC	7,951	0.47	0.00	0.20	0.50	0.80	1.00

Units: 2010 Euros. All statistics are computed using survey weights.

URE: $Y - T - C + A - L$**.** To construct my measure of unhedged interest rate exposure, I use net annual disposable income (which includes taxes, transfers and capital returns) as my measure of income net of taxes $Y - T$. My consumption measure C includes expenditures on both durables and non durables goods, but it does not include interest payments, which are not reported separately from principal payments, and which I therefore count as part of L.

For assets maturing in the year (A), I consider the amounts held in checking accounts, savings accounts, certificates of deposits, and repurchase agreements (maturing equities and bonds are already included as the dividend income part of $Y - T$). I consider various scenarios for maturities. Given an assumed maturity of N_j years for a given asset or liability j, I scale the observed amounts by $\frac{1}{N_j}$ to obtain an annual measure of maturing flows. In my benchmark scenario, I assume that these assets have a duration of two quarters ($N_j = \frac{1}{2}$).

As part of liabilities maturing in the year (L), I include payments on all loans. The SHIW records up to three mortgages for each households. I add to principal payments the principal *balance* outstanding on adjustable rate mortgages, assuming a duration of three quarters ($N_j = \frac{3}{4}$). I also include all debt outstanding on credit cards, assuming a duration of two quarters.

Section B performs a sensitivity analysis around these maturity assumptions.

NNP and Income. To construct my measure of net nominal position, I include in nominal assets the full amount held in checking accounts, savings accounts, certificates of deposits and repurchase agreements. I also include the full amounts held in bonds from Italian banks and firms, with the exception of inflation-indexed BTP bonds. I assume that two-thirds of foreign bonds are denominated in euros, and count that amount in nominal assets. I then include all the shares of money market mutual funds and bonds mutual funds, in keeping with Doepke and Schneider (2006). For shares held at 'mixed' mutual funds, I assume that half of those are indirectly invested in bonds. Finally, I count all credit originating from commercials or private party loans.

For nominal liabilities, my measure includes all debt due to banks, other financial institutions, and other households, as well as commercial loans.

My results are not influenced in any meaningful way by altering the share of 'mixed' mutual funds invested in bonds, the share of foreign bonds that are euro-denominated, or by excluding commercials and private party loans from both nominal assets and liabilities.

For my income exposure measure Y, since the SHIW does not provide information on government taxes and transfers, I assume that the exposure is based on net rather than gross income. Other assumptions, such as assuming a constant tax rate, only have a minor influence on the size of the relevant covariance.

B.2 Panel Study of Income Dynamics

For the Panel Study of Income Dynamics website at the University of Michigan, I assemble a dataset with household-level information on consumption, income, assets and liabilities. This base file, together with a data dictionary, is included in the replication folder. The procedure to identify MPC out of transitory income shocks that I employ in this section originates from the contribution of Blundell, Pistaferri and Preston (2008) (BPP). It has been used by Kaplan, Violante and Weidner (2014) to estimate the MPCs of hand-to-mouth households, and by Berger et al. (2015) to estimate MPCs at different levels of housing wealth. My sample selection closely follows these papers. Since the PSID only starts recording detailed consumption information in 1999, my sample period starts with the 1999 wave, and ends in 2013. I use the core sample of the PSID (made up of the SCR, SEO and Immigrant samples) and drop households with missing information on the head's race, education or the state of residence. I then drop households whose income or consumption grows more than 500%, falls by more than 80%, or is below $100 in any period. I treat top-coded income or consumption data as missing data.

While the literature usually restricts the sample to working-age households, in my benchmark scenario I keep all families whose head is between 20 and 90 years old, in order to have a more accurate picture of the cross-sectional distribution of UREs and NNPs by age.[49] This sample selection leaves me with 38,143 observations from 9,620 different households.

[49] As we know from Doepke and Schneider (2006), young and old households tend to have the largest net nominal positions, with opposite signs. Since households' income processes tend to change upon entering retirement, however, including older households could lead to noisier estimates of MPCs. For this reason, in section B.5.5 I provide results for my elasticity estimates that restrict the PSID sample to households aged 25 to 55 years old, as is common in the literature.

Table B.2: Summary statistics in the PSID

	N	mean	p5	p25	p50	p75	p95
Net Income	38,143	60,753	13,722	29,546	47,914	74,659	138,136
Consumption	38,143	28,556	9,640	16,937	24,604	34,939	60,662
Maturing assets	38,143	41,639	0	828	6,593	25,761	175,813
Maturing liabilities	38,143	23,008	0	2,289	10,308	22,638	72,213
URE	38,143	50,828	-40,105	1,642	18,945	53,658	225,344
Nominal assets	38,143	40,221	0	600	5,000	25,761	185,000
Nominal liabilities	38,143	77,546	0	382	27,632	118,499	292,862
Net Nominal Position	38,143	-37,324	-260,753	-94,686	-11,973	1,740	127,464
Pre-govtt income	38,143	76,302	8,912	31,045	56,961	94,577	187,254

Units: 2009 USD. All statistics are computed using survey weights.

B.2.1 Exposure measures

Just as for the SHIW, I follow the structure of the PSID survey and do all my measurement at annual frequency. The PSID groups assets and liabilities into coarse categories, so I sometimes need to take a stand on their internal composition. I deflate all monetary variables to 2009 dollars using the CPI in order to ensure comparability over time. Table B.2 reports summary statistics in 2009 dollars.

URE: $Y - T - C + A - L$**.** For URE, I use an annual measure of net disposable income for $Y - T$ (which includes capital returns), and an annual consumption measure C that includes only the consumption categories continuously available in the survey since 1999 (my first sample year). Those consists of expenditures on food, rent, property taxes, home insurance, utilities, telecommunications, transportations, education, childcare and healthcare. Loan repayments are included in L, since—just like in the SHIW—interest expenses are not reported separately from principal payments.

For assets maturing in the year (A), the PSID contains a variable that groups together checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T–bills. In my benchmark scenario, I assume a duration of two quarters for this asset category.

For the remainder of liabilities (L), the PSID reports up to two mortgages for each household. In my benchmark scenario I assume that the duration for ARMs is three quarters. The PSID also contains a variable that includes credit cards debt, student loans, medical bills, legal debt and loan from relatives. From 2011 onwards, a breakdown of categories is available, and credit cards account for an average of 40% of the total. I assume that this fraction has been constant over time, and maintain my assumption of two quarter duration for credit card debt.

NNP and Income. To construct a household's net nominal position, I count as nominal assets all the amount held in checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T–bills. The PSID contains another variable that includes bonds, trusts, estates, cash value of life insurance and collection. I assume that half of that is constituted by bonds, and include this in nominal assets as well. I include the whole amount in IRAs invested in bonds, and half the amount in IRAs invested in a mix of stocks and bonds.

For nominal liabilities, I count the principal balance outstanding on each mortgage and the whole amount due in the form of credit cards debt, student loans, medical bills, legal debt and loan from relatives.

For my income exposure measure, I use the PSID's measure of gross income before taxes and government transfers.

B.2.2 Identification of MPC

As mentioned in main text, the literature exploits the panel dimension of the data in PSID in order to estimate the MPC out of transitory income shocks. I follow BPP and construct my consumption measure for MPC using all non durable consumption categories.[50] For my income measure, I use labor income plus government transfers, as in Kaplan, Violante and Weidner (2014). Following BPP, I first regress the log of consumption and the log of income on observables characteristics of the households, including dummy variables for year, year of birth, education, race, family structure, employment status and region, as well as dummies for interactions between year with education, race, employment status and region. I then use the residuals of these regressions (call them y_{it} and c_{it}) to estimate the MPC out of transitory income shocks. Specifically, for each exposure measure, in each year, I stratify the population in J bins. I then estimate $\psi_j = \frac{\text{Cov}_j(\triangle c_t, \triangle y_{t+1})}{\text{Cov}_j(\triangle y_t, \triangle y_{t+1})}$ as the pass-through coefficient of log income on log consumption, pooling all years together.[51] I finally recover a measure of the marginal propensity to consume MPC_j by multiplying ψ_j by the ratio of average consumption to average income in each bin j.

Next, for each exposure measure, I calculate the average value of exposure in each bin, EXP_j, normalized by average consumption in the sample. I finally compute my estimators as[52]

[50] This is also consistent with Kaplan et al. (2014) and Berger et al. (2015). In section B.5.1, I report instead an MPC calculated using all consumption expenditures available in the PSID.

[51] See Blundell et al. (2008) and Kaplan et al. (2014) for the structural assumptions under which this procedure correctly recovers the MPC out of transitory income shocks. The estimate can, of course, be recovered with an instrumental variable regression of $\triangle c_t$ on $\triangle y_t$, using $\triangle y_{t+1}$ as an instrument.

[52] Note that I simply take $\widehat{S}$ to be the sample counterpart to $1 - \mathbb{E}_I [MPC]$. The procedure cannot simultaneously recover an estimate of the covariance between MPC and consumption. In the SHIW data, the difference between average MPC and consumption-weighted MPC is small, so this is unlikely to significantly affect the value of S.

$$\widehat{\mathcal{E}_{EXP}^{NR}} = \frac{1}{J}\sum_{j=1}^{J} MPC_j EXP_j$$

$$\widehat{\mathcal{E}_{EXP}} = \widehat{\mathcal{E}_{EXP}^{NR}} - \left(\frac{1}{J}\sum_{j=1}^{J} MPC_j\right)\left(\frac{1}{J}\sum_{j=1}^{J} EXP_j\right)$$

$$\widehat{S} = 1 - \left(\frac{1}{J}\sum_{j=1}^{J} MPC_j\right)$$

In order to take into account sampling uncertainty, I compute the distribution of these estimators using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section B.5.4 considers robustness to using $J = 3$ to 8 bins to stratify the sample.

B.3 Consumer Expenditure Survey, 2001-2002 (JPS sample)

My data for the Consumer Expenditure Survey comes from the Johnson, Parker and Souleles (2006) (JPS) dataset, which I merged with the main survey data and detailed expenditure files to obtain additional information on households's consumption expenditures, financial assets and liabilities. The dataset covers households with interviews between February 2001 and March 2002. Relative to the full CE sample, JPS drop the bottom 1% of nondurable expenditure in levels, households living in student housing, those with age less than 21 or greater than 85, those with age changing by more than a unit or by a negative amount between quarters, and those whose family size changes by more than three members between quarters. Since the 2001 CE survey has several observations with missing values for income—which is a crucial component of URE and a measure of exposure in its own right—I do not consider observations with incomplete income information when analyzing the interest rate exposure or the earnings heterogeneity channel. My sample is therefore made of 9,983 observations from 4,833 different households when computing statistics relevant to these two channels, and contains 12,227 observations from 5,900 households when analyzing the Fisher channel.

B.3.1 Exposure measures

Following the structure of the dataset, all my CE measurement is performed at a quarterly frequency. Table B.3 presents summary statistics in dollars.

URE: $Y - T - C + A - L$**.** In order to construct my quarterly measure of URE, I use one fourth of the annual net disposable income as my measure of income $Y - T$, while for

Table B.3: Summary statistics in the CE

	N	mean	p5	p25	p50	p75	p95
Net income	9,983	11,621	1,445	4,550	8,995	15,643	29,706
Consumption	9,983	9,993	2,187	4,633	7,625	12,514	26,205
Maturing assets	9,983	4,796	0	0	60	1,769	21,000
Maturing liabilities	9,983	5,334	0	0	782	2,954	35,502
URE	9,983	1,582	-28,561	-2,874	596	4,916	26,890
Nominal assets	12,227	19,006	0	0	9	5,000	100,000
Nominal liabilities	12,227	49,671	0	0	12,786	73,951	200,794
Net Nominal Position	12,227	-27,859	-174,318	-58,441	-6,801	0	58,078
Pre govt. income	9,983	12,520	1,731	4,814	9,500	16,750	32,500

Units: 2001 USD. All statistics are computed using survey weights.

consumption C I use a quarterly measure of total expenditures that include both durables and non durables goods.

For maturing financial assets A, as in the other two surveys, I assume that checking accounts and savings accounts have a duration of two quarters.

For maturing liabilities, as in the SHIW and PSID, my benchmark assumptions is that the duration of an adjustable rate mortgages is three quarters, and that credit card debt has a two quarter duration. Relative to those surveys, the CE also contains information on adjustable-rate home equity loans, for which I also assume a three quarter duration. As before, I also include all principal payments carried out in the period towards my measure of L.

NNP and Income. To construct my NNP measure, I include in nominal assets all the amount in savings and checking accounts. I then assume that the variable 'securities', which contains the amount held in stocks, mutual funds, private sector bonds, government bonds or Treasury notes, contains a 50% share of bonds, and include those in my measurement. I also count all the amount held in US savings bonds and in private party loans owed.

Using the supplemental expenditure files, my measure of nominal liabilities is fairly detailed. I take the sum of principal balances outstanding on mortgages, home equity loans, home equity line of credit, loans on vehicles, personal debt and credit card debt.

For my income exposure measure, I use an annual measure of gross income before taxes, converted to quarterly value.

B.3.2 MPC identification strategy

JPS identified the propensity to consume out of the 2001 tax rebate by exploiting random variation in the timing of its receipt across households. In this section I closely follow their procedure for analyzing responses to the rebate among different exposure groups. Specifically, for each of my redistribution channels, I rank households in equally-sized bins according to their measure of exposure as at the time of the first interview. I then regress changes in the level of consumption expenditures (ΔC_{it} in JPS's notation) on the amount of the tax rebate ($Rebate_{it}$). I follow their instrumental-variable specification, instrumenting $Rebate_{it}$ with a dummy indicator for whether the debate was received. I include month effects and control for age and changes in family composition, and I allow both the intercept and the rebate coefficients to differ across households bins.

My benchmark estimate uses food consumption expenditures as dependent variable. This allows for substantially more precise estimates, as it does in JPS. Section B.5.1 below reports all results using total consumption expenditures as dependent variable instead. Section B.5.4 considers using different numbers of bins.

The procedure to compute estimators is then the same one as the PSID, with confidence intervals again constructed using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section B.5.4 reports redistribution elasticities using between 3 and 8 bins to stratify the sample.

B.4 Evaluating the quality of the financial information in U.S. surveys

In order to shed light on the quality of financial data in the PSID and the CE, tables B.4 and B.5 compare the median value of each class of assets and liabilities for households holding these instruments with the comparable number from the Survey of Consumer Finance. All three surveys are analyzed in 2001, the year in which they all overlap. As discussed above, the CE and the PSID group assets and liabilities into coarse categories, making a precise comparison difficult. However, table B.4 illustrates that liabilities in both the CE and the PSID appear to be aligned with numbers from the SCF as far as medians are concerned. This is especially true in the CE. Regarding financial assets, PSID and SCF data are fairly comparable. By contrast, the CE appears to considerably underreport assets, confirming claims in the literature.

Table B.4: Median values for financial liabilities — CE v. PSID v. SCF

Liabilities	SCF	CE	PSID	CE/SCF	PSID/SCF
Mortgages on primary residence	72	72.3	78	1.00	1.08
HELOC on primary residence	15	18.9	-	1.26	-
Other residential debt	40	37.9	18	0.95	0.45
Credit cards	1.9	2		1.05	
Vehicle loans	9.2	10.4	6	1.13	0.6
Education loans, personal loans, other	5	1.2		0.24	
Any debt	38.7	40.1	50	1.04	1.29

Units: Thousands of 2001 USD.
Households holding those liabilities in 2001. Medians computed using survey weights.

Table B.5: Median values for financial assets — CE v. PSID v. SCF

Financial Assets	SCF	CE	PSID	CE/SCF	PSID/SCF
Transaction accounts	3.9	1		0.26	
Certificates of deposit	15	3	3.5	0.2	0.7
Savings bonds	1	0.8		0.8	
Retirement accounts	29.4	-	25	-	0.85
Stocks	20	25	20	0.64	1
Bonds, mutual funds, life insurance, other	20		12		0.6
Any financial asset	28.3	4.5	8	0.16	0.28

Units: Thousands of 2001 USD.
Households holding those assets in 2001. Medians computed using survey weights.

B.5 Sensitivity analysis

In this section I perform several robustness checks. As a general matter, my results in the SHIW and PSID are remarkably stable across all scenarios.

B.5.1 Using total expenditure to estimate MPC

Figure B.1 replicates the right two columns of figure 2 when *all* available consumption expenditures are used to estimate MPC in the PSID and in the CE, instead of my benchmark scenario (which uses non durable consumption in the PSID and food consumption in the CE). This involves a minor change of consumption measure for the PSID, but a much more substantial change in the CE.

As a result, for the PSID, figure B.1 delivers qualitatively similar patterns as figure 2. Conversely, for the CE, the patterns are different, but the confidence intervals are extremely

Table B.6: Using total expenditures to estimate MPC in the PSID and CE

Survey	PSID		CE	
	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}_R}$	-0.04	[-0.12,0.03]	-0.41	[-2.00,1.17]
$\widehat{\mathcal{E}_R^{NR}}$	-0.01	[-0.08,0.06]	-0.41	[-1.98,1.16]
$\widehat{S}$	0.98	[0.95,1.01]	0.99	[-0.02,2.01]
$\widehat{\mathcal{E}_P}$	-0.07	[-0.15,0.01]	-0.44	[-7.03,6.15]
$\widehat{\mathcal{E}_P^{NR}}$	-0.11	[-0.19,-0.03]	-1.67	[-9.13,5.80]
$\widehat{\mathcal{M}}$	0.06	[-0.01,0.12]	-0.03	[-1.93,1.88]
$\widehat{\mathcal{E}_Y}$	-0.03	[-0.08,0.02]	-0.31	[-1.37,0.74]

This figure recomputes the right two columns of table 3, but uses total expenditures to estimate MPC.

wide, and the point estimates tend to give implausible values, either very close to 1 or below 0.

Table B.6 replicates the right two columns of table 3 with this alternative definition of MPC. In the PSID, the point estimates are little changed, though confidence intervals are larger. In the CE, by contrast, the signs are the same, but the magnitudes are larger than in my benchmark scenario. However, the confidence bands are very wide. Moreover, income-weighted MPC is now negative on average, though again with very large confidence intervals. I conclude that this measure of MPC, while theoretically more appealing, is too imprecise to be able to draw definitive conclusions.

B.5.2 Excluding durable consumption from the URE calculation

Section 2.2 shows that, if relative durable goods prices have an elasticity ϵ with respect to the real interest rate, then a theoretically-consistent measure of URE counts a fraction $1-\epsilon$ of nondurable expenditures. Figure B.2 plots my estimated $\widehat{\mathcal{E}_R}$ against ϵ in all three datasets. The left-most part of the graph corresponds to $\epsilon = 0$, which is my benchmark scenario. As is clear from the graphs, the magnitudes are not altered dramatically by the choice of ϵ. If anything, excluding a larger fraction of durable goods tends to make the estimated value of $\mathcal{E}_R$ more negative.

B.5.3 Alternative maturity assumptions

Here, I consider the sensitivity of my estimates of the redistribution elasticity with respect to the real interest rate, $\mathcal{E}_R$, to my assumptions regarding maturities for short-term assets and liabilities that I am counting as part of A_i or L_i. Table B.7 reports results. In the first column, I assume that all assets and liabilities have a duration of one quarter. In

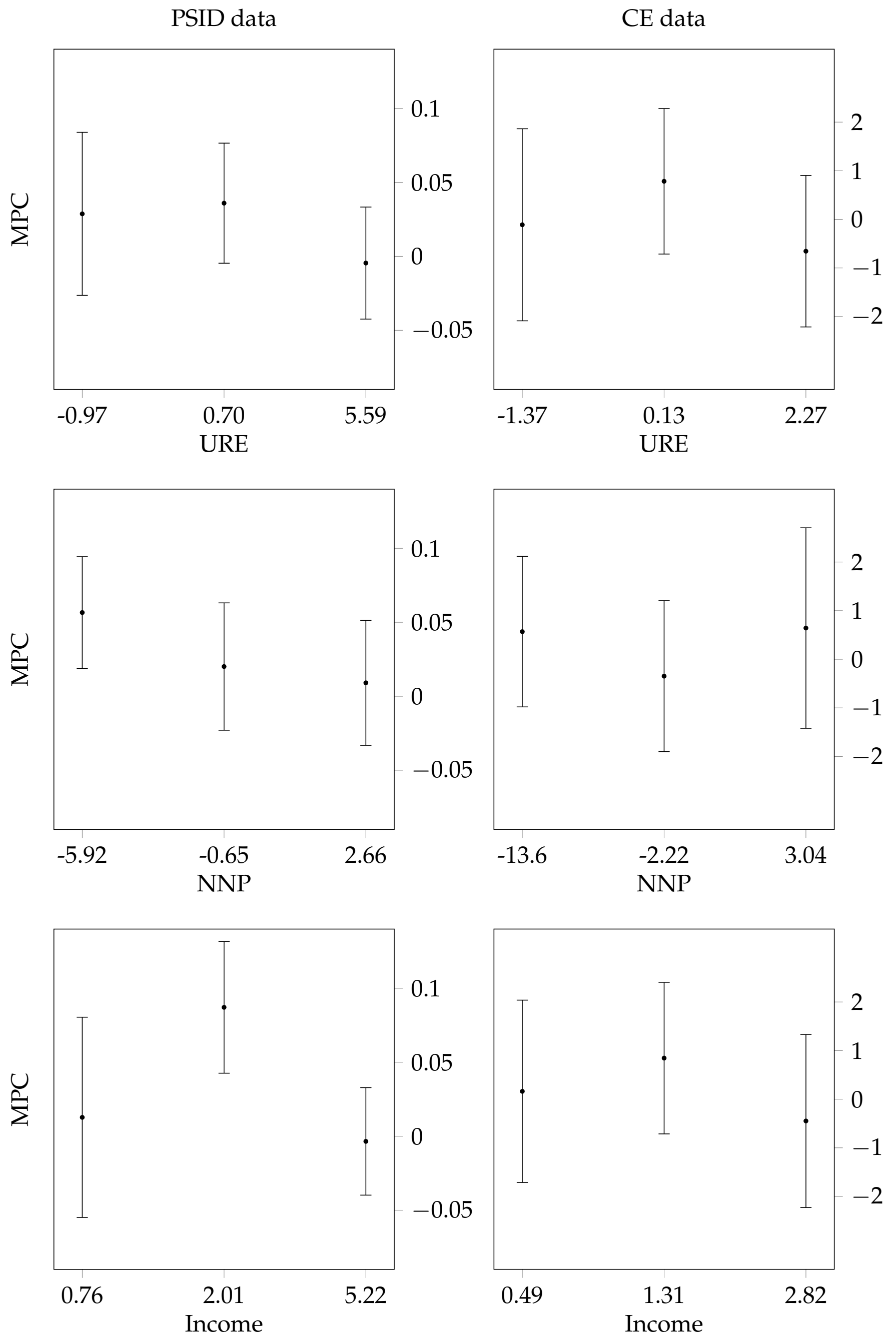

Figure B.1: Using total expenditures to estimate MPC in the PSID and the CE

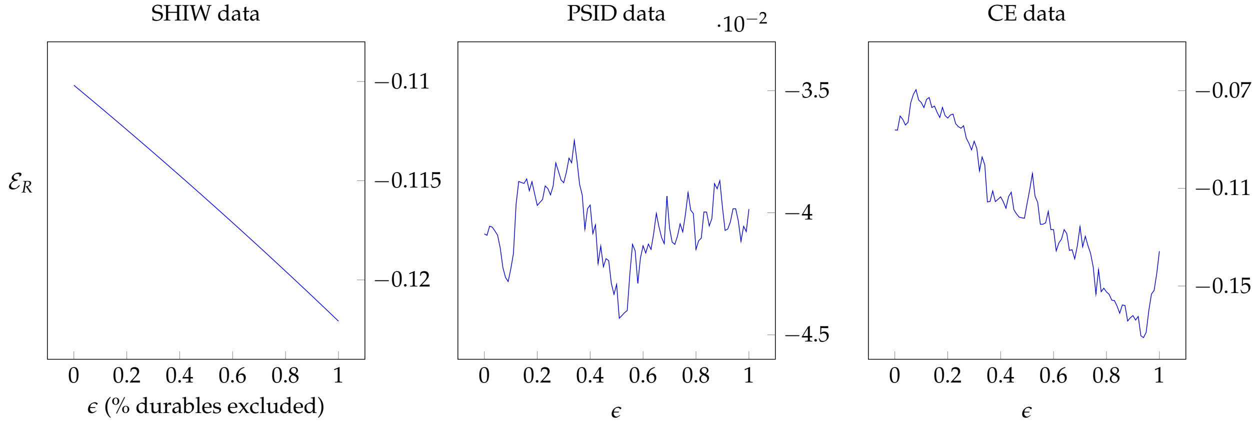

Figure B.2: Estimating $\widehat{\mathcal{E}_R}$ assuming alternative values of ϵ.

Table B.7: Estimated redistribution elasticity $\mathcal{E}_R$ for five duration scenarios

		Duration scenario				
		Quarterly	Short	Benchmark	Long	Annual
$\widehat{\mathcal{E}_R}$	SHIW	-0.18 [-0.29,-0.06]	-0.21 [-0.29,-0.13]	-0.11 [-0.16,-0.06]	-0.08 [-0.11,-0.04]	-0.06 [-0.09,-0.03]
	PSID	-0.10 [-0.20,-0.00]	-0.10 [-0.19,-0.01]	-0.05 [-0.10,-0.00]	-0.04 [-0.09,0.00]	-0.03 [-0.07,0.01]
	CE	-0.19 [-0.55,0.17]	-0.13 [-0.41,0.15]	-0.09 [-0.26,0.09]	-0.07 [-0.20,0.06]	-0.06 [-0.17,0.06]

the second, I maintain these assumptions, but increase ARM mortgage durations to two quarters. The third column is my benchmark scenario (two quarter duration for deposits, three for ARMs, two for credit card debt). My fourth scenario increases the duration of deposits to three quarters, the ARM durations to one year, and credit card debt durations to three quarters. Finally, the last column reports results assuming durations of one year.

A stylized fact emerging from these results is that, the longer the durations, the closer to zero $\widehat{\mathcal{E}_R}$ becomes. This is consistent with the predictions from my model in section 5, and in particular the left panel of figure 3. The point estimates do not vary dramatically across scenarios, and remain negative in all scenarios across all three datasets, suggesting that my benchmark estimates are robust to maturity assumptions.

B.5.4 Number of bins in the PSID and CE

Recall that my estimates of MPCs in the PSID and the CE are obtained by I stratifying the population in three equally-sized groups. Table B.8 reports the full redistribution elasticities of all three channels by progressively increasing the number of bins from 3 to 8 bin in

Table B.8: Redistribution elasticities using 3 to 8 bins in the PSID and the CE

		Number of bins					
		3	4	5	6	7	8
PSID	$\widehat{\mathcal{E}_R}$	-0.05	-0.04	-0.04	-0.05	-0.05	-0.05
		[-0.10,-0.00]	[-0.09,0.02]	[-0.10,0.02]	[-0.11,0.01]	[-0.12,0.01]	[-0.11,0.01]
	$\widehat{\mathcal{E}_P}$	-0.02	-0.04	-0.03	-0.02	-0.03	-0.03
		[-0.08,0.04]	[-0.10,0.02]	[-0.10,0.03]	[-0.08,0.04]	[-0.10,0.04]	[-0.09,0.04]
	$\widehat{\mathcal{E}_Y}$	-0.04	-0.04	-0.05	-0.05	-0.05	-0.05
		[-0.08,-0.00]	[-0.08,-0.00]	[-0.09,-0.01]	[-0.09,-0.00]	[-0.09,-0.00]	[-0.09,0.00]
CE	$\widehat{\mathcal{E}_R}$	-0.09	-0.10	-0.14	-0.10	-0.12	-0.10
		[-0.26,0.09]	[-0.29,0.09]	[-0.36,0.08]	[-0.33,0.14]	[-0.37,0.13]	[-0.35,0.16]
	$\widehat{\mathcal{E}_P}$	-0.11	-0.18	-0.28	-0.20	-0.42	-0.60
		[-0.83,0.60]	[-0.93,0.58]	[-1.17,0.61]	[-1.18,0.78]	[-1.50,0.66]	[-1.66,0.46]
	$\widehat{\mathcal{E}_Y}$	-0.05	-0.04	-0.06	-0.05	-0.09	-0.06
		[-0.15,0.06]	[-0.14,0.06]	[-0.17,0.05]	[-0.17,0.07]	[-0.21,0.03]	[-0.19,0.06]

both samples.

In the PSID results are fairly robust, both in terms of point estimates (which remaining negative and close to benchmark-scenario values) and in terms of confidence intervals (which remain relatively narrow). Signs are also stable in the CE, though for $\widehat{\mathcal{E}_P}$ magnitudes increase and confidence bands also widen considerably.

B.5.5 Age sample selection in PSID

Table B.9 assesses the impact of including very young and very old households in my baseline PSID sample, which departs from the benchmark assumption in the literature. The first set of columns recall my benchmark estimates for all my key cross sectional moments. The second one reports results obtained when restricting the sample to households whose head is between 25 and 55 years old, as in Kaplan, Violante and Weidner (2014) and others. As evident from the table, results are broadly consistent for both samples, suggesting that age is not an important driver of my results.

B.6 Empirical drivers of MPC, URE, NNP and income

This section complements section 4.4 by providing other perspectives on the empirical drivers of my main objects of interest.

Table B.9: Sensitivity with respect to age sample selection in PSID

Survey	Benchmark		[25,55]	
	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}_R}$	-0.05	[-0.10,-0.00]	-0.03	[-0.08,0.01]
$\widehat{\mathcal{E}_R^{NR}}$	0.01	[-0.05,0.06]	0.03	[-0.01,0.07]
$\widehat{S}$	0.97	[0.95,0.98]	0.96	[0.94,0.98]
$\widehat{\mathcal{E}_P}$	-0.02	[-0.08,0.04]	-0.02	[-0.08,0.04]
$\widehat{\mathcal{E}_P^{NR}}$	-0.07	[-0.13,-0.01]	-0.10	[-0.17,-0.03]
$\widehat{\mathcal{M}}$	0.08	[0.03,0.13]	0.10	[0.04,0.15]
$\widehat{\mathcal{E}_Y}$	-0.04	[-0.08,-0.00]	-0.04	[-0.07,-0.00]

B.6.1 The role of age

This section examines the distribution of exposures and MPC by age in each survey. I divide the population in eight equally-sized age bins. This allows me to assess life-cycle dynamics. It also helps to visualize clearly the relative strengths and weaknesses of each survey.

Exposure measures. Figure B.3 reports the average value of URE, NNP and income in each age bin, normalized by average consumption in the survey. Average URE (the blue line in the first row of graphs) is increasing in age across all three surveys, with a pattern of decline after retirement in the SHIW. This pattern is mostly due to a decumulation of financial assets in that survey (as represented by the green line). In terms of magnitudes, average URE is always positive in the SHIW and in the PSID, while in the CE average URE is negative for most working-age households. However, this is clearly driven by the different data flaws in each survey: the SHIW and the PSID greatly underreport consumption relative to income—notice the difference between the black and the red line. This tends to overestimate URE. By contrast, as documented above, the CE severely underreports assets, underestimating URE.

Regarding net nominal positions (the blue line in the second row of graphs), the life-cycle pattern in the SHIW is also increasing in age. By contrast, the PSID and the CE display an interesting U shape, with a minimum around age 40. In particular, in the SHIW, nominal liabilities are declining almost monotonically with age, while nominal assets are sharply increasing until age 60 and then decline rapidly. By contrast, in the PSID and in the CE, nominal liabilities are increasing in age for young households, and then start to decline steadily after age 40—while nominal assets are almost monotonically increasing in age. In terms of magnitudes, average NNP is negative for most of working age population

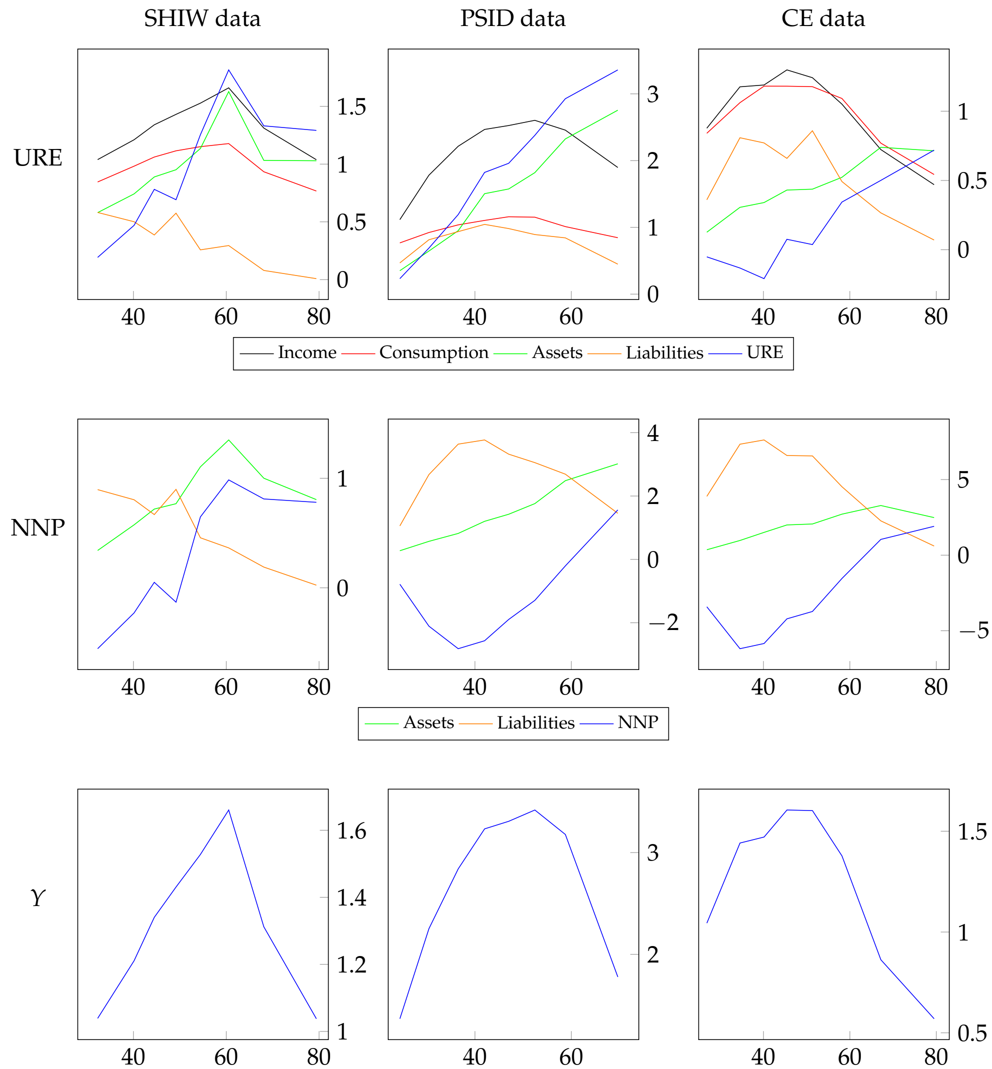

Figure B.3: Exposure measures by age bins in all three datasets

in the SHIW, while it is very negative in the CE and PSID for all households cohorts except the oldest ones. This highlights, once again, the issue that these surveys cover liabilities better than they cover assets.

For income, we observe the classic inverted-U shape in age across all three datasets.

MPC. Figure B.4 reports marginal propensities to consume by age bins in all three surveys. There is an overall declining pattern in age, except for a spike for the oldest cohort in the CE. Interestingly, all three surveys also suggest a rise in MPC around middle age. This pattern is not sensitive to the number of bins employed to stratify the population. Combining this graph with figure B.3, it appears that age is indeed a driver of the negative correlation between MPC and my exposure measures—as already apparent in table 4.

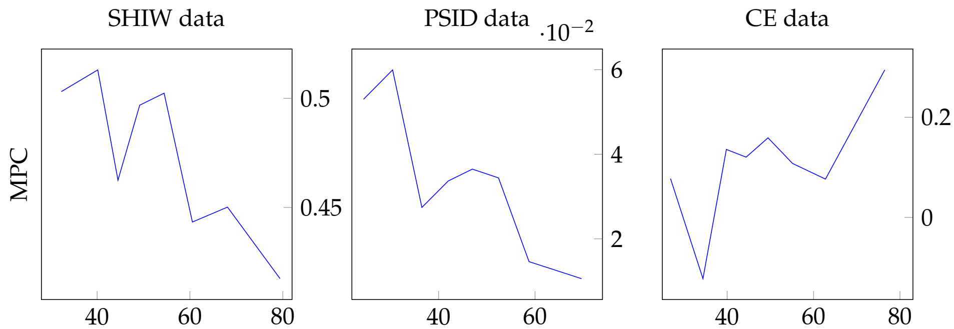

Figure B.4: MPCs by age bins in all three datasets

B.6.2 The role of income

Figure B.5 examine the distribution of URE and NNP in all three surveys, when the population is grouped into eight income bins. Unsurprisingly, average URE is increasing in income, especially in the SHIW and the PSID. In these surveys, average URE increases more than one for one with income at the top of the distribution, owing an increase in maturing assets. Interestingly, maturing liabilities (the orange line) also increase in income across all three surveys.

For net nominal position, patterns are different in Italy and in the United States. In the SHIW, net nominal position is initially flat, and then increases with income, owing to an increase in assets at the top of the income distribution. By contrast, in the PSID and in the CE, net nominal position initially declines in income, and then flattens out. This is because nominal liabilities initially increase strongly with income, while nominal assets only increase mildly.

B.6.3 A general covariance decomposition

In section 4.4, I presented a covariance decomposition that projected observables on a single covariate. This approach can of course be generalized to include any number of covariates. The procedure is in two steps: first, run an OLS regression

$$\begin{aligned} MPC_i &= (\beta^m)' \mathbf{Z_i} + \epsilon_i^m \\ URE_i &= (\beta^u)' \mathbf{Z_i} + \epsilon_i^u \end{aligned}$$

where $\mathbf{Z_i} = (1, Z_{i1}, \cdots, Z_{iJ})'$ is now a vector of covariates. Then, recover fitted values

$$\begin{aligned} \widehat{MPC_i} &= \left(\widehat{\beta^m}\right)' \mathbf{Z_i} \\ \widehat{URE_i} &= \left(\widehat{\beta^u}\right)' \mathbf{Z_i} \end{aligned}$$

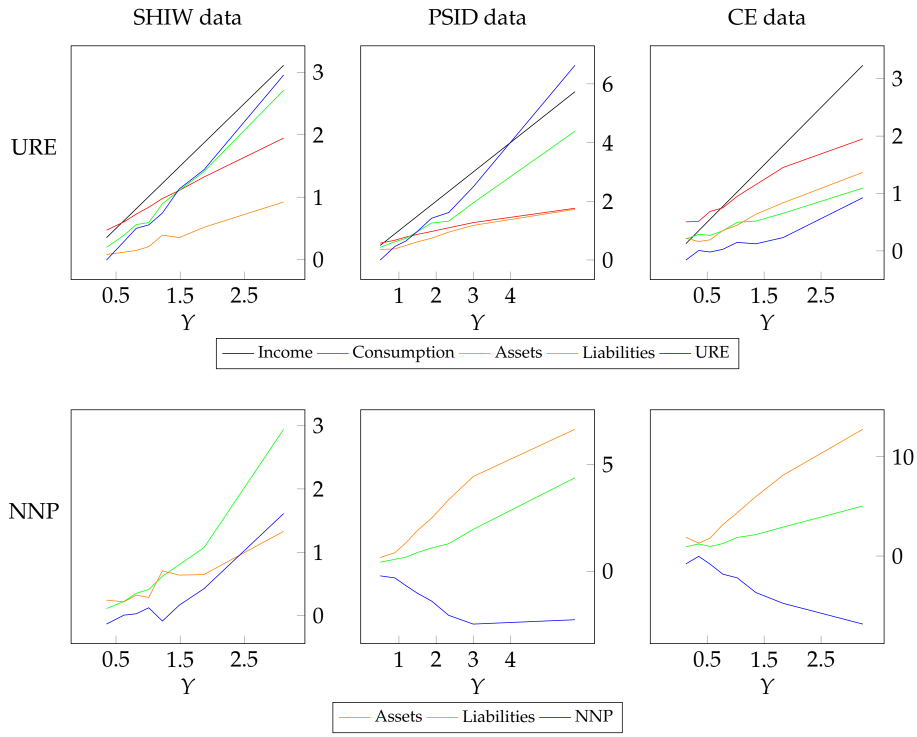

Figure B.5: URE and NNP components by income bins in all three datasets

and residuals $\widehat{\epsilon_i^m}, \widehat{\epsilon_i^u}$. The law of total covariance can now be expressed as

$$\text{Cov}\left(MPC_i, URE_i\right) = \text{Cov}\left(\widehat{MPC_i}, \widehat{URE_i}\right) + \text{Cov}\left(\widehat{\epsilon_i^m}, \widehat{\epsilon_i^u}\right) \tag{B.1}$$

The first term gives the component of explained covariance, and the second the component of unexplained covariance. The explained part of the covariance can be further decomposed as

$$\text{Cov}\left(\widehat{MPC_i}, \widehat{URE_i}\right) = \text{Cov}\left(\sum_{j=1}^{J} \widehat{\beta_j^m} Z_{ij}, \sum_{k=1}^{J} \widehat{\beta_k^m} Z_{ik}\right) = \sum_{j=1}^{J} \sum_{k=1}^{J} \widehat{\beta_j^m} \widehat{\beta_k^m} \text{Cov}\left(Z_{ij}, Z_{ik}\right) \tag{B.2}$$

Of course, the 'share of explained covariance' attributed to one particular covariate through this procedure depends on which other covariates are included in $\mathbf{Z_i}$.

Implementation. Tables B.10 reports the full matrix described by equation (B.2) for each of my three main covariances $\mathcal{E}_R$, $\mathcal{E}_P$, and $\mathcal{E}_Y$ in the SHIW, when all covariates from table 4 are included simultaneously. In the PSID and the CE, this exercise is less interesting since MPCs are only available at the group level, but it is possible to do by using the average value of explanatory variables in each bin. These results can easily be generated using the code provided online.

Table B.10: Fraction of $\mathcal{E}_R$ explained by each pair of SHIW covariates

	Age	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	9.03	0.20	-0.03	-2.36	0.13	-0.02	0.01	0.13
Male	0.83	1.78	0.13	0.25	-0.03	0.09	0.00	0.04
Married	-0.27	0.29	0.22	0.28	-0.09	-0.04	0.00	0.01
Years of ed.	-3.04	0.08	0.04	7.61	-0.05	0.49	-0.03	0.00
Family size	2.65	-0.15	-0.19	-0.82	0.38	0.34	-0.00	0.08
Res. South	-0.34	0.34	-0.06	6.09	0.27	10.53	-0.00	0.39
City size	0.59	0.08	0.04	-2.37	-0.01	-0.01	0.45	0.01
Unemployed	1.62	0.13	0.01	0.03	0.05	0.31	0.00	1.07

Table B.11: Fraction of $\mathcal{E}_P$ explained by each pair of SHIW covariates

	Age	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	13.29	0.22	-0.06	-2.56	1.52	-0.01	-0.02	-0.15
Male	1.22	1.98	0.28	0.28	-0.35	0.04	-0.01	-0.05
Married	-0.40	0.33	0.49	0.30	-1.06	-0.02	-0.02	-0.01
Years of ed.	-4.47	0.09	0.08	8.27	-0.60	0.22	0.12	-0.00
Family size	3.89	-0.16	-0.43	-0.89	4.48	0.15	0.01	-0.10
Res. South	-0.50	0.37	-0.13	6.62	3.12	4.84	0.00	-0.46
City size	0.87	0.08	0.10	-2.58	-0.14	-0.00	-1.73	-0.01
Unemployed	2.39	0.15	0.03	0.03	0.60	0.14	-0.00	-1.26

Table B.12: Fraction of $\mathcal{E}_Y$ explained by each pair of SHIW covariates

	Age	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	8.63	0.21	-0.14	-4.65	-2.33	-0.07	-0.04	0.48
Male	0.79	1.90	0.65	0.50	0.53	0.29	-0.02	0.17
Married	-0.26	0.31	1.12	0.55	1.63	-0.12	-0.03	0.04
Years of ed.	-2.90	0.08	0.19	15.00	0.93	1.62	0.19	0.01
Family size	2.53	-0.15	-0.98	-1.61	-6.89	1.13	0.02	0.31
Res. South	-0.32	0.36	-0.30	12.01	-4.79	35.21	0.01	1.49
City size	0.56	0.08	0.22	-4.68	0.22	-0.03	-2.66	0.04
Unemployed	1.55	0.14	0.07	0.05	-0.92	1.04	-0.01	4.09

C Details on the structural model of section 5

C.1 Additional details on the model

In the model, every household i has felicity function $u(c) = \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}}$ and picks the sequence $\{c_t^i\}$ to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\left(\prod_{\tau=0}^{t}\beta_\tau^i\right) u\left(c_t^i\right)\right] \tag{C.1}$$

by choice of a portfolio of nominal bonds Λ_t and real bonds χ_t, with

$$P_t c_t^i + Q_t\left(\Lambda_{t+1}^i - \delta\Lambda_t^i\right) + q_t P_t\left(\chi_{t+1}^i - \delta\chi_t^i\right) = P_t y_t\left(e_t^i\right) + \Lambda_t^i + P_t\chi_t^i \tag{C.2}$$

and borrowing constraint

$$Q_t\Lambda_{t+1}^i + q_t P_t\chi_{t+1}^i \geq -\overline{D}_t P_t \tag{C.3}$$

Given Λ_t^i and χ_t^i, define the equivalent real bond position as

$$\lambda_t^i \equiv \frac{\Lambda_t^i}{P_{t-1}} + \frac{q_{t-1}}{Q_{t-1}}\chi_t^i$$

Along any perfect-foresight path with a constant price level $P_t = P$, no arbitrage between nominal and real bonds implies

$$\frac{1+\delta Q_t}{Q_{t-1}} = \frac{1+\delta q_t}{q_{t-1}}$$

and therefore $\frac{q_t}{Q_t} = \frac{1}{P}$. The consumer is then indifferent between holding nominal or real bonds. I resolve the indeterminacy by assuming that a constant share κ of the portfolio is invested in real (indexed) bonds, so that the household's portfolio allocation is

$$\begin{aligned}\frac{\Lambda_t^i}{P_{t-1}} &= (1-\kappa)\lambda_t \\ \frac{q_{t-1}}{Q_{t-1}}\chi_t^i &= \kappa\lambda_t\end{aligned}$$

With this notation, the budget constraint (A.55) and borrowing constraint (C.3) rewrite

$$\begin{aligned}c_t^i + Q_t\lambda_{t+1}^i &= y_t\left(e_t^i\right) + (1+\delta Q_t)\left[\frac{1-\kappa}{\Pi_t} + \kappa\right]\lambda_t \\ Q_t\lambda_{t+1}^i &\geq -\overline{D_t}\end{aligned} \tag{C.4}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is inflation. Along any perfect-foresight paths I consider, $\Pi_t = 1$ and the budget constraint simplifies to

$$c_t^i + Q_t\lambda_{t+1}^i = y_t\left(e_t^i\right) + (1+\delta Q_t)\lambda_t^i$$

Equation (C.4) to determine portfolio losses in case of a deviation of Π_t from its perfect-foresight value.

This problem has the following recursive formulation. The consumer's idiosyncratic state is given by the combination of $\mathbf{s}_t^i$ and his real bond position $\lambda_t^i \equiv \frac{\Lambda_t^i}{P_{t-1}}$. From his point of view, the relevant components of the aggregate state are $\left(y_t, Q_t, \Pi_t, \overline{D}_t\right)$, where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the inflation rate at t. Hence his optimization problem is characterized by the Bellman equation:

$$\begin{aligned} V_t(\lambda, \mathbf{s}) = \quad & \max_{c,\lambda'} \quad u(c) + \beta(\mathbf{s})\,\mathbb{E}\left[V_{t+1}\left(\lambda', \mathbf{s}'\right)|\mathbf{s}\right] \\ & \text{s.t.} \qquad c + Q_t\lambda' = y(\mathbf{s}) + (1+\delta Q_t)\,\lambda \\ & \qquad\qquad Q_t\lambda' \geq -\overline{D}_t \end{aligned} \tag{C.5}$$

I calibrate the model such that, at the initial steady-state distribution $\Psi(\mathbf{s}, \lambda)$, aggregate consumption is equal to the aggregate endowment, so

$$C_t \equiv \int c_t(\mathbf{s}, \lambda)\, d\Psi_t(\mathbf{s}, \lambda) = Y_t$$

This can be interpreted as the flexible-price equilibrium of a Huggett model with no government debt.

C.2 Behavior of constrained agents after real interest rate shocks

I specify that the borrowing limit $\left\{\overline{D_t}\right\}$ adjusts in response to such a shock so as to hold the *real coupon payment in the next period fixed*: $\overline{D_t} = Q_t\overline{d}$, or equivalently

$$\frac{\Lambda_{t+1}^i}{P_t} \geq -\overline{d} \tag{C.6}$$

In addition to being a natural one, the specification of the adjustment process for borrowing limits in (C.6) implies that theorem 1 holds exactly, including for agents at a binding borrowing limit. It is crucial to understand how these agents are affected depending on the maturity of the debt in the economy, δ. In the experiments I consider inflation is $\Pi_t = 1$, so that nominal and real interest rates are equal. Consider an agent with income Y_t^i who maintains himself at the borrowing limit in an initial steady-state where the real interest rate is R and the bond price is constant at $Q = \frac{1}{R-\delta}$. His consumption is equal to his income, minus the interest payment on the value of the borrowing limit $\overline{D} = Q\overline{d}$:

$$c_t^i = Y_t^i - (R-1)\,\overline{D}$$

Across economies with different debt maturities δ, $\overline{D}$ is a constant, so that the steady-state payments are the same, but the exposure of these payments to real interest rate changes differ. Indeed we can decompose:

$$(R-1)\,\overline{D} = (R-\delta)\,\overline{D} - \overline{D}\,(1-\delta) = \overline{d} + \underline{URE}$$

where $\overline{d} \equiv (R-\delta)\,\overline{D}$ is the part that is precontracted and $\underline{URE} \equiv -\overline{D}\,(1-\delta)$ the part that is subject to interest changes. Hence, economies with different δ involve very different levels of unhedged interest rate exposures for borrowing-constrained agents, ranging from the full principal $-\overline{D}$ when $\delta = 0$ to none when $\delta = 1$. In the benchmark calibration with

$\delta = 0.95$, highly-indebted low-income agents use their full income for interest payments and amortization $\overline{d}$, and then borrow, as on a home equity line of credit, to maintain their consumption level. Hence they are only mildly affected by changes in interest rates. On the other hand, when all debt is short-term, we instead have $c_t^i = Y_t^i - (R_t - 1)\overline{D}$ for constrained agents, leading to large swings in their consumption as interest rates change.

C.3 Computational method

C.3.1 Method of endogenous gridpoints

I use the method of endogenous gridpoints (Carroll 2006) to solve for consumer policy functions. This is a computationally efficient solution method based on policy function iteration, which avoids costly root-solving operations and is applicable to any standard incomplete market problem with CRRA utility functions (see for example Guerrieri and Lorenzoni 2015). The computation involves finding the policy function for consumption $c_t(\lambda, \mathbf{s})$ on a fine grid for λ (2000 points) and a discrete grid for $\mathbf{s}$ (20 points: 2 states for β and 10 states for z).

When the borrowing constraint binds, which happens for $\lambda \leq \lambda_t^*$ for some λ_t^*, the policy function is given by

$$c_t(\lambda, \mathbf{s}) = y_t(\mathbf{s}) + \lambda(1 + Q_t\delta) + \overline{D_t} \tag{C.7}$$

For $\lambda > \overline{\lambda_t}$ the borrowing constraint is not binding, and defining the real interest rate by

$$R_t = \frac{1 + \delta Q_{t+1}}{Q_t}$$

the solution is characterized by the Euler equation

$$c_t^{-\sigma^{-1}} = \beta_t R_t \mathbb{E}_t\left[(c_{t+1})^{-\sigma^{-1}}\right] \tag{C.8}$$

The idea behind endogenous gridpoints is to start from a given state today $\mathbf{s}$ and a *target* bond level in the next period λ'. The budget constraint

$$\lambda' = \frac{1}{Q_t}\left(y_t(\mathbf{s}) + \lambda(1 + Q\delta) - c\right) \tag{C.9}$$

implies that the pairs (λ, c) that are consistent with λ' are on a straight line. Moreover, given a guess for the policy function $c_{t+1}(\cdot,\cdot)$, there is a unique value of g consistent with an optimal choice of λ' tomorrow, given by

$$c = \left(\beta_t R_t \mathbb{E}\left[c_{t+1}(\lambda', \mathbf{s}')^{-\sigma^{-1}} |\mathbf{s}\right]\right)^{-\sigma} \tag{C.10}$$

Hence by varying the target bond level λ', one traces out the policy function $c_t(\lambda, \mathbf{s})$ in the region $\lambda > \overline{\lambda_t}$. This is very efficient computationally since it can be performed on the grid for λ' which is used to store $c_{t+1}(\lambda', \mathbf{s}')$. The calculation only involves:

a) Finding c using (C.10), which only involves power operations and linear combina-

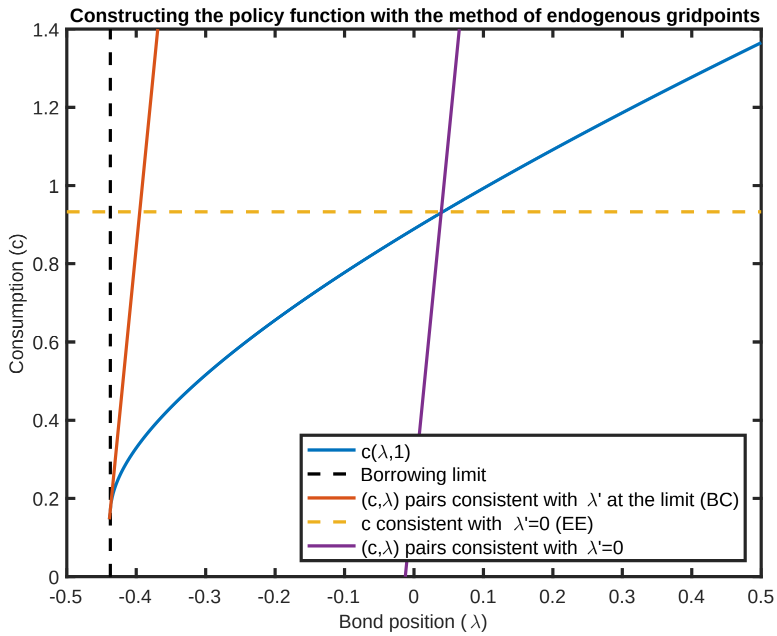

Figure C.1: Constructing the policy function $c(\lambda, \mathbf{s} = 1)$ (model calibration)

tions using the Markov transition matrix for $\mathbf{s}$

b) Finding λ by solving one linear equation in one unknown in (C.9)

c) Defining λ_t^* as the bond value today that corresponds to $\lambda' = \frac{\overline{D}_t}{Q_t}$, since this is the highest level of bonds for which the consumer chooses to be at the borrowing limit tomorrow with his Euler equation holding with equality

d) If $\lambda_t^* > \frac{\overline{D}_{t-1}}{Q_{t-1}}$, completing the policy function on an arbitrary grid for $\left[\frac{\overline{D}_{t-1}}{Q_{t-1}}, \lambda_t^*\right]$ using (C.7)

e) Interpolating the resulting policy function back to the grid for λ

Figure C.1 illustrates the construction of the policy function for state $\mathbf{s} = 1$ in the model calibration. Consider targeting a bond level $\lambda' = 0$. This yields a value for consumption through the Euler Equation (C.8) indicated by the dashed yellow line. It also yields a set of pairs (c, λ) consistent with $\lambda' = 0$ through the budget constraint (C.9), as indicated by the solid purple line. The intersection of these two lines yields a new point of the policy function over λ. Varying λ' in this way we trace out this policy function (solid blue line) over the range where the Euler equation holds. The policy function is completed by the set of points consistent with borrowing at the limit (solid red line).

C.3.2 Flexible price steady-state

In a steady-state, the consumer faces a constant sequence $\left(Q_t, \Pi_t, \overline{D}_t\right) = \left(\frac{1}{R-\delta}, 1, \overline{D}\right)$ where R is the interest rate that prevails in steady-state.

I find the steady-state interest rate R using the following classic bisection procedure:

a) Start with a guess for R and for the consumption policy function $c^0(\lambda, \mathbf{s})$

b) Iterate on $c_t(\lambda, \mathbf{s})$ using the procedure described in C.3.1 until $c_{t+1} - c_t$ is sufficiently small. By construction, $c = c^{SS}(b, y)$ then satisfies the functional equation

$$c(\lambda, \mathbf{s})^{-\sigma^{-1}} = \beta(\mathbf{s}) R\mathbb{E}\left[c\left(\frac{1}{Q}\left(y(\mathbf{s}) + \lambda(1+\delta) - c(\lambda, \mathbf{s})\right), \mathbf{s}'\right)^{-\sigma^{-1}} |\mathbf{s}\right] \quad \text{(C.11)}$$

c) Use the inverse policy function for next period bonds $\lambda(\lambda', \mathbf{s}) = [\lambda']^{-1}(\lambda', \mathbf{s})$, which is computed as part of the endogenous gridpoints method, to find the stationary conditional distribution for bonds $\Psi(\lambda|\mathbf{s})$, as the fixed point of the operator mapping Ψ_t to Ψ_{t+1},

$$\Psi_{t+1}(\lambda'|\mathbf{s}') = \sum_{\mathbf{s}} \Psi_t\left([\lambda']^{-1}(\lambda', \mathbf{s})|\mathbf{s}\right) \frac{\Pr(\mathbf{s}_t = \mathbf{s})}{\Pr(\mathbf{s}_{t+1} = \mathbf{s}')} \Pi(\mathbf{s}'|\mathbf{s})$$

d) Check that goods market clear, $\int c(\lambda; \mathbf{s})\, d\Psi(\lambda, \mathbf{s}) = Y^*$. If they do not, adjust R in the direction of market clearing and repeat (one must first determine whether steady-state consumption is locally increasing or decreasing in R)

C.3.3 Transitional dynamics following a shock

Here I describe how to compute perfect-foresight transition paths following a change in the path for real interest rates $\{R_t\}$, such as at described by equation (29). Assume that the economy returns to steady-state by time T (in my computations, $T = 200$ when the shock has persistence $\rho = 0.5$)

a) Using the path of R_t, compute the bond price path

$$Q_t = \frac{1 + \delta Q_{t+1}}{R_t}$$

backwards starting from $Q_T = \frac{1}{R-\delta}$.

b) Given the paths for $\left(Q_t, \Pi_t = 1, \overline{D}_t\right)$, compute policy functions backwards, starting from $c_T = c^{SS}$, using the method of endogenous gridpoints described above.

c) Starting from the conditional bond distribution that prevails in the initial steady-state, and using the transitional inverse policy function for next period bonds computed as part of step b), compute the conditional bond distributions along the transition using

$$\Psi_{t+1}\left(\lambda'|\mathbf{s}'\right)=\sum_{\mathbf{s}}\Psi_t\left(\left[\lambda_t'\right]^{-1}\left(\lambda',\mathbf{s}\right)|\mathbf{s}\right)\frac{\Pr\left(\mathbf{s}_t=\mathbf{s}\right)}{\Pr\left(\mathbf{s}_{t+1}=\mathbf{s}'\right)}\Pi\left(\mathbf{s}'|\mathbf{s}\right)$$

d) Finally, compute aggregate consumption as $C_t=\int c_t\left(\lambda;\mathbf{s}\right)d\Psi_t\left(\lambda,\mathbf{s}\right)$.

References

Barsky, Robert B., Christopher L. House, and Miles S. Kimball, "Sticky-Price Models and Durable Goods," *American Economic Review*, June 2007, *97* (3), 984–998.

Berger, David, Veronica Guerrieri, Guido Lorenzoni, and Joseph Vavra, "House Prices and Consumer Spending," Working Paper 21667, National Bureau of Economic Research October 2015.

Blundell, Richard, Luigi Pistaferri, and Ian Preston, "Consumption Inequality and Partial Insurance," *American Economic Review*, December 2008, *98* (5), 1887–1921.

_ , _ , and Itay Saporta-Eksten, "Consumption Inequality and Family Labor Supply," *American Economic Review*, February 2016, *106* (2), 387–435.

Carroll, Christopher D., "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *Economics Letters*, 2006, *91* (3), 312–320.

Doepke, Matthias and Martin Schneider, "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, December 2006, *114* (6), 1069–1097.

Galí, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press, February 2008.

Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings, and the Liquidity Trap," *Manuscript*, January 2015.

Jappelli, Tullio and Luigi Pistaferri, "Fiscal Policy and MPC Heterogeneity," *American Economic Journal: Macroeconomics*, October 2014, *6* (4), 107–136.

Johnson, David S., Jonathan A. Parker, and Nicholas S. Souleles, "Household Expenditure and the Income Tax Rebates of 2001," *American Economic Review*, December 2006, *96* (5), 1589–1610.

Kaplan, Greg, Giovanni L. Violante, and Justin Weidner, "The Wealthy Hand-to-Mouth," *Brookings Papers on Economic Activity*, 2014, *2014* (1), 77–138.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, August 2003.