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Online Appendix for Influencing Connected Legislators

Abstract

In this appendix we present omitted proofs and tables for "Influencing Connected Legislators."

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1 Proof of Lemma 1

Let $\boldsymbol{\varphi}$ be the n dimensional column vector of voting probabilities with ith element equal to φ_i. Define $\boldsymbol{\eta} : R^n \to R^n$ as $\boldsymbol{\eta}(\boldsymbol{\varphi}, \mathbf{s}) = \boldsymbol{\varphi} - \mathbf{F}(\boldsymbol{\varphi}, \mathbf{s})$, where $\mathbf{s} = (\mathbf{s_A}, \mathbf{s_B})$ and $\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s})$ is a column vector with ith element equal to $1/2 + \Psi\left(\omega(s^i_A) - \omega(s^i_B) + v^i q^i(\boldsymbol{\varphi}) + \phi \sum_j g_{i,j}\left(2\varphi_j - 1\right)\right)$, as defined in (5) in Section 3.1. The equilibrium probabilities $\boldsymbol{\varphi}(\mathbf{s})$ are defined as the solution of $\boldsymbol{\eta}(\boldsymbol{\varphi}^*, \mathbf{s}) = 0$. The fact that the solution of this system exists follows from Brouwer's fixed-point theorem as argued in Section 3. Since, by Assumption 1, $\Psi\left(\overline{v} + \phi + \omega(2W)\right) < 1/2$, the solution is interior in $(0,1)$. To show uniqueness of an equilibrium of the voting stage with policy motivated legislators for Ψ sufficiently small, let $\|x\|$ be the norm $\|x\| = \sum_i |x_i|$ for any $x \in R^n$. We have:

$$\begin{aligned} \left\|\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s}) - \mathbf{F}(\boldsymbol{\varphi}', \mathbf{s})\right\| &\leq \Psi\left(\overline{v} \cdot \sum_l \left|\int_{\varphi'_l}^{\varphi_l} \sum_i q^i_l(\mathbf{x})\right| dx + 2\phi \sum_j \left(\sum_i g_{i,j}\right)\left|\varphi_j - \varphi'_j\right|\right) \\ &\leq \Psi\left(n\overline{v} \cdot \sum_l \left(|\varphi_l - \varphi'_l|\right) + 2\phi \sum_j \left|\varphi_j - \varphi'_j\right|\right) \leq \Psi\left(n\overline{v} + 2\phi\right)\|\boldsymbol{\varphi} - \boldsymbol{\varphi}'\| \end{aligned}$$

where we use the fact that $\left|q^i_j(\mathbf{x})\right| < 1$ for any $\mathbf{x}$, and $\sum_i g_{i,j} \leq 1$ for any j. For any $\eta < 1$, we therefore have $\left\|\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s}) - \mathbf{F}(\boldsymbol{\varphi}', \mathbf{s})\right\| \leq \eta \cdot \|\boldsymbol{\varphi} - \boldsymbol{\varphi}'\|$ for Ψ sufficiently small. We can therefore conclude that there is a Ψ_1 such that $\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s})$ is a contraction in $[0,1]$ with a unique fixed-point in $(0,1)$ for $\Psi \leq \Psi_1$.

We now turn to the derivatives of the voting probabilities. The implicit function theorem implies that the solution φ_i is differentiable in s^j_A at $\mathbf{s}_A, \mathbf{s}_B$ if $(D\boldsymbol{\eta})_\varphi$ is invertible in a neighborhood of $(\mathbf{s}_A, \mathbf{s}_B, \boldsymbol{\varphi}(\mathbf{s_A}, \mathbf{s_B}))$, where $\boldsymbol{\varphi}(\mathbf{s_A}, \mathbf{s_B})$ solves $\boldsymbol{\eta}(\boldsymbol{\varphi}, \mathbf{s}_A, \mathbf{s}_B) = 0$ (the expression $(D\boldsymbol{\eta})_{\boldsymbol{\varphi}}$ represents the Jacobian of $\boldsymbol{\eta}$ with respect to $\boldsymbol{\varphi}$). It is easy to verify that $(D\boldsymbol{\eta})_{\boldsymbol{\varphi}} = \left[I - \phi\Psi 2\widetilde{G}\right]$, where $\widetilde{G}$ is a $n \times n$ matrix with i,j element equal to $\widetilde{g}_{i,l} = g_{i,l}\left(g_{i,l} + v^i q^i_l(\boldsymbol{\varphi})\right)$. Let r^* be the largest eigenvalue of $\widetilde{G}$ achieved for some $\boldsymbol{\varphi}$ (this is well defined and bounded since $r(\widetilde{G})$ is continuos in $\boldsymbol{\varphi}$ in and the space of feasible $\boldsymbol{\varphi}$ is compact). Theorem III* of Debreu and Herstein [1953] implies that $\left[I - \phi\Psi 2\widetilde{G}\right]^{-1}$ exists and is nonegative for $\Psi \leq (2\phi r^*)^{-1} = \Psi_2$. The Jacobian of φ is then

$$D_j\left[\varphi\right] = \Psi \cdot \omega'(s^l_A)\left[I - \phi\Psi 2\widetilde{G}\right]^{-1}\mathbf{1}_j,$$

where $\mathbf{1}_j$ is a n-dimensional vector equal to zero except at the ith dimension in which it is equal to one. Since $\left[I - \phi\Psi 2\widetilde{G}\right]^{-1}$ is nonnegative with at least one strictly positive element for $\Psi \leq \Psi_2$, it follows that $\sum_i \partial\varphi_i/\partial s^j_A = D_j\left[\varphi\right]^T \cdot \mathbf{1} > 0$ for n large enough.

To verify concavity with respect to $\mathbf{s}_A$, let $D^2\varphi_i$ be the Hessian of φ_i. Consider first its diagonal entries $\partial^2\varphi_i/\partial s^j_A \partial s^j_A$ for any j. We can write:

$$\frac{\partial^2\varphi_i}{\partial s^j_A \partial s^j_A} = \Psi\left[\begin{array}{c} \frac{\partial^2\omega_j(s^i_A)}{\partial s^j_A \partial s^j_A} + 2\phi\sum_l g_{i,l}\frac{\partial^2\varphi_l}{\partial s^j_A \partial s^j_A} \\ +v^i\sum_l\left(\sum_k q^i_{lk}(\boldsymbol{\varphi})\left(\frac{\partial\varphi_l}{\partial s^j_A}\right)\left(\frac{\partial\varphi_k}{\partial s^j_A}\right) + q^i_l(\boldsymbol{\varphi})\frac{\partial^2\varphi_l}{\partial s^j_A \partial s^j_A}\right)\end{array}\right]. \quad (1)$$

We can write:

$$\left[I-\phi\Psi 2\widetilde{G}\right]\left(D^2\boldsymbol{\varphi}\right)_{jj}=\Psi\omega''(s_A^l)\left(\mathbf{1}_j+\Psi^2\frac{\left(\omega'(s_A^l)\right)^2}{\omega''(s_A^l)}V\left(\mathbf{z}^j\right)^T D^2q^i(\boldsymbol{\varphi})\left(\mathbf{z}^j\right)\right), \tag{2}$$

where $\left(D^2\boldsymbol{\varphi}\right)_{jj}=\left(\frac{\partial^2\varphi_1}{\partial s_A^j\partial s_A^j},...,\frac{\partial^2\varphi_n}{\partial s_A^j\partial s_A^j}\right)^T$, the $n\times n$ matrix $D^2q^i(\boldsymbol{\varphi})$ is the Hessian of $q^i(\boldsymbol{\varphi})$, and $\mathbf{z}^j=\left[I-\phi\Psi 2\widetilde{G}\right]^{-1}\mathbf{1}_j$. The Hessian $\left(D^2\boldsymbol{\varphi}\right)_{jj}$ exists if $\left[I-\phi\Psi 2\widetilde{G}\right]$ is invertible: a property that, as shown above, is verified if $\Psi\leq\Psi^*$. Since $\left(\omega'(s_A^l)\right)^2/\omega''(s_A^l)$ is bounded for any feasible s_A^l, and $\mathbf{z}^j$ is a positive column vector with l element $z_l^j\leq\overline{z}$ for some finite $\overline{z}$, the ith term of the second term in the parenthesis in the right hand side of (2) is bounded above by $\Psi^2\frac{\omega''(s_A^l)}{\omega'(s_A^l)}\overline{vz}^2\sum_v\sum_k q_{v,k}^i(\boldsymbol{\varphi})$. It follows that:

$$\left(D^2\boldsymbol{\varphi}\right)_{jj}=\left[I-\phi\Psi 2\widetilde{G}\right]^{-1}\Psi\omega'(s_A^l)\left(\mathbf{1}_j+o(\Psi^2)\right), \tag{3}$$

where $o(\Psi^2)$ converges to zero as $\Psi\to 0$ at the speed of Ψ^2. By the fact that $\left[I-\phi\Psi 2\widetilde{G}\right]^{-1}$ is positive and $\omega''(s_A^l)<0$, it follows that $\sum_i\partial^2\varphi_i/\partial s_A^j\partial s_A^j=\left(D^2\boldsymbol{\varphi}\right)_{jj}\cdot 1<0$ for a sufficiently small Ψ. We conclude that the diagonal of the Hessian of $\sum_i\varphi_i$ has all strictly negative values. Following the same steps as above it we can also show that for any ε there is a Ψ_3 such that the absolute values of the off diagonal elements of the Hessian of $\sum_i\varphi_i$ are lower than ε for $\Psi\leq\Psi_3$. This implies that there is a Ψ^* such that $\sum_i\varphi_i$ is increasing and strictly concave in respectively s_A^j and $\mathbf{s}_A$ for $\Psi\leq\Psi^*$. ■

2 Proof of Lemma 3.1

The fact that all agents of the same type have the same Bonacich centrality is immediate from the definition. We can write:

$$b_i(\phi^*,G^T)=1+\phi^*\textstyle\sum_{l=1}^n g_{l,i}\cdot b_l(\phi^*,G^T)=1+\phi^*\sum_{\tau=1}^m n_\tau h_{\tau,\iota(i)}\cdot\overline{b}_\tau$$

where $\overline{b}_\tau$ be the Bonacich centrality of an agent of type τ. Since, again, $b_i(\phi^*,G)=\overline{b}_{\iota(i)}$, we have: $\overline{b}_{\iota(i)}=1+\phi^*\sum_{\tau=1}^m\widetilde{h}_{\tau,\iota(i)}\cdot\overline{b}_\tau$ where $\widetilde{h}_{i,j}=n_\tau h_{\tau,\iota(i)}=\alpha_j h_{i,j}/(\sum_l\alpha_l h_{i,l})$, since $\sum_l\alpha_l h_{i,l}=\sum_l g_{i,l}/n=1/n$. We therefore have that $\overline{\mathbf{b}}=\left[I+\phi^*\widetilde{H}^T\right]^{-1}\cdot\mathbf{1}$, implying that $b_i(\phi^*,G)$ is defined by (30) as stated. ■

3 Proof of Lemma 3.2

We first note that by Assumption 1 $\varphi_j\leq\overline{\varphi}$, $\varphi_j\geq\underline{\varphi}$ for some $\overline{\varphi}$ and $\underline{\varphi}$ in $(0,1)$, any legislator j and any $\mathbf{s}_A,\mathbf{s}_B$. Given this, we proceed in two steps.

Step 1. We prove here that $\lim_{n\to\infty} q^{n,j} = 0$ for all $j = 1, ..., n$. Consider the pivot probability of a player j of type i. There are two cases to consider.

Case 1.1. Suppose first that $\alpha_i^n \to \alpha_i > 0$. Let M_{-i}^n be the profile of votes of all types different from i. Let P_i^n be the probability that there is a profile of votes M_{-i}^n such that j can be pivotal for some profile $m_i^{-j,n}$ of players of type i different than j. Let $p_j^n(M_{-i}^n)$ be the probability of $m_i^{-j,n}$ such that j is pivotal given M_{-i}^n and let $\overline{p}_j^n = \max_{M_{-i}^n} p_j(M_{-i}^n)$. Associated to $\overline{p}_j^n$ there is a number $\widehat{l}_j^n$ of legislators of type i that must vote for A in order for j to be pivotal. Let $\eta_j^n = \widehat{l}_j^n / (n_i - 1)$ the share of types i other than j that are needed to make j pivotal. If $\eta_j^n \to 1$ or $\eta_j^n \to 0$ then $\overline{p}_j^n$ converges to zero, so j's pivot probability converges to zero. Assume $\eta_j^n \to \eta_j \in (0, 1)$. Given this, j's pivot probability can be bounded above as follows. To keep the formulas simple, let $z_i(n) = \alpha_i n - 1$

$$
\begin{aligned}
\lim_{n\to\infty} q^{n,i} &\leq P_i^n \cdot \lim_{n\to\infty} b(\eta_j^n z_i(n); z_i(n), \varphi^i) \\
&\leq \lim_{n\to\infty} \begin{pmatrix} z_i(n) \\ \\ \eta_j^n z_i(n) \end{pmatrix} \left((\varphi^i)^{\eta_j^n z_i(n)} \cdot (1-\varphi^i)^{(1-\eta_j^n) z_i(n)} \right) \\
&\leq \lim_{n\to\infty} \frac{\left(\sqrt{2\pi z_i(n)} \cdot (z_i(n))^{z_i(n)} e^{-z_i(n)}\right) \cdot \left((\varphi^i)^{\eta_j^n z_i(n)} \cdot (1-\varphi^j)^{(1-\eta_j^n) z_i(n)} \right)}{\left(\begin{array}{c} \left(\sqrt{2\pi \eta_j^n z_i(n)} \cdot (\eta_j^n z_i(n))^{\eta_j^n z_i(n)} e^{-\eta_j^n z_i(n)}\right) \\ \cdot \sqrt{2\pi (1-\eta_j^n) z_i(n)} \cdot \left(\left(1-\eta_j^n\right) z_i(n)\right)^{(1-\eta_j^n) z_i(n)} e^{-(1-\eta_j^n) z_i(n)} \end{array} \right)} \\
&= \lim_{n\to\infty} \frac{1}{\left(\left(\sqrt{2\pi\eta_j^n}\right) \cdot \sqrt{(1-\eta_j^n)}\right)} \cdot \left(\frac{\left((\varphi^i)^{\eta_j^n} \cdot (1-\varphi^j)^{1-\eta_j^n} \right)}{(\eta_j^n)^{\eta_j^n} \left(1-\eta_j^n\right)^{1-\eta_j^n}} \right)^{z_i(n)} \cdot \frac{1}{\sqrt{z_i(n)}} \\
&\leq \frac{1}{\left(\left(\sqrt{2\pi\eta_j}\right) \cdot \sqrt{(1-\eta_j)}\right)} \lim_{n\to\infty} \frac{1}{\sqrt{z_i(n)}} = 0,
\end{aligned}
$$

where the third inequality follows from the Stirling formula and the last follows from the fact that $\eta_j^n \in \arg\max_\varphi \left((\varphi)^{\eta_j^n} (1-\varphi)^{1-\eta_j^n} \right)$.

Case 1.2. Consider now that case in which $\alpha_i^n \to 0$. Let M_{-jk}^n the profile of votes of: 1) all types i but different than agent j; and 2) of all other types $t \neq i, k$, where k is a type such that $\alpha_k^n \to \alpha_k > 0$. Let P_{-jk}^n be the probability that there is a profile of votes M_{-jk}^n such that j can be pivotal for some profile m_k^n of players in k. Let $p_j^n(M_{-jk})$ be the probability of m_k^n such that j is pivotal given M_{-jk} and let $\overline{p}_j^n = \max_{M_{-jk}^n} p_j(M_{-jk}^n)$. As above the pivot probability $q^{n,i}$ can be bounded above by $P_{-jk}^n \cdot \overline{p}_{jk}^n$. Proceeding as in the previous case, we can prove that this upper bound converges to zero as $n \to \infty$, implying the result.

Step 2. Consider now $\sum_j \left| q_j^{n,i} \right|$. For any two distinct legislators i and j, let N^{-ij} and $\boldsymbol{\varphi}^{-ij}$ be, respectively, the set of all legislators except i and j and the associated vector of probabilities of choosing P. Let moreover $S(N^{-i}, s)$ be the set of all s-combinations of N^{-ij}. We have that for any $j \neq i$, $q^{n,i} = \varphi_j E_n + (1-\varphi_j) F_j$ where:

$$E_n = \sum_{A \in S(N^{-ij}, qn-2)} \prod_{k \in A} \varphi_k^{-ij} \cdot \prod_{l \in A^C} (1 - \varphi_l^{-ij})$$
$$F_n = \sum_{A \in S(N^{-ij}, qn-1)} \prod_{k \in A} (\varphi_k^{-ij}) \cdot \prod_{l \in A^C} \left(1 - \varphi_l^{-ij}\right)$$

We can therefore write: $q_j^{n,i} = (E_n - F_n)$. From Step 1 we know that $q^{n,i} \to 0$ as $n \to \infty$ for all i. It follows from (5) in Section 3.1, that $\varphi_i \to 1/2$ for all legislators. This implies that, for all i, $|E_n - F_n|$ can be bounded above by: $K_n = \Theta \begin{pmatrix} n \\ qn \end{pmatrix} ((1+\delta)/2)^n$ where $\Theta > 1$, and $\delta > 0$ is a parameter that can be chosen arbitrarily close to 0 for n sufficiently large. It follows that $\sum_j \left| q_j^{n,i} \right|$ is bounded above by nK_n. Using again the Stirling formula we have:

$$\lim_{n\to\infty} nK_n = \lim_{n\to\infty} \frac{n\Theta\sqrt{2\pi n} n^n e^n}{\left[\begin{array}{c} \sqrt{2\pi qn}\,(qn)^{qn}\, e^{qn} \cdot \\ \sqrt{2\pi(1-q)n}\,[(1-q)n]^{(1-q)n}\, e^{(1-q)n} \end{array} \right]} ((1+\delta)/2)^n$$
$$= \frac{\Theta}{\sqrt{2\pi q(1-q)}} \lim_{n\to\infty} \left(n^{1/2} \left(\frac{2 \cdot q^q (1-q)^{1-q}}{1+\delta} \right)^{-n} \right) = \frac{\Theta}{\sqrt{2\pi q(1-q)}} \lim_{n\to\infty} \left(n^{1/2} (1-\epsilon)^n \right)$$

for some $\epsilon > 0$, where the last equality follows from the fact since, δ is arbitrarily small, $2 \cdot q^q (1-q)^{1-q} / (1+\delta) > 1$ for any $q \in (1/2, 1)$. Since $\lim_{n\to\infty} \left(n^{1/2} (1-\epsilon)^n \right) = 0$, we have that $\sum_j \left| q_j^{n,i} \right|$ converge to zero. ■

4 Derivation of Equation 19 in Section 4.1

The necessary and sufficient condition (17) in Section 4.1 for interest group $l = A, B$ is

$$\sum_j \left(\partial \varphi_j(\mathbf{s}_A, \mathbf{s}_B) / \partial s_l^i \cdot \theta_j \right) = \lambda,$$

where λ is the Lagrangian multiplier associated to the constraint. In matrix form as $D\boldsymbol{\varphi}^T \cdot \boldsymbol{\theta} = \lambda$ and using (9) in Section 3.2, we have:

$$D\boldsymbol{\varphi}^T \cdot \boldsymbol{\theta} = \Psi \cdot D\boldsymbol{\omega}^T \cdot \left(I - \phi^* \cdot G^T \right)^{-1} \boldsymbol{\theta} = \lambda. \quad (4)$$

Let $\mathbf{b}^\theta(\boldsymbol{\phi}^*, \mathbf{G}) = \left(I - \phi^* \cdot G^T \right)^{-1} \cdot \boldsymbol{\theta}$ be the weighted Bonacich centrality measure, with $\mathbf{b}^\theta(\boldsymbol{\phi}^*, \mathbf{G}) = \left(b_1^\theta(\phi^*, G), .., b_n^\theta(\phi^*, G) \right)$. The first order condition (4) can then be written as: $b_j^\theta(\phi^*, G)\omega \left(s_A^j \right) = \lambda_*$, where $\lambda_* = \lambda / \Psi$. ■

5 Proof of the result stated in Section 5.3

Let us define $\beta_l^n(s_A, s_B)$ as the probability that threshold l is passed for $l = 1, ..., J$, $\beta_l^n(s_A, s_B) = \Pr\left(\sum_i x_i^n(A) > z_l \,|\mathbf{s}_A, \mathbf{s}_B\right)$. With preferences that depend on reaching the threshold z_j, interest group A's expect utility can be written as: $W_n^{\mathbf{z},\mathbf{u}}(\mathbf{s}_A, \mathbf{s}_B) = u_0 + \sum_{l=0}^{J}(u_l - u_{l-1})\,\beta_l^n(\mathbf{s}_A, \mathbf{s}_B)$. The equilibrium contributions are characterized by the first order necessary condition of:

$$\max_{(\mathbf{s}_A, \mathbf{s}_B) \in S} W_n^{\mathbf{z},\mathbf{u}}(\mathbf{s}_A, \mathbf{s}_B). \tag{5}$$

The necessary condition of the corresponding Lagrangian with respect to s_A^j where j is an agent of type i:

$$\frac{\partial W_n^{\mathbf{z},\mathbf{u}}(\mathbf{s}_A, \mathbf{s}_B)}{\partial s_A^j} = \sum_k \left(\sum_l (u_l - u_{l-1}) \frac{\partial \beta_l^n}{\partial \varphi_k} \right) \cdot \frac{\partial \varphi_k^n}{\partial s_A^j} = \lambda^n \tag{6}$$

where $\partial \beta_l^n / \partial \varphi_k$ and $\partial \varphi_k^n / \partial s_A^j$ are the derivatives of $\beta_l^n(s_A, s_B)$ and $\varphi_k^n(s_A, s_B)$ with respect to, respectively, φ_k^n and $s_A^{n,j}$ evaluated at $\widetilde{\mathbf{s}}$ and λ^n is chosen to satisfy the budget constraint. It is easy to verify that $\partial \beta_l^n / \partial \varphi_k$ is equal to the probability that legislator k is "pivotal" in having threshold l passed, that is $\partial \beta_l^n / \partial \varphi_k = \beta_l^{-k,n}$ where $\beta_l^{-k,n} = \Pr\left(\sum_{i \neq k} x_i^n(A) = z_l \,|\mathbf{s}_*, \mathbf{s}_*\right)$. We can rewrite (6) as:

$$\frac{\sum_{k=1}^n (R_k^n / R_1^n) \cdot \partial \varphi_k^n / \partial s_A^j}{\sum_{k=1}^n (R_k^n / R_1^n) \cdot \partial \varphi_k^n / \partial s_A^l} = 1,$$

where $R_k^n = \sum_l \left[(u_l - u_{l-1}) \cdot \partial \beta_l^n / \partial \varphi_k\right]$. Note that, by Lemma 3.2, $q_n^i \to 0$ as $n \to \infty$, so by (5) in section 3.1 we must have that the probability that i votes for A is $\varphi_{i,n} \to 1/2$ as $n \to \infty$. This implies that $\beta_l^{-k} / \beta_1^{-k} \to 1$ and so $R_j^n / R_1^n \to 1$ for any $j = 1, .., m$. It follows that

$$\begin{aligned}
\lim_{n\to\infty} \frac{\sum_{k=1}^n (R_k^n / R_1^n) \cdot \partial \varphi_k^n / \partial s_A^j}{\sum_{k=1}^n (R_k^n / R_1^n) \cdot \partial \varphi_k^n / \partial s_A^l} &= \lim_{n\to\infty} \frac{\sum_{k=1}^n \partial \varphi_k^n / \partial s_A^j}{\sum_{k=1}^n \partial \varphi_k^n / \partial s_A^l} = \lim_{n\to\infty} \frac{b_j^{\mathcal{M}}(\phi^*, V, G^T)\omega'(s_A^j)}{b_l^{\mathcal{M}}(\phi^*, V, G^T)\omega'(s_A^l)} \\
&= \lim_{n\to\infty} \frac{b_j(\phi^*, G^T)}{b_l(\phi^*, G^T)} \frac{\omega'(s_A^j)}{\omega'(s_A^l)} = 1 \quad \forall j, l
\end{aligned}$$

where the second equality follows from the analysis of $D\boldsymbol{\varphi}^T \cdot 1$ in Section 7.2 and $\left(b_i(\phi^*, G^T)\right)_{i=1}^n$ are the limit Bonacichs. We conclude that for a large n, we have $\frac{\omega'(s_A^j)}{\omega'(s_A^l)} \simeq \frac{b_l(\phi^*, G^T)}{b_j(\phi^*, G^T)}$, or $b_j(\phi^*, G^T)\omega'(s_A^j) \simeq \lambda$ for all $j = 1, ..., n$. Assuming log utility as in Section 4 of the paper, we have $s_A^j \simeq b_j(\phi^*, G^T)$ for all $j = 1, ..., n$. ■

References

Debreu G. and I. N. Herstein [1953]: "Nonnegative Square Matrices," *Econometrica*, 21(4): 597-607.

TABLE A.1. Summary statistics

	Variable definition	Committee network		Alumni network		*p-value*
		Mean	**St. Dev**	**Mean**	**St. Dev**	
PAC Contributions ($Mil)	PAC Contributions to a member of Congress, excluding contributions from individuals and Super PACs, source: http://opensecrets.org.	886,284	989,801.6	891,450.8	1,021,031	*0.8883*
Party (1=Republican)	Dummy variable taking value of one if the member of Congress is a Republican.	0.5061	0.5000	0.4734	0.4999	*0.2608*
Gender (1=Female)	Dummy variable taking value of one if the member of Congress is female.	0.1738	0.3790	0.1732	0.3786	*0.9635*
Chair (1=Yes)	Dummy variable taking value of one if the member of Congress is a chair of at least one committee.	0.0469	0.2116	0.0497	0.2175	*0.7261*
Seniority	Maximum consecutive years in the same committee	7.6433	6.2492	7.7581	6.4334	*0.6207*
Margin of Victory	Election Margin of Victory	0.3518	0.2496	0.3622	0.2585	*0.2634*
Per capita Income	Mean Per Capita Income in Political District	26,815.48	8,377.558	26,772.33	8,480.09	*0.8884*
DW_ideology	Distance to the center in terms of ideology of each member of Congress measured using the absolute value of the first dimension of the dw-nominate score created by McCarty et al. (1997)	0.5012	0.2221	0.4993	0.2292	*0.8182*
Relevant Committee (1=Yes)	Dummy variable taking value of one if the member of Congress sits on one of the powerful committees (Appropriations, Energy and Commerce, Financial Services, Rules or Ways and Means).	0.5446	0.4981	0.4485	0.4975	*0.7071*
Joint Committee (1=Yes)	Dummy variable taking value of one if the member of Congress is in a joint committee.	0.0559	0.2298	0.0643	0.2454	*0.3368*
Top 10 university (1=Yes)	Top 10 universities according to the 2014 ranking of http://www.usnews.com/education	0.0657	0.2479	0.1140	0.3180	*0.000*
N. obs		2,128	2,128	1,166	1,166	

Notes: We report the p-values of the T-tests for equality in means between the committee network and alumni network samples.

TABLE A.2. Estimation results
Increasing set of control variables
-Committee network-

	Dep. Var.: PAC contributions ($mil)						
	MLE (1)	**MLE (2)**	**MLE (3)**	**MLE (4)**	**MLE (5)**	**MLE (6)**	**MLE (7)**
Φ	0.3649*** (0.0671)	0.2309*** (0.0714)	0.2894*** (0.0703)	0.22143*** (0.0679)	0.2084*** (0.0697)	0.20884*** (0.0697)	0.2165*** (0.0703)
Party (1=Republican)			-0.0874** (0.0430)	0.1519*** (0.0569)	0.1399** (0.0570)	0.1443** (0.0573)	0.1473*** (0.0011)
Gender (1=Female)			-0.1341** (0.0561)	-0.0986* (0.0534)	-0.0975* (0.0534)	-0.0950* (0.0535)	-0.09472*** (0.001)
Chair (1=Yes)			0.3774*** (0.1016)	0.3966*** (0.0969)	0.3992*** (0.097)	0.4006*** (0.0967)	0.3959*** (0.0020)
Seniority			-0.0249*** (0.0035)	-0.0168*** (0.0034)	-0.0154*** (0.0034)	-0.0154*** (0.0034)	-0.0153*** (0.00001)
Margin of Victory				-0.8428*** (0.0867)	-0.8991*** (0.088)	-0.8972*** (0.0885)	-0.8959*** (0.0019)
Per capita Income				0.0075*** (0.0025)	0.0064** (0.0025)	0.0061** (0.0025)	0.0062*** (0.00004)
DW_ideology				-1.0876*** (0.124)	-1.0771*** (0.1241)	-1.0774*** (0.1241)	-1.0817*** (0.0031)
Relevant Committee (1=Yes)					0.10437** (0.0413)	0.1037** (0.0413)	0.0998*** (0.0007)
Joint Committee (1=Yes)					0.1695** (0.0861)	0.1694** (0.0861)	0.1669*** (0.0016)
Top 10 university (1=Yes)						0.0581 (0.0809)	0.0579*** (0.0011)
Unobservables (ψ)							-0.1132*** (0.0016)
Intercept	0.5628*** (0.0631)	0.5767*** (0.0711)	0.7881*** (0.0781)	1.3219*** (0.1071)	1.3032*** (0.1072)	1.3019*** (0.1072)	1.2949*** (0.0629)
Time dummies	No	Yes	Yes	Yes	Yes	Yes	Yes
N. obs.	2,128	2,128	2,128	2,128	2,128	2,128	2,128

Notes: ML estimated coefficients and standard errors (in parentheses) are reported. In column (7) standard errors are bootstrapped with 1000 replications. A precise definition of control variables can be found in Table A.1. *, **, *** indicate statistical significance at the 10, 5 and 1 percent levels.

TABLE A.3. Estimation results
Increasing set of control variables
-Alumni network-

	Dep. Var.: PAC contributions ($mil)					
	MLE (1)	**MLE (2)**	**MLE (3)**	**MLE (4)**	**MLE (5)**	**MLE (6)**
Φ	0.1025***	0.0819***	0.0743***	0.0837***	0.0858***	0.0837***
	(0.0273)	(0.0273)	(0.0271)	(0.0261)	(0.0261)	(0.0262)
Party (1=Republican)			-0.0629	0.2243***	0.2112***	0.2212***
			(0.0608)	(0.0791)	(0.0792)	(0.0801)
Gender (1=Female)			-0.1422*	-0.0743	-0.0731	-0.0685
			(0.0793)	(0.076)	(0.076)	(0.0761)
Chair (1=Yes)			0.4377***	0.4733***	0.4736***	0.4759***
			(0.1382)	(0.1322)	(0.1321)	(0.1321)
Seniority			-0.0289***	-0.0186***	-0.0170***	-0.0169***
			(0.0047)	(0.0046)	(0.0047)	(0.0047)
Margin of Victory				-0.7281***	-0.7835***	-0.7793***
				(0.1174)	(0.1202)	(0.1202)
Per capita Income				0.0080**	0.0073**	0.0067*
				(0.0034)	(0.0035)	(0.0035)
DW_ideology				-1.1363***	-1.1167***	-1.1171***
				(0.1669)	(0.1670)	(0.1670)
Relevant Committee (1=Yes)					0.1143**	0.1135**
					(0.0575)	(0.0575)
Joint Committee (1=Yes)					0.0792	0.0810
					(0.1128)	(0.1128)
Top 10 university (1=Yes)						0.0790
						(0.0900)
Intercept	0.80009***	0.66081***	0.93568***	1.33895***	1.29062***	1.2895***
	(0.0383)	(0.0711)	(0.0893)	(0.1309)	(0.1331)	(0.1330)
Time dummies	No	Yes	Yes	Yes	Yes	Yes
N. obs.	1,166	1,166	1,166	1,166	1,166	1,166

Notes: ML estimated coefficients and standard errors (in parentheses) are reported. A precise definition of control variables can be found in Table A.1. *, **, *** indicate statistical significance at the 10, 5 and 1 percent levels.