

Diagnostic Expectations and Credit Cycles

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Abstract

We present a model of credit cycles arising from diagnostic expectations – a belief formation mechanism based on Kahneman and Tversky’s (1972) representativeness heuristic. In this formulation, when forming their beliefs agents overweight future outcomes that have become more likely in light of incoming data. The model reconciles extrapolation and neglect of risk in a unified framework. Diagnostic expectations are forward looking, and as such are immune to the Lucas critique and nest rational expectations as a special case. In our model of credit cycles, credit spreads are excessively volatile, over-react to news, and are subject to predictable reversals. These dynamics can account for several features of credit cycles and macroeconomic volatility.

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1.Introduction

The financial crisis of 2008-2009 revived economists' and policymakers' interest in the relationship between credit expansion and subsequent financial and economic busts. According to an old argument (e.g., Minsky 1977), investor optimism brings about the expansion of credit and investment, and leads to a crisis when such optimism abates. Stein (2014) echoes this view by arguing that policy-makers should be mindful of credit market frothiness and consider countering it through policy. In this paper, we develop a behavioral model of credit cycles with micro-founded expectations, which yields the Minsky narrative but is also consistent with a great deal of evidence.

Recent empirical research has developed a number of credit cycle facts. Schularick and Taylor (2012) demonstrate, using a sample of 14 developed countries between 1870 and 2008, that rapid credit expansions forecast declines in real activity. Jorda, Schularick, and Taylor (2013) further find that more credit-intensive expansions are followed by deeper recessions. Mian, Sufi, and Verner (2015) show that the growth of household debt predicts economic slowdowns. Baron and Xiong (2014) establish in a sample of 20 developed countries that bank credit expansion predicts increased crash risk in both bank stocks and equity markets more broadly. And Fahlenbrach, Prilmeier, and Stulz (2016) find, in a cross-section of U.S. banks, that fast loan growth predicts poor loan performance and low bank returns in the future.

Parallel findings emerge from the examination of credit market conditions. Greenwood and Hanson (2013) show that credit quality of corporate debt issuers deteriorates during credit booms, and that high share of risky loans in the total forecasts low, and even negative, corporate bond returns. Gilchrist and Zakrajsek (2012) and Krishnamurthy and Muir (2015) relatedly establish that eventual credit tightening correctly anticipates the coming recession. Lopez-Salido, Stein, and

Zakrajsek (hereafter LSZ 2015) find that low credit spreads predict both a rise in credit spreads and low economic growth afterwards. They stress predictable mean reversion in credit market conditions². We show further that survey forecasts of future credit spreads are excessively optimistic when these spreads are low, and that both errors and revisions in forecasts are predictable. This evidence is inconsistent with rational expectations, and suggests a need for a behavioral approach to modeling credit cycles.

In this paper, we propose a psychological model of investor expectations and credit cycles that accounts for the evidence described above, and articulates in a fully dynamic setup the phenomenon of credit market overheating. It implies that in a boom investors are excessively optimistic and systematically become more pessimistic in the future, leading to a crisis even without deteriorating fundamentals. The model unifies the phenomena of extrapolation (Cagan 1956, Cutler et al. 1990, DeLong et al. 1990, Barberis and Shleifer 2003, Greenwood and Shleifer 2014, Barberis et al. 2015a, b, Gennaioli, Ma, and Shleifer 2015) and the neglect of risk (Gennaioli, Shleifer, and Vishny 2012, Coval, Pan, and Stafford 2014, Arnold, Schuette, and Wagner 2015). Critically, households in our model are forward looking, and recognize policy shifts. As such, the model is not vulnerable to the Lucas critique. Indeed, for any data generating process, rational expectations emerge a special case of our model.

Our principal contribution is to write down a psychologically-founded model of beliefs and their evolution in light of new data.³ The model we propose is taken from a very different context and adapted to macroeconomic problems, rather than just

² An older literature on financial asset prices and economic activity includes Bernanke (1990), Friedman and Kuttner (1992), and Stock and Watson (2003), among others.

³ Many models of beliefs in finance are motivated by psychological evidence, but often use specifications specialized to financial markets (e.g., Muth 1961, Barberis, Shleifer, and Vishny 1998, Rabin and Vayanos 2010, Fuster, Laibson, and Mendel 2010, Hirshleifer et al 2015, Greenwood and Hanson 2015, Barberis et al. 2015a,b). Fuster et al (2010) review evidence from lab and field settings documenting deviations from rational expectations.

designed to match credit cycle facts. It is portable in the sense of Rabin (2013). Our model of belief evolution is based on Gennaioli and Shleifer's (2010) formalization of Kahneman and Tversky's (KT 1972, TK 1983) representativeness heuristic describing how people judge probabilities. According to KT, people estimate types with a given attribute to be excessively common in a population when that attribute is representative or diagnostic for these types, meaning that it occurs more frequently among these types than in the relevant reference class. For instance, beliefs about the Irish exaggerate the share of red haired people among them because red hair is much more common among the Irish than in the average national group, even though the true share of red-haired Irish is small. Similarly, after seeing a patient test positive on a medical test for a disease doctors overestimate the likelihood that he has it because being sick is representative of testing positive, even when it remains unlikely despite a positive test (Casscells et al. 1978). Our formalization of representativeness accounts for several well-documented judgment biases, such as the conjunction and disjunction fallacies and base rate neglect. It also delivers a model of stereotypes consistent with empirical evidence (Bordalo, Coffman, Gennaioli, and Shleifer (BCGS) 2016).

This formalization of representativeness can be naturally applied to modeling expectations. Analogously to the medical test example, agents focus on, and thus overweight in their beliefs, the future states whose likelihood increases the most in light of current news *relative to what they know already*. Just as doctors overestimate the probability of sickness after a positive test result, agents overestimate the probability of a good (bad) future state when the current news is good (bad). Following TK (1983)'s description of the representativeness heuristic as overweighting diagnostic information, we refer to such beliefs as diagnostic expectations.

This approach has significant implications. For example, a path of improving news leads to excess optimism, and a path of deteriorating news to excess pessimism, even when these paths lead to the same fundamentals. There is a kernel of truth in assessments: revisions respond to news, but excessively. When change slows down, the agent no longer extrapolates. This in itself leads to a reversal. Excessively volatile expectations drive cyclical fluctuations in both financial and economic activity.

We construct a neoclassical macroeconomic model in which the only non-standard feature is expectations. In particular, we do not include financial or any other frictions. The model accounts for many empirical findings, some of which also obtain under rational expectations, but some do not. In our model:

- 1) In response to good news about the economy, credit spreads decline, credit expands, the share of high risk debt rises, and investment and output grow.
- 2) Following this period of narrow credit spreads, these spreads predictably rise on average, credit and the share of high risk debt decline, while investment and output decline as well. Larger spikes in spreads predict lower GDP growth.
- 3) Credit spreads are too volatile relative to fundamentals and their changes are predictable in a way that parallels the cycles described in points 1) and 2).
- 4) Investors commit predictable forecast errors and forecast revisions. Bond returns are also predictable in a way that parallels points 1) and 2).

Prediction 1) can obtain under rational expectations, and the same is true about prediction 2) provided fundamentals are mean reverting. Predictions 3) and 4), in contrast, critically depend on our model of diagnostic expectations.

Our paper is related to four strands of research. First, the prevailing approach to understanding the link between financial markets and the real economy is financial frictions, which focus on the transmission of an adverse shock through a leveraged

economy (Bernanke and Gertler 1989, Kiyotaki and Moore 1997)⁴. The adverse shock in such models is either a drop in fundamentals, or a “financial shock” consisting of the tightening of collateral constraints or an increase in required returns. These models do not usually explain the sources of “financial shocks”. As importantly, because they assume rational expectations, these models do not explain predictable negative or low abnormal returns on debt in over-heated markets or systematic errors in expectations. Our model explains both sudden market collapses and abnormal returns.

Second, our paper relates to the growing body of research on extrapolative expectations in financial markets. Barberis, Shleifer, and Vishny (1998) were the first to connect representativeness and extrapolation, but did not have a micro-founded model of representativeness. Our contribution is to provide such a micro-foundation of expectation formation, and to use it to simultaneously account for extrapolation and the closely related phenomenon of the neglect of risk in a unified framework.

Third, our paper is related to recent work on limited attention (e.g., Sims 2003, Gabaix 2014). In general, these models predict sluggish expectations and under-reaction to information, consistent with empirical evidence for inflation in particular (Coibion and Gorodnichenko 2012, 2015). Also related is research on momentum and slow reaction to information in financial markets (Jegadeesh and Titman 1993, and Stein 1999, Bouchard et al. 2016). Our model most naturally delivers over-reaction to information, although we discuss briefly how the two approaches can be unified.

Finally, our paper continues the small literature on behavioral credit cycles, initiated by Minsky (1977) but with very few models available so far. Gennaioli, Shleifer, and Vishny (2012) present an early formulation, focusing on the neglect of risk.

⁴ Some papers add to financial frictions Keynesian elements, such as the zero lower bound on interest rates or aggregate demand effects (e.g., Eggertson and Krugman 2012, Rognlie et al. 2015).

Gennaioli, Shleifer, and Vishny (2015) sketch a model of credit cycles exhibiting both under-reaction and over-reaction based on the BCGS (2016) model of stereotypes. Jin (2015) models extrapolation in credit markets. Greenwood, Hanson, and Jin (2016) present a model of extrapolation of default rates which also delivers many of the credit cycle facts. Our contribution is to unify several theories, as well as a good deal of evidence on credit cycles and credit spreads, in a micro-founded model of beliefs.

In section 2, we present some evidence that both errors in forecasts of credit spreads, and revisions in these forecasts, can be predicted from information available at the time initial forecasts are made. Section 3 introduces diagnostic expectations, describes how they evolve, and relates our formulation to extrapolation and neglect of risk. Section 4 presents our model of credit cycles, and examines some initial implications of diagnostic expectations. Section 5 develops the predictions of the model for the behavior of credit spreads, expectations about credit spreads, and the link between credit spreads and economic activity. Section 6 concludes. An Appendix discusses some alternative specifications of the diagnostic expectations model.

2. Some Evidence on Expectations and Credit Spreads

We start by offering some motivating evidence on analysts' expectations of the Baa bond – Treasury credit spread. Both here and in our theory we focus on the credit spread, a commonly used indicator of credit market conditions (Greenwood and Hanson 2013). We do not have enough data to test rational expectations, but can illustrate how the analysis of credit cycles can be deepened with expectations data, and to establish some facts that a model of expectations formation should account for⁵.

⁵ One may worry that Blue Chip professional forecasts are distorted for signaling or entertainment reasons since participants are not anonymous. However, these forecasts tend to be very similar to the

We use data from Blue Chip Financial Forecasts, a monthly survey of around 40 panelists' forecasts of various interest rates for the current quarter and for each of 6 quarters ahead. We average forecasts over four quarters ahead to obtain 12-month forecasts; we then construct consensus forecasts by averaging expectations across analysts. We use data from the March, June, September, and December publications. We construct implied forecasts of the Baa spread as the difference between the forecasts of the Baa corporate bond yield and of the 10Y Treasury yield. We also construct a spread using the 5Y Treasury yield. Forecasts of Baa yields start in 1999Q1 and end in 2014Q4, which is thus the period we focus on.

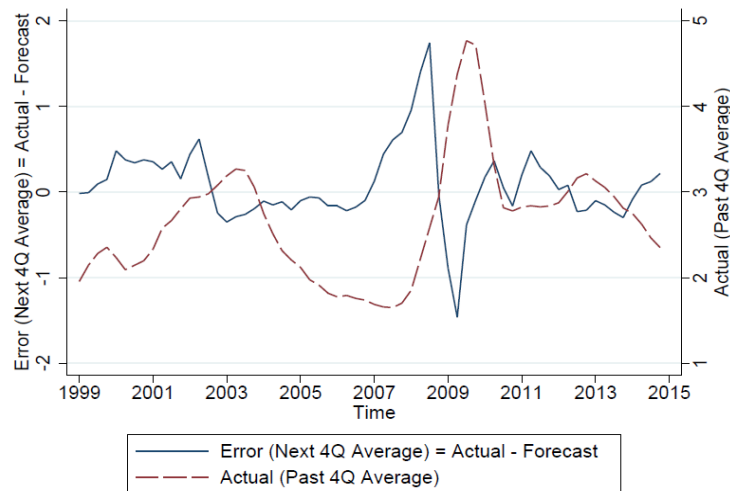
2.1 Predictability in Forecast Errors

Under the joint assumptions of rational expectations and knowledge of the data generating process, analysts' forecast error should be unpredictable from past information. Figure 1 plots, over time, the current spread against the error in the forecast of the future spread. The data suggest predictability: when the current spread is low, the expected spread is too low (forecast errors are systematically positive). Likewise, when the current spread is high, the expected spread is too high. The 1999-2000 and 2005-2008 periods witness low spreads and excessive optimism, the early 2000s and the recent crisis witness high spreads and excess pessimism.

Table 1 reports an econometric test of predictability. Column 1 estimates an AR(1) process for the Baa-10Y spread, column 2 regresses analysts' forecast on the current spread, column 3 regresses the future forecast error on the current spread. Columns 4, 5 and 6 repeat the analysis using the 5Y treasury yield.

anonymous forecasts collected by the Philadelphia Fed Survey of Private Forecasters. Moreover, unlike in the case of stock analysts, there is no unconditional bias in the Blue Chip forecasts we study here.

Figure 1: Predictable Errors in Forecasts of Credit Spread



	Baa-10Y spread			Baa-5Y spread		
	Actual	Forecast	Error	Actual	Forecast	Error
Avg. spread past year	0.3927 [1.67]	0.6519 [4.62]	-0.2592 [-2.20]	0.5711 [2.66]	0.7815 [6.06]	-0.2104 [-2.13]
Constant	1.6280 [2.56]	0.8596 [2.25]	0.7684 [2.40]	1.4822 [2.01]	0.6793 [1.54]	0.8029 [2.36]
Observations	64	64	64	64	64	64
R ²	0.158	0.472	0.161	0.348	0.646	0.172

Table 1: Actual, Forecast, and Error of Next Year Average Credit Spread
(Actual Forecast Error=Actual-Forecast, Newey West t-statistics in brackets)

Table 1 confirms the message of Figure 1. In column 3, the higher the current spread, the higher is the forecast relative to the realization. This may occur because analysts see excessive persistence in current conditions: in Column 1 the estimated persistence of the actual Baa-10Y spread is about 0.4, but in column 2 forecasts follow the current spread with a coefficient of about 0.6, and similarly for the Baa-5Y spread.

2.2 Tests of Expectations' Revisions

The above results may be consistent with rationality if the assumed AR(1) model is misspecified, and spreads are also influenced by an unobservable but persistent shock. In this case, after receiving good news fully rational analysts become optimistic

about the unobservable shock. Forecasts can then overshoot because good news might instead be due to the transitory shock. To address this possibility, we examine analysts' forecast revisions. Under rational inference these revisions should be unpredictable.

Figure 2 plots the current spread against the future forecast revision, defined as the difference between the forecast for the spread in quarter $t + 4$ made in quarter $t + 3$ and the current (quarter t) forecast of the same spread. The evidence again suggests predictability: when the current spread is low, forecasts are subsequently revised upwards, when the current spread is high, forecasts are revised downward.

Figure 2: Predictable Revisions in Forecasts of Credit Spread

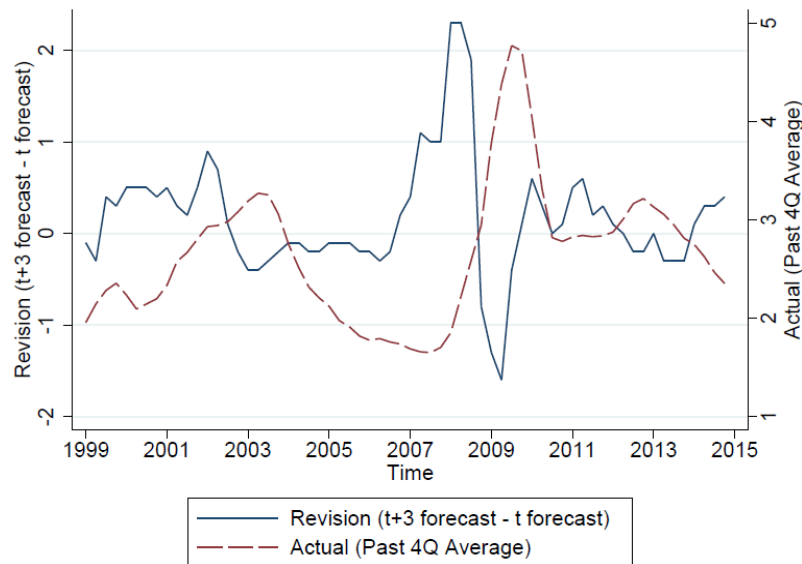


Table 2 shows that this predictability is statistically robust.

	Revision of Baa-10Y spread	Revision of Baa-5Y spread
Avg. spread past year	-0.3636 [-2.13]	-0.3115 [-1.89]
Constant	1.1334 [2.44]	1.2865 [2.27]
Observations	64	64
R ²	0.152	0.156

Table 2: Forecast Revisions in Credit Spreads
(Revision=Forecast($t+3$)-Forecast(t), Newey West t-statistics in brackets)

This evidence is difficult to reconcile with rational expectations, even allowing for model misspecification. It instead suggests that analysts' forecasts follow a boom bust pattern. During booming bond markets (low spreads), expectations are too optimistic and systematically revert in the future, creating the seeds for a cooling of bond markets. Although we cannot draw sharp conclusions due to small sample size, the extrapolative dynamics of forecasts that we document here are in line with other studies, such as with Greenwood and Hanson's (2013) evidence of systematic reversal in bond spreads, as well as with the extrapolative nature of CFO's expectations about their company's earnings growth (Gennaioli et al. 2015). In the next section we present a model that accounts for these dynamics of beliefs as a consequence of Kahneman and Tversky's representativeness heuristic.

3. Diagnostic Expectations

3.1 A Formal Model of Representativeness

We build our model of expectations from first principles, starting with research on heuristics and biases in human decision-making. One of Kahneman and Tversky's most universal heuristics is representativeness, which they define as follows: "an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in the relevant reference class (TK 1983)." KT argue that individuals often assess likelihood by representativeness, thus estimating types or attributes as being likely when they are instead representative, and present a great deal of experimental evidence to support this claim. Gennaioli and Shleifer (2010) build a model in which judgment biases arise because decision makers overweight events that are representative precisely in the sense of KT's definition. To

motivate our model of diagnostic expectations, we summarize some of this work as well as a related application to stereotype formation by Bordalo et al. (BCGS 2016).

A decision maker judges the distribution of a trait T in a group G . The true distribution of the trait is $h(T = t|G)$. GS (2010) formally define the representativeness of the trait $T = t$ for group G to be:

$$\frac{h(T = t|G)}{h(T = t|-G)}$$

where $-G$ is a relevant comparison group. As in KT's quote, a trait is more representative if it is *relatively* more frequent in G than in $-G$. GS (2010) assume that representative types are easier to recall. Due to limited working memory, the agent overweighs these types in his assessment. Crucially, in this model beliefs about a group G are context dependent, namely they depend on features of the comparison group $-G$.

To illustrate, consider an individual assessing the distribution of hair color among the Irish. The trait T is hair color, the conditioning group G is the Irish. The comparison group $-G$ is the world at large. The true relevant distributions are:⁶

	$T = red$	$T = blond/light brown$	$T = dark$
$G \equiv Irish$	10%	40%	50%
$-G \equiv World$	1%	14%	85%

The most representative hair color for the Irish is red because it is associated with the highest likelihood ratio among hair colors:

$$\frac{Pr(red\ hair|Irish)}{Pr(red\ hair|World)} = \frac{10\%}{1\%} = 10.$$

Our model thus predicts that assessments exaggerate the frequency of red haired Irish.

⁶ Shares of red hair are accurate, shares of brown and dark are approximate. See http://www.eupedia.com/genetics/origins_of_red_hair.shtml and http://www.eupedia.com/europe/genetic_maps_of_europe.shtml.

After hearing the news “Irish”, the representative red-haired type quickly comes to mind and its likelihood is inflated. It is not that blond or dark haired types are not considered, but the agent discounts their probability because these types are less available when thinking about the Irish.

This example also illustrates context dependence of beliefs. It is the paucity of red haired people in the “rest of the world” that renders red hair so distinctive for the Irish. The judgment bias would be smaller if the share of red haired people in the rest of the world were to rise, or equivalently if the agent was primed to think about the Irish in the context of a more similar group (e.g., $-G = Scots$). BCGS (2016) show that this model explains many empirical features of stereotypes, including evidence that they contain a “kernel of truth” (Judd and Park 1993) as well as their context dependence.

Consider next base rate neglect, another well documented bias in information processing. A doctor must assess the health of a patient in light of a positive medical test. Here $T = \{healthy, sick\}$, G is patients who tested positive, while $-G$ is untested patients. Being sick is representative of a patient who tested positive provided:

$$\frac{\Pr(T = sick|G = positive)}{\Pr(T = sick|-G = untested)} > \frac{\Pr(T = healthy|G = positive)}{\Pr(T = healthy|-G = untested)}$$

This condition is always satisfied provided the test is minimally informative, in the sense that a positive test raises the likelihood of having the disease. After a positive test, the “sick” type jumps to mind and the doctor inflates its probability. Consistent with the evidence collected by Casscells et al. (1978), physicians may greatly inflate the probability of diseases that are very rare, committing a form of base rate neglect.⁷ A significant literature in psychology explores this finding, and the mechanism above

⁷ Casscells et al (1978) asked physicians: “If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5 per cent, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person's symptoms or signs?” While the correct answer is about 2%, they found an average response of 56% and a modal answer of 95%.

captures TK's (1974) verbal account of base rate neglect.

The general mechanism in both examples is that representativeness causes the agent to inflate the likelihood of types whose *objective* probability rises the most in G relative to the reference *context* – G . This is “red hair” in the Irish example, and “sick” in the medical example. In both cases, representativeness causes diagnostic information to be overweighed and beliefs to depend on context – G .

In GS (2010) and BCGS (2016) we show that this model of the representativeness heuristic offers a unified account of widely documented judgment biases, of key features of social stereotypes, and of context dependent beliefs. The same logic can be naturally applied to studying the evolution of beliefs in macroeconomics.

3.2 Diagnostic Expectations

We next extend our model to belief formation about aggregate economic conditions. Time is discrete $t = 0, 1, \dots$. The state of the economy at t is captured by a random variable $\Omega_t \in \mathbb{R}$, whose realization is denoted by ω_t . This random variable follows the AR(1) process $\omega_t = b\omega_{t-1} + \epsilon_t$, with $\epsilon_t \sim N(0, \sigma^2)$, $b \in [0, 1]$. The model is easily generalized to richer AR(N)-normal processes.

When forming a forecast the agent assesses the distribution of a future state, say Ω_{t+1} , entailed by current conditions $\Omega_t = \omega_t$. This is similar to the medical test example, where the doctor assesses the health of the patient conditional on a positive test outcome. Pursuing the analogy, the agent must predict future prospects Ω_{t+1} in a group $G \equiv \{\Omega_t = \omega_t\}$ that summarizes current conditions.

The rational agent solves this problem by using the true conditional distribution $h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)$. The agent judging by representativeness has this true distribution in the back of his mind, but selectively retrieves and thus overweighs the

likelihood of future states ω_{t+1} that are representative or diagnostic of $G \equiv \{\Omega_t = \omega_t\}$ relative to the background context $-G$. But what is $-G$ here?

In the Irish example, representativeness assesses $G = \textit{Irish}$ against $-G = \textit{Rest of the world}$. In the medical test example, the information $G = \textit{Positive test}$ is assessed against its absence $-G = \textit{Not taking the test}$, so that context captures absence of new information. Here we adopt this dynamic perspective by taking context at time t to reflect only information held at $t - 1$. Specifically, we take context to be the state prevailing if there is no news. Formally, under the assumed AR(1), we take context to be $-G \equiv \{\Omega_t = b\omega_{t-1}\}$. A future state ω_{t+1} is thus more representative at t if it is more likely under the realized state $G \equiv \{\Omega_t = \omega_t\}$ than under the reference state capturing past information $-G \equiv \{\Omega_t = b\omega_{t-1}\}$. Representativeness of ω_{t+1} is then given by:

$$\frac{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)}{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = b\omega_{t-1})}. \quad (3)$$

The most representative state is the one exhibiting the largest increase in its likelihood based on recent news. Definition (3) captures two psychologically relevant features: first, in the absence of news, the true distribution $h(\Omega_{t+1} = \omega_{t+1} | G)$ coincides with the reference distribution $h(\Omega_{t+1} = \omega_{t+1} | -G)$ so that no state is particularly representative. Second, good (bad) news render states in the right tail strictly more (less) representative.⁸ There are other ways of specifying the comparison group $-G$: it could be slow moving – including more remote recollections – or it may be specified in terms of diagnostic expectations. In the Appendix, we discuss these cases.

⁸ These properties would not hold if context was defined as the past state $-G \equiv \{\Omega_{t-1} = \omega_{t-1}\}$, making this otherwise similar specification much less tractable. In particular, the distributions $h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)$ and $h(\Omega_{t+1} = \omega_{t+1} | \Omega_{t-1} = \omega_{t-1})$ have different variances, so measuring representativeness relative to the past state would distort not only the mean, but also the variance, of $h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)$, even in the absence of news. The representation obtained in (4) below – and the tractability it entails – extends to several classes of distributions, including lognormal and exponential.

The psychology of diagnostic expectations works as follows. After seeing current news $\Omega_t = \omega_t$, the most representative future states immediately come to mind. Memory limits then imply that the agent over samples representative states from the true distribution $h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)$, which is stored in memory. As a consequence, beliefs inflate the probability of more representative states and deflate the probability of less representative states. In light of Equation (3), we formalize overweighting of representative states “as if” the agent uses the distorted density:

$$h_t^\theta(\omega_{t+1}) = h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t) \cdot \left[\frac{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)}{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = b\omega_{t-1})} \right]^\theta \frac{1}{Z},$$

where the normalizing constant Z ensures that $h_t^\theta(\omega_{t+1})$ integrates to one, and $\theta \in (0, +\infty)$ measures the severity of judging by representativeness. When $\theta = 0$, the agent has no memory limits and he appropriately uses all information, forming rational expectations. When $\theta > 0$, memory is limited and the agent does not fully correct for the fact that some states are less available. Thus, the distribution $h_t^\theta(\omega_{t+1})$ inflates the likelihood of representative states and deflates the likelihood of non-representative ones. Because they overweight the most representative, or diagnostic, future outcomes, we call the expectations formed in light of $h_t^\theta(\omega_{t+1})$ diagnostic.

Parameter θ captures the limits of working memory. In what follows we take θ to be fixed. In principle, however, memory can also depend on the agent’s deliberate effort and attention. This possibility may cause θ to vary across situations.⁹

In this formulation, news do not just alter the objective likelihood of certain states. They also change the extent to which the agent focuses on them. An event that increases the likelihood of a future state ω_{t+1} also makes it more representative, so

⁹ Another possibility is that the distortions caused by θ may arise – and be particularly strong – when the agent does not know the true data generating process and data are very noisy. It is possible to extend our formalization to learning problems of this sort, but here we use a simpler formulation to illustrate the implications of representativeness more starkly.

$h_t^\theta(\omega_{t+1})$ overshoots. The reverse occurs when the likelihood of ω_{t+1} decreases. If the likelihood ratio in (2') is monotone increasing, “rationally” good news cause the agent to overweight high future states, and to underweight low future states (the converse is true if news are bad). In this sense, good news cause neglect of downside risk.

Proposition 1 *When the process for ω_t is AR(1) with normal $(0, \sigma^2)$ shocks, the diagnostic distribution $h_t^\theta(\omega_{t+1})$ is also normal, with variance σ^2 and mean:*

$$\mathbb{E}_t^\theta(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]. \quad (4)$$

Diagnostic expectations are represented by a linear combination of the rational expectations of ω_{t+1} held t and at $t - 1$. It is not that decision-makers compute rational expectations and combine them according to Proposition 1. Rather, oversampling representative future states yields the linear combination in (4). This formula reflects a “kernel of truth” logic: diagnostic expectations differ from rational expectations by a shift in the direction of the information received at t , given by $[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]$.

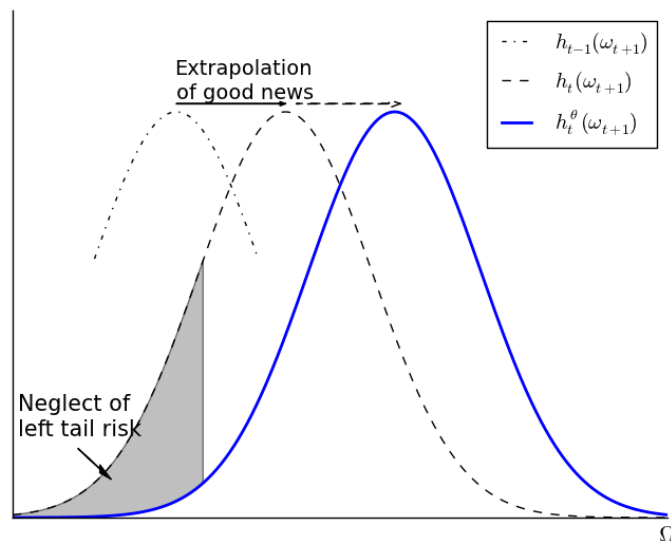


Figure 3: Neglect of risk and Extrapolation

Figure 3 illustrates the entailed neglect of risk. After good news, the diagnostic distribution of ω_{t+1} is a right shift of the objective distribution. Due to the monotone increasing and unbounded likelihood ratio of normal densities, good news cause under-estimation of probabilities in the left tail (the shaded area). In fact, neglect of risk and extrapolation are connected by the same psychological mechanism. For the AR(1) process as $\omega_t = b\omega_{t-1} + \epsilon_t$, with persistence parameter b , Equation (4) becomes:

$$\mathbb{E}_t^\theta(\omega_{t+1}) - \omega_t = [\mathbb{E}_t(\omega_{t+1}) - \omega_t] + b \cdot \theta \cdot [\omega_t - \mathbb{E}_{t-1}(\omega_t)],$$

namely the current shock $\omega_t - \mathbb{E}_{t-1}(\omega_t)$ is extrapolated into the future, but only if the data are serially correlated, $b > 0$. Diagnostic expectations exaggerate the role of new information, consistent with Kahneman's (2011) view that "our mind has a useful capability to focus spontaneously on whatever is odd, different, or unusual."

Because diagnostic expectations are forward looking, they address the Lucas critique.¹⁰ Since diagnostic expectations distort the true distribution $h_t(\omega_{t+1})$, they respond to policy shifts that affect $h_t(\omega_{t+1})$. The diagnostic distribution $h_t^\theta(\omega_{t+1})$ incorporates changes in the objective frequency (as do rational expectations) but also changes in representativeness. Thus, if the government commits to inflating the economy, inflation expectations will also react upwards.¹¹

It is straightforward to extend diagnostic expectations to longer term forecasts.

Corollary 1 *When the process for ω_t is AR(1) with normal $(0, \sigma^2)$ shocks, the diagnostic expectations for ω_{t+T} is given by:*

¹⁰ The Lucas critique holds that mechanical models of expectations cannot be used for policy evaluation because expectation formation in such models does not respond to changes in policy. Indeed, empirical estimates of adaptive expectations processes revealed parameter instability to policy change. This instability led researchers to prefer rational expectations, which account for regime shifts.

¹¹ Muth (1961) generalizes rational expectations to allow for systematic errors in expectations. His formula is precisely of the linear form of Equation (4): relative to rationality, expectations distort the effect of recent news. Muth's formulation naturally follows from the psychology of representativeness.

$$\mathbb{E}_t^\theta(\omega_{t+T}) = \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})], \quad (5)$$

Furthermore, we have that $\mathbb{E}_t^\theta(\omega_{t+T}) = \mathbb{E}_t^\theta(\mathbb{E}_{t'}^\theta(\omega_{t+T}))$ for any $t < t' < t + T$.

Longer term forecasts can also be represented as a linear combination of past and present rational expectations. Furthermore, diagnostic expectations obey the law of iterated expectations with respect to the distorted expectations \mathbb{E}_t^θ , so that forecast revisions are unpredictable from the vantage point of the decision maker.

However, forecast revisions are predictable using the true probability measure, because errors in expectations correct on average in the future. Using (4) we find:

$$\mathbb{E}_{t-1}[\mathbb{E}_t^\theta(\omega_{t+T})] = \mathbb{E}_{t-1}(\omega_{t+T}),$$

which is the rational forecast. On average, diagnostic expectations revert to rational expectations because in our model the distortion of expectations is a linear function of news, and the average news is zero by definition. Even if expectations are inflated at $t - 1$, they return to rationality on average at t . As we show in Section 5, this behavior allows us to account for the empirical findings in Section 2.

4. A Model of Credit Cycles

We next introduce diagnostic expectations into a simple macroeconomic model and show that the psychology of representativeness generates excess volatility in expectations about credit spreads, over-heating and over-cooling of credit markets, as well as predictable reversals in credit spreads and economic activity that are consistent with the evidence of Section 2, as well as with many other features of credit cycles.

4.1. Production

A measure 1 of atomistic firms uses capital to produce output. Productivity at t depends on the state ω_t , but to a different extent for different firms. A firm is identified by its risk $\rho \in \mathbb{R}$. Firms with higher ρ are less likely to be productive in any state ω_t . If a firm ρ enters period t with invested capital k , its current output is given by:

$$y(k|\omega_t, \rho) = \begin{cases} k^\alpha & \text{if } \omega_t \geq \rho \\ 0 & \text{if } \omega_t < \rho \end{cases} \quad (6)$$

where $\alpha \in (0,1)$. The firm produces only if it is sufficiently safe, $\rho < \omega_t$. The safest firms, for which $\rho = -\infty$, produce k^α in every state of the world. The higher is ρ , the better the state ω_t needs to be for the firm to pay off. At the same capital k , two firms produce the same output if they are both active, namely if $\omega_t \geq \rho$ for both firms.

A firm's riskiness is common knowledge and it is distributed across firms with density $f(\rho)$. Capital for production at $t + 1$ must be installed at t , before ω_{t+1} is known. Capital fully depreciates after usage. At time t each firm ρ demands funds from a competitive financial market to finance its investment. The firm issues risky debt that promises a contractual interest rate $r_{t+1}(\rho)$. Debt is repaid only if the firm is productive: if at t the firm borrows $k_{t+1}(\rho)$ at the interest rate $r_{t+1}(\rho)$, next period it produces and repays $r_{t+1}(\rho)k_{t+1}(\rho)$ provided $\omega_{t+1} \geq \rho$, and defaults otherwise.

Because there are no agency problems and each firm's output has a binary outcome, the model does not distinguish between debt and equity issued by the firm. Both contracts are contingent on the same outcome and promise the same rate of return. For concreteness, we refer to the totality of capital invested as debt.

4.2 Households

A risk neutral, infinitely lived, representative household discounts the future by a

factor $\beta < 1$. At each time t , the household allocates its current income between consumption and investment by maximizing its expectation of the utility function:

$$\sum_{s=t}^{+\infty} \beta^{s-t} c_s.$$

The household consumes and purchases the claims issued by firms, which then pay out or default in the next period. Its income consists of the payout of debt bought in the previous period, the profits of firms (which are owned by the household), and a fixed endowment w that we assume to be large enough:¹²

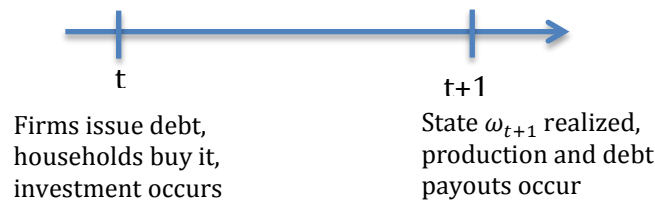
A.1 $w \geq (\alpha\beta)^{\frac{1}{1-\alpha}}$.

At each time s and state ω_s , then, the household's budget constraint is:

$$c_s + \int_{-\infty}^{+\infty} k_{s+1}(\rho) f(\rho) d\rho = w + \int_{-\infty}^{+\infty} I(\rho, \omega_s) [r_s(\rho) k_s(\rho) + \pi_s(\rho)] f(\rho) d\rho,$$

where c_s is consumption, $k_{t+1}(\rho)$ is capital supplied to firm ρ , $I(\rho, \omega_s)$ is an indicator function equal to one when firm ρ repays, namely when $\omega_s \geq \rho$, and $\pi_s(\rho)$ is the profit of firm ρ when active. The household's income depends, via debt repayments, on the state of the economy: the worse is the current state (the lower is ω_s), the higher is the fraction of defaulting firms and thus the lower is the household's income.

The timeline of an investment cycle in the model is illustrated below.



Investment decisions by households and firms depend on the *perceived* probability with which each firm type ρ repays its debt in the next period. Under the

¹² As we show later, this condition ensures that the equilibrium expected return is equal to β^{-1} .

assumed normal shocks, at time t the perceived probability with which firm ρ repays at time $t + 1$ (i.e. the assessed probability of positive output at $t + 1$) is given by:

$$\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = \int_{\rho}^{+\infty} h_t^\theta(x) dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{\rho}^{+\infty} e^{-\frac{(x-\mathbb{E}_t^\theta(\omega_{t+1}))^2}{2\sigma^2}} dx. \quad (7)$$

The perceived probability of default is then $1 - \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)$. A perfectly safe firm $\rho \rightarrow -\infty$ never defaults, since $\lim_{\rho \rightarrow -\infty} \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = 1$. When $\theta = 0$, we are in the case of rational expectations and the perceived probability of default is computed according to the true conditional distribution $h(\Omega_{t+1} = \omega | \Omega_t = \omega_t)$. When $\theta > 0$, the distortions of diagnostic expectations affect the perceived safety of different firms. In what follows, we refer to $\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)$ as the “perceived creditworthiness” of firm ρ .

4.3 Capital Market Equilibrium and Credit Spreads

At time t firm ρ demands capital $k_{t+1}(\rho)$ at the market contractual interest rate $r_{t+1}(\rho)$ so as to maximize its expected profit at $t + 1$:

$$\max_{k_{t+1}(\rho)} (k_{t+1}(\rho))^\alpha - k_{t+1}(\rho) \cdot r_{t+1}(\rho) \cdot \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right). \quad (8)$$

The first order condition for the profit maximization problem is given by:

$$k_{t+1}(\rho) = \left[\frac{\alpha}{r_{t+1}(\rho)} \right]^{\frac{1}{1-\alpha}}, \quad (9)$$

which is the usual downward sloping demand for capital.

Households are willing to supply any amount of capital to firm ρ provided the interest rate $r_{t+1}(\rho)$ makes the household indifferent between consuming and saving:

$$r_{t+1}(\rho) \cdot \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = \beta^{-1} \Leftrightarrow r_{t+1}(\rho) = \frac{1}{\beta \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}. \quad (10)$$

In equilibrium, this condition must hold for all firms ρ . On the one hand, no arbitrage requires all firms to yield the same expected return. On the other hand, such expected return cannot be below β^{-1} . If this were the case, the household would not invest and the marginal product of capital would be infinite, leading to a contradiction. But the expected return of debt cannot be above β^{-1} either. If this were the case, the household would invest the totality of its income. Under A.1, however, this implies that the marginal product of capital would fall below β^{-1} , again leading to a contradiction.

From Equation (10), we can compute the spread obtained on the debt of risky firm ρ at time t as the difference between the equilibrium $r_{t+1}(\rho)$ and the safe rate β^{-1} :

$$S(\rho, \mathbb{E}_t^\theta(\omega_{t+1})) = \left(\frac{1}{\mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))} - 1 \right) \beta^{-1}. \quad (11)$$

Risky firms must compensate investors for bearing their default risk by promising contractual interest rates above β^{-1} . The spread at t depends on the firm's riskiness ρ and of current expectations of the aggregate economy. Greater optimism $\mathbb{E}_t^\theta(\omega_{t+1})$ lowers spreads by improving perceived creditworthiness $\mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))$. Greater riskiness ρ enhances spreads by reducing perceived creditworthiness.

By combining Equations (11) and (9) we obtain:

$$k_{t+1}(\rho) = \left[\frac{\alpha\beta}{1 + \beta S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))} \right]^{\frac{1}{1-\alpha}}, \quad (12)$$

which links expectations to credit spreads, investment and output. When times are good, households are optimistic and $\mathbb{E}_t^\theta(\omega_{t+1})$ is high. As a consequence, spreads are compressed, firms issue debt and expand investment. When times turn sour, households become pessimistic, spreads rise and firms cut debt issuance and

investment. As we show later, Equation (12) can be aggregated across different values of ρ to obtain aggregate investment at time t and output at $t + 1$.

We can now generate some testable implications of our model. Using Equation (11), define the average spread at time t as:

$$S_t = \int_{-\infty}^{+\infty} S(\rho, \mathbb{E}_t^\theta(\omega_{t+1})) f(\rho) d\rho. \quad (13)$$

Here S_t represents an inverse measure of optimism, which is strictly monotonically decreasing in the expectations argument. When investors are more optimistic, i.e. $\mathbb{E}_t^\theta(\omega_{t+1})$ is higher, average perceived creditworthiness is higher, and hence the average spread S_t charged on risky debt is lower.

We can substitute this inverse measure of optimism for expectations $\mathbb{E}_t^\theta(\omega_{t+1})$ in Equations (11) and (12). An increase in the average spread S_t corresponds to an increase in the spreads, as well as a decrease in investment, of all firms ρ . We obtain the following result for the cross section of firms:

Proposition 2. *Lower optimism $\mathbb{E}_t^\theta(\omega_{t+1})$ and thus higher spread S_t at time t causes:*

i) a disproportionate rise in the spread of riskier firms:

$$\frac{\partial^2 S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial S_t \partial \rho} > 0.$$

ii) a disproportionate decline in debt issuance and investment by riskier firms:

$$\frac{\partial k_{t+1}(\rho_1)}{\partial S_t k_{t+1}(\rho_2)} < 0 \quad \text{for all } \rho_1 > \rho_2.$$

Because it is more sensitive to aggregate conditions, investment by riskier firms fluctuates more with expectations and displays more co-movement with credit markets.

These predictions of the model are consistent with the evidence of Greenwood and Hanson (2013). They document that when the Baa-credit spread falls, bond

issuance increases and the effect is particularly strong for firms characterized by higher expected default rates. As a consequence, the share of non-investment grade debt over total debt (the “junk share”) increases, as has also been documented by LSZ (2015).

This behavior of the junk share follows directly from property ii) above, which implies that the share of debt issued by firms riskier than an arbitrary threshold $\hat{\rho}$:

$$\frac{\int_{\hat{\rho}}^{+\infty} k_{t+1}(\rho) f(\rho) d\rho}{\int_{-\infty}^{+\infty} k_{t+1}(\rho) f(\rho) d\rho},$$

unambiguously increases as spreads become compressed (for any $\hat{\rho}$).

The qualitative effects described in Proposition 2 do not rely on diagnostic expectations and obtain even if households are fully rational. Diagnostic expectations have distinctive implications for the behavior over time of equilibrium credit spreads as well as of their expectations by market participants. We now turn to this analysis.

5. Diagnostic Expectations and Equilibrium Credit Spreads

We investigate the link between expectations and the dynamics of the equilibrium credit spread by considering a linearized version of Equation (11). A first order expansion of Equation (11) with respect to investors’ expectations $\mathbb{E}_t^\theta(\omega_{t+1})$ around the long run mean of zero yields:

$$S(\rho, \mathbb{E}_t^\theta(\omega_{t+1})) \approx \frac{1}{\beta} \cdot \left[\frac{1}{\mu(\rho, 0)} - 1 \right] - \frac{\mu'(\rho, 0)}{\beta \mu(\rho, 0)^2} \cdot \mathbb{E}_t^\theta(\omega_{t+1})$$

The spread drops as expectations improve (since $\mu'(\rho, 0) > 0$), but more so for riskier firms (the slope coefficient increases in ρ). Aggregating this equation across all firms ρ and denoting by $\sigma_0, \sigma_1 > 0$ the average intercept and slope, we find that the average spread at time t approximately satisfies:

$$S_t = \sigma_0 - \sigma_1 \mathbb{E}_t^\theta(\omega_{t+1}). \tag{14}$$

Inserting into (14) the expression for $\mathbb{E}_t^\theta(\omega_{t+1})$ of Equation (4), under the maintained assumption of AR(1) fundamentals $\omega_t = b\omega_{t-1} + \epsilon_t$, we establish:

Proposition 3. *The average credit spread S_t follows an ARMA(1,1) process given by:*

$$S_t = (1 - b)\sigma_0 + b \cdot S_{t-1} - (1 + \theta)b\sigma_1\epsilon_t + \theta b^2\sigma_1\epsilon_{t-1}. \quad (15)$$

Under rational expectations ($\theta = 0$) the equilibrium spread, like fundamentals, follows an AR(1) process characterized by persistence parameter b . Starting from the long run spread σ_0 , after a positive fundamental shock $\epsilon_t > 0$ expectations improve and the spread declines. After this initial drop, the spread on average gradually returns to σ_0 . The reverse occurs after a negative piece of news $\epsilon_t < 0$: spreads go up on impact and then monotonically return to σ_0 .

Under diagnostic expectations, $\theta > 0$, credit spreads continue to have an autoregressive parameter b but now also contain a moving average component. The spread at time t now depends also on the shock experienced at $t - 1$. If the news received in the previous period were good, $\epsilon_{t-1} > 0$, so that S_{t-1} was low, there is a discrete hike in the spread at time t . If the news received in the previous period were bad, $\epsilon_{t-1} < 0$, so that S_{t-1} was high, there is a discrete drop in the spread at time t .

These delayed corrections occur on average (controlling for mean reversion in fundamentals) and correspond to the systematic correction of errors in diagnostic forecasts described in Section 3. In particular, the over-reaction $\theta\epsilon_{t-1}$ at $t - 1$ reverses on average at t . Reversal of optimism about fundamentals contaminates the spread, which exhibits the predictable non-fundamental reversal of Equation (15). When at $t - 1$ news are good, $\epsilon_{t-1} > 0$, optimism is excessive and the spread S_{t-1} drops too far. Next period this excess optimism wanes on average, so that S_t is corrected upwards.

The reverse occurs if at $t - 1$ news are bad. In this sense, the making and un-making of expectational errors cause boom bust cycles and mean reversion in spreads.

We next show that the equilibrium behavior of credit spreads in (15) accounts for the findings on expectational errors of Section 2. In section 5.2, we show that the model also accounts for the evidence on the link between credit spreads and economic activity that is hard to explain under rational expectations.

5.1 Credit Spread Forecasts

In Section 2, we provided some evidence concerning expectations of credit spreads, namely that forecasts of credit spreads exhibit predictable errors due to an extrapolative nature of expectations, and that these forecasts exhibit systematic reversals. To connect the model to this evidence, we now describe how agents with diagnostic expectations form forecasts of credit spreads.

Investors forecast future credit spreads using the structural equation (14). As a consequence, the forecast made at t for the spread at $t + T$ is given by:

$$\mathbb{E}_t^\theta(S_{t+T}) = \sigma_0 - \sigma_1 \mathbb{E}_t^\theta[\mathbb{E}_{t+T}^\theta(\omega_{t+T+1})].$$

By exploiting Proposition 1 and Corollary 1 we obtain:

Lemma 1 *The T periods ahead diagnostic forecast of the spread is given by:*

$$\mathbb{E}_t^\theta(S_{t+T}) = \sigma_0(1 - b^T) + b^T S_t. \quad (16)$$

Diagnostic expectations project the current spread into the future via the persistence parameter b . The more persistent is the process for fundamentals, the greater is the influence of the current spread S_t on forecasts of future spreads.

Critically, unlike the equilibrium process for the spread in (15), the forecast process in (16) does not exhibit reversals. The intuition is simple: diagnostic forecasters

fail to anticipate the systematic reversal in the equilibrium spread realized when their extrapolation of current news turns out to be incorrect.

This idea can help account for the evidence of Section 2.

Proposition 4. *If the equilibrium spread follows (15) and expectations follow (16):*

i) the forecast error at $t + 1$ is predictable in light of information available at t :

$$\mathbb{E}_t[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})] = \theta b^2 \sigma_1 \epsilon_t. \quad (17)$$

ii) the revision of expectations about S_{t+T} occurring between t and $t + s$ is predictable in light of information available at time t :

$$\mathbb{E}_t[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})] = \theta b^{T+1} \sigma_1 \epsilon_t. \quad (18)$$

Forecast errors and forecast revisions are predictable because expectations neglect the systematic reversal of excess optimism or pessimism. As a consequence, current good news about fundamentals, $\epsilon_t > 0$, predict both that the realized spread next period is on average above the forecast (Equation 17) and that longer term forecasts of spreads will be revised upward in the future (Equation 18). The reverse pattern of predictability occurs after bad news $\epsilon_t < 0$.

Equations (17) and (18) can thus account for the evidence of Section 2. In good times $\epsilon_t > 0$ and the spread S_t is excessively low. This reflects excess optimism about the future, so $\mathbb{E}_t[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})] > 0$. The reverse occurs in bad times. Overall, the spread S_t is negatively correlated with the forecast error $\mathbb{E}_t[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})]$, as documented in Column 3 of Table 1.¹³ Similarly, good times in which the spread S_t is

¹³ This negative correlation comes about precisely because, as shown in columns 1 and 2 of Table 1, the coefficient of forecasts $\mathbb{E}_t^\theta(S_{t+1})$ on the current spread S_t is higher than the persistence parameter obtained by fitting an AR(1) to the path of equilibrium spreads. Formally, suppose that expectations are diagnostic. Then, if one fits the AR(1) $S_{t+1} = \gamma + \delta S_t + v_{t+1}$ to the equilibrium spread of Equation (15), the estimated autoregressive coefficient is:

$$\hat{\delta}(\theta) = \frac{\mathbb{E}(S_t S_{t-1})}{\mathbb{E}(S_{t-1} S_{t-1})} = b \frac{(1 + \theta)[1 - \theta b^2] + (\theta b)^2}{(1 + \theta)[1 + \theta - 2\theta b^2] + (\theta b)^2}$$

low predict upward revisions of future forecasts of spreads $\mathbb{E}_t[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})] > 0$. The spread S_t is thus negatively correlated with the forecast revision $\mathbb{E}_t[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})]$ as shown in Table 2. Predictability of both forecast errors and revisions obtains because agents' expectations over-react and extrapolate current spreads into the future without realizing that such over-reaction eventually reverses.

These predictions are illustrated in Figure 4 below. For a simulated path of fundamentals following an AR(1) process, Panel A shows the time series of next-period forecasts of credit spreads, under rational expectations ($\mathbb{E}_{t-1}(S_t)$, dashed blue), and under diagnostic expectations ($\mathbb{E}_{t-1}^\theta(S_t)$, solid red line).¹⁴ For the same simulation, Panel B shows the negative correlation between the current spread S_t and forecast errors $S_{t+1}^\theta - \mathbb{E}_t^\theta(S_{t+1})$.

We can draw another useful comparison between our model and “Natural Expectations” (Fuster et al 2010). Both models share the feature that forecast errors are predictable because individuals underestimate the possibility of reversals. The underlying mechanism is however very different. The natural expectations framework assumes exogenous long-term reversals, and errors in expectations arise because agents fit a simpler AR(1) model to the data. In our model, in contrast, both the process for spreads and the agents' forecast errors are endogenous to diagnostic expectations. Agents extrapolate current news too far into the future, which in turn endogenously generates unanticipated reversals in the equilibrium process for spreads.

If instead one fits the linear expectations model $\mathbb{E}_t^\theta(S_{t+1}) = \varphi + \mu S_t + e_{t+1}$, the estimated coefficient on the current spread is equal to $\hat{\mu} = b$. It is easy to check that the estimated persistence of actuals $\hat{\delta}(\theta)$ is lower than b , the estimated persistence of forecasts. This is because the AR(1) specification picks reversals in the moving average component of actuals by estimating a lower persistence parameter.

¹⁴ The simulated process is $\omega_t = 0.7\omega_{t-1} + \varepsilon_t$, with shocks $\varepsilon_t \sim \mathcal{N}(0,1)$ i.i.d. across time. The simulation started at $\omega_t = 0$ (the long-term mean of the process), and was run for 75 periods. The diagnostic expectation parameter was set at $\theta = 1$.

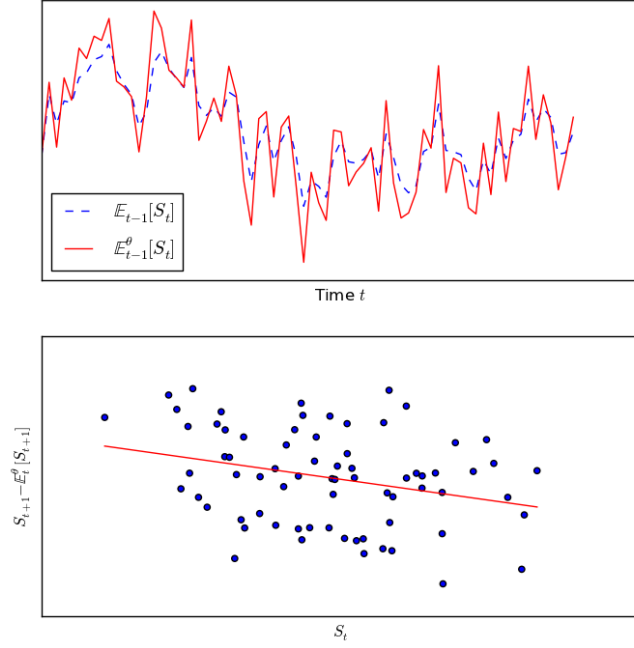


Figure 4: Predictable Errors in Forecasts of Credit Spreads

5.2 Predictability of Returns, Volatility of Spreads, and Economic Activity

Our model can account for some additional evidence. We first consider the evidence on abnormal bond returns and on excess volatility of credit spreads. For our purposes, it is convenient to define the “rational spread” S_t^r as one that would prevail at time t under rational expectations ($\theta = 0$). This spread is the compensation for default risk demanded by rational investors. Proposition 3 then implies:

Corollary 2. *Under diagnostic expectations, $\theta > 0$, the following properties hold:*

i) investors earn predictably low (resp. high) average returns after good (resp. bad) news:

$$S_t - S_t^r = -\theta b \sigma_1 \epsilon_t.$$

ii) credit spreads exhibit excess volatility:

$$\text{Var}[S_t | \omega_{t-1}] = (1 + \theta)^2 \text{Var}[S_t^r | \omega_{t-1}].$$

Predictability of returns comes from errors in expectations. After good news $\epsilon_t > 0$, investors are too optimistic and demand too little compensation for default risk,

$S_t < S_t^r$. The average realized return on bonds is thus below the riskless rate β^{-1} . After bad news $\epsilon_t < 0$, investors are too pessimistic and demand excessive compensation for default risk, $S_t > S_t^r$. The average realized return is above β^{-1} .¹⁵

Expectational errors also underlie excess volatility of spreads. Equilibrium spreads vary too much relative to objective measures of default risk, which are captured by S_t^r , because spreads also reflect investor over-reaction to recent news. Over-reaction to good or bad news causes investors' risk perceptions to be too volatile, which in turn introduces excess volatility into market prices.¹⁶

Greenwood and Hanson (2013) document the pattern of return predictability in Corollary 2. They find that high levels of the junk share predict anomalously low, and even negative, excess returns, and that this occurs precisely after good news, measured by drops in expected default rates (point i).¹⁷ They consider conventional explanations for this finding, such as time varying risk aversion and financial frictions, but conclude that the evidence (particularly the observed frequency of negative returns) is more consistent with the hypothesis that the junk share is a proxy for investor sentiment and extrapolation. Diagnostic expectations offer a psychological foundation for this account.

Additionally, several papers document that credit spreads appear too volatile relative to what could be explained by the volatility in default rates or fundamentals (Collin-Dufresne et al. 2001, Gilchrist and Zakrasjek 2012). For instance, Collin-

¹⁵ Return predictability can also be gauged from Equation (10). The investor's average return at t is in fact equal to $\left[\mu(\rho, \mathbb{E}_t(\omega_{t+1})) / \mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1})) \right] \beta^{-1}$. This is below the risk free rate in good times, when investors are too optimistic $\mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1})) > \mu(\rho, \mathbb{E}_t(\omega_{t+1}))$, and above the risk free rate during periods of pessimism.

¹⁶ Specifically, excess volatility is due to the fact that beliefs do not just depend on the *level* of the current fundamentals ω_t (as would be the case under rational expectations). They also depend on the magnitude ϵ_t of the recently observed news, which corresponds roughly speaking to the *change* in fundamentals.

¹⁷ One intuitive way to see this is to note (see Equation (16)) that the credit terms obtained by riskier firms are more sensitive to the biases caused by diagnostic expectations than those obtained by safer firms. Periods of excess optimism witness an abnormal increase in the junk share and disappointing subsequent returns. Periods of excess pessimism see the reverse pattern.

Dufresne et al. (2001) find that credit spreads display excess volatility relative to measures of fundamentals such as realized default rates, liquidity, or business conditions. They argue this excess volatility can be explained by a common factor that captures aggregate shocks in credit supply and demand. Our model suggests that investors' excessive reaction to changing news can offer an account of these shocks.

The boom-bust cycles in credit spreads shape investment (see Equation 12) and cause in turn overbuilding, underbuilding, and excess volatility in the real economy. Gennaioli, Ma and Shleifer (2015) find that CFOs with more optimistic earnings expectations invest more. Greenwood and Hanson (2015) study empirically investment cycles in the ship industry. Consistent with our model, they find that returns to investing in dry bulk ships are predictable and tightly linked to boom-bust cycles in industry investment. High current ship earnings are associated with higher ship prices and higher industry investment, but predict low future returns on capital.

We next consider the implications of our model for the link between credit markets and economic activity. Krishnamurthy and Muir (2015) and LSZ (2015) document that a tightening of credit spreads at t induces an output contraction in period $t + 1$. Our model yields this pattern as the result of a drop in confidence. A reduction in optimism $\mathbb{E}_t^\theta(\omega_{t+1})$ raises the current spread. Tighter financial conditions in turn cause current debt issuance and investment to decline, leading to a decline in aggregate output at $t + 1$.

There is also growing evidence of systematic reversion in credit conditions and of subsequent output drops. In particular, LSZ (2015) show that low credit spreads at $t - 1$ systematically predict higher credit spreads at t and then a drop in output at $t + 1$. This evidence bears directly on the possibility for our model to generate full-fledged credit cycles. LSZ (2015) do not try to tease out whether the cycle in credit spreads is

due to fundamentals (e.g., mean reversion in the state of the economy) or to fluctuations in investor sentiment. According to the sentiment account, which they seem to favor, a period of excessive investor optimism is followed by a period of cooling off, which they refer to as “unwinding of investor sentiment”. This reversal contributes to a recession over and above the effect of changes in fundamentals.

Diagnostic expectations can account for this “unwinding of investor sentiment”, thereby reconciling predictable reversals in market conditions with abnormal returns and excess volatility of credit spreads.¹⁸ Diagnostic expectations yield predictable unwinding of sentiment through the stochastic ARMA for the equilibrium spread in Proposition 3. Using Equation (15), the rational expectation of credit spreads at time t is given by:

$$\mathbb{E}_{t-1}(S_t) = [(1 - b)\sigma_0 + b \cdot S_{t-1}] + \theta b^2 \sigma_1 \epsilon_{t-1} \quad (19)$$

There are two terms in expression (19). The first term is mean reversion: conditions at t can be predicted to deteriorate if the current spread is lower than the true long run value σ_0 . The second term captures reversals of past sentiment, which is a function of past news $\epsilon_{t-1} = \mathbb{E}_{t-1}(\omega_t) - \mathbb{E}_{t-2}(\omega_t)$. Good news $\epsilon_{t-1} > 0$ at $t - 1$ lead to overheated markets in the same period and to non-fundamental reversal at t . This result has the following implication:

Proposition 5. *Suppose that expectations are diagnostic, $\theta > 0$, and at $t - 1$ credit spreads are too low due to recent good news, namely $\epsilon_{t-1} > 0$. Then:*

i) Controlling for fundamentals at $t - 1$, credit spreads predictably rise at t .

¹⁸ In related settings, Jorda, Schularick and Taylor (2012) document that strong growth of bank loans forecasts future financial crises and output drops. Baron and Xiong (2014) show that credit booms are followed by stock market declines. They document that in good times banks expand their loans, and this expansion predicts future negative returns on bank equity. The negative returns to equity might reflect the unwinding of initial investor optimism, or might be caused by abnormally low realized performance on the bank’s credit decisions (as per Proposition 4). See also Fahlenbrach et al. (2016).

ii) Controlling for fundamentals at $t - 1$, there is a predictable drop in aggregate investment at t and in aggregate production at $t + 1$.

Diagnostic expectations drive a cycle around fundamentals: over-reaction to good news causes credit markets and the economy to overshoot at $t - 1$. The subsequent reversal of such over-reaction causes a drop in credit and economic activity that is more abrupt than could be accounted for by mean reversion in fundamentals. In fact, investor psychology can itself be a cause of volatility in credit, investment, and business cycles, even in the absence of mean reversion in fundamentals, for example if the process for aggregate productivity ω_t is a random walk ($b = 1$).

In sum, diagnostic expectations lead to short-term extrapolative behavior and systematic reversals. This is in line with a large set of recent empirical findings on both financial markets and production, including: i) excess volatility of spreads relative to measures of fundamentals, ii) excessive spread compression in good times and excessive spread widening in bad times (and a similar pattern in the junk share), iii) excessively volatile investment and output, iv) good times predicting abnormally low returns, and finally v) non-fundamental boom bust cycles in credit spreads, driven by transient overreaction to news.

6. Conclusion

We have presented a new approach to modeling beliefs in economic models, diagnostic expectations, based on Kahneman and Tversky's representativeness heuristic. Our model of expectations is portable in Rabin's sense, meaning that the same framework accounts for many experimental findings, the phenomenon of stereotyping, but also critical features of beliefs in financial markets such as extrapolation, over-

reaction, and neglect of risk. Diagnostic expectations are also forward-looking, which means that they are invulnerable to the Lucas critique of mechanical backward looking models of beliefs. We applied diagnostic expectations to a straightforward macroeconomic model of investment, and found that it can account for several empirical findings regarding credit cycles without resort to financial frictions.

Two aspects of our research most obviously require further investigation. First, we have assumed away financial frictions. Indeed, in our model debt is indistinguishable from equity, in that there are no costs of financial distress and no differential legal rights of alternative financial claims, and in particular no collateral constraints. Furthermore, investors are risk neutral, so that debt does not have a special role in meeting the needs of risk averse investors (see Gennaioli, Shleifer, and Vishny 2012). The absence of financial frictions and of risk averse investors leads to completely symmetric effects of positive and negative news. Introducing a more realistic conception of debt might be extremely useful, particularly in the context of analyzing financial crises. In particular, diagnostic expectations may interact with collateral constraints to give rise to additional consequences of tightening credit.

When the economy is hit by a series of good news, investors holding diagnostic expectations become excessively optimistic, fueling as in the current model excessive credit expansion. During such a credit expansion households would pay insufficient attention to the possibility of a bust. As fundamentals stabilize, the initial excess optimism unwinds, bringing this possibility to investors' minds. The economy would appear to be hit by a "financial shock": a sudden, seemingly unjustified, increase in credit spreads. Agents would appear to have suddenly become more risk averse: they now take into account the crash risk they previously neglected.

In the presence of financial frictions, the economy will not go back to its normal course. When excessive leverage is revealed, debt investors try to shed the excessive risk they have taken on, depressing debt prices and market liquidity, particularly if they are very risk averse as in Gennaioli, Shleifer and Vishny (2012, 2015). The tightening of debt constraints causes fire-sales and corporate investment cuts, leaving good investment opportunities unfunded. Such a crisis does not occur because of deteriorating fundamentals, but because the initial excess optimism bursts. Years of bonanza plant the seeds for a financial crisis. A combination of diagnostic expectations and financial frictions could thus lead to models of financial crises that match both the expectations data and the reality of severe economic contractions.

The second set of open questions relates to expectation formation, and the ability of diagnostic expectations and other models in explaining the data. Using data on survey expectations greatly expands the possibilities for building and testing new models. As we have already noted, some economic time series, such as inflation, appear to exhibit under-reaction to data, while others exhibit over-reaction or the combination of the two. This raises both empirical and theoretical challenges. First, it seems critical to understand what are the series, and types of news, where we see under- and over-reaction. This may help determine whether the data are best explained by models of inattention (or slow arrival of information), representativeness, a combination of the two, or neither. Second, as we illustrated in this paper for the case of credit spreads, combining the data on time series with data on survey expectations can provide a good deal of information about the expectation process and its influence on decisions and aggregate outcomes. Building models of beliefs and expectations starting from psychological primitives seems to be a promising area of research in both macroeconomics and finance.

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Proofs

Proposition 1. Let ω_t be an AR(1) process, $(\omega_t - \bar{\omega}) = b(\omega_{t-1} - \bar{\omega}) + \epsilon_t$, with i.i.d. normal $(0, \sigma^2)$ shocks ϵ_t . We now compute diagnostic expectations at a generic horizon $T > 1$. Writing ω_{t+T} as a function of ω_t plus subsequent shocks, we find

$$\omega_{t+T} = b^T \omega_t + (1 - b)\bar{\omega} \sum_{s=0}^{T-1} b^s + \sum_{s=0}^{T-1} b^s \epsilon_{t+s+1}$$

so that the true distribution of ω_{t+T} given ω_t , namely $h(\omega_{t+T}|\omega_t)$, is a normal distribution $\mathcal{N}(\mathbb{E}_t(\omega_{t+T}), \sigma_{t,t+T}^2)$ with mean and variance given by:

$$\mathbb{E}_t(\omega_{t+T}) = b^T \omega_t + (1 - b)\bar{\omega} \sum_{s=0}^{T-1} b^s, \quad \text{and} \quad \sigma_{t,t+T}^2 \equiv \sigma_T^2 = \sigma^2 \sum_{s=0}^{T-1} b^{2s} = \sigma^2 \frac{1 - b^{2T}}{1 - b^2}$$

The reference distribution is $h(\omega_{t+T}|b\omega_{t-1} + (1 - b)\bar{\omega})$, characterized by:

$$\mathbb{E}_t(\omega_{t+T}) = b^{T+1}\omega_{t-1} + (1 - b)\bar{\omega} \sum_{s=0}^T b^s, \quad \text{and} \quad \sigma_{t,t+T}^2 \equiv \sigma_T^2$$

The diagnostic distribution then reads (up to normalization constants):

$$h_t^\theta(\omega_{t+T}) \sim \exp - \frac{1}{2\sigma_T^2} \left[(\omega_{t+1} - \mathbb{E}_t(\omega_{t+T}))^2 (1 + \theta) - \theta (\omega_{t+T} - \mathbb{E}_{t-1}(\omega_{t+T}))^2 \right]$$

The quadratic and linear terms in ω_{t+T} are:

$$\exp - \frac{1}{2\sigma_T^2} \left[\omega_{t+1}^2 - 2\omega_{t+1}(\mathbb{E}_t(\omega_{t+1})(1 + \theta) - \theta \mathbb{E}_{t-1}(\omega_{t+1})) \right]$$

It follows that the diagnostic distribution $h_t^\theta(\omega_{t+T})$ is also a normal distribution $\mathcal{N}(\mathbb{E}_t^\theta(\omega_{t+T}), \sigma_T^2)$ with mean:

$$\mathbb{E}_t^\theta(\omega_{t+T}) = \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})]$$

In particular, for $T = 1$ we get

$$\mathbb{E}_t^\theta(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]$$

■

Corollary 1. Equation (5) follows from the general proof given for Proposition 1. We now compute iterated diagnostic expectations. We start by computing $\mathbb{E}_t^\theta(\mathbb{E}_{t'}(\omega_{t+T}))$ when $t < t' < t + T$. From the perspective of period t , the expectation $\mathbb{E}_{t'}(\omega_{t+T}) = b^{t+T-t'}\omega_{t'} + (1-b)\bar{\omega} \sum_{s=0}^{t+T-t'-1} b^s$ is a normal variable with mean $b^T \omega_t + (1-b)\bar{\omega} \sum_{s=0}^{T-1} b^s$ and variance $\sigma_{t+T-t'}^2$. A calculation parallel to Proposition 1's then shows that a similar representation holds:

$$\mathbb{E}_t^\theta(\mathbb{E}_{t'}(\omega_{t+T})) = \mathbb{E}_t(\mathbb{E}_{t'}(\omega_{t+T})) + \theta[\mathbb{E}_t(\mathbb{E}_{t'}(\omega_{t+T})) - \mathbb{E}_{t-1}(\mathbb{E}_{t'}(\omega_{t+T}))]$$

The law of iterated expectations of the rational operator \mathbb{E}_t then yields:

$$\mathbb{E}_t^\theta(\mathbb{E}_{t'}(\omega_{t+T})) = \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})]$$

In particular, $\mathbb{E}_t^\theta(\mathbb{E}_{t'}(\omega_{t+T}))$ is independent of t' , so that

$$\mathbb{E}_t^\theta(\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})) = 0$$

Intuitively, future distortions are in the kernel of the diagnostic expectations operator, because on average there is no news. Then, from repeated application of Equation (5) we get:

$$\begin{aligned} \mathbb{E}_t^\theta(\mathbb{E}_{t'}^\theta(\omega_{t+T})) &= \mathbb{E}_t^\theta(\mathbb{E}_{t'}(\omega_{t+T}) + \theta[\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})]) \\ &= \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})] = \mathbb{E}_t^\theta(\omega_{t+T}) \end{aligned}$$

■

Proposition 2. For point i), write

$$\frac{\partial^2 S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial S_t \partial \rho} = \frac{\partial^2 S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial \mathbb{E}_t^\theta(\omega_{t+1}) \partial \rho} \cdot \frac{\partial \mathbb{E}_t^\theta(\omega_{t+1})}{\partial S_t}$$

where the last term is negative. The first term reads

$$\frac{\partial^2 S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial \mathbb{E}_t^\theta(\omega_{t+1}) \partial \rho} = \frac{\partial^2 \frac{1}{\beta \mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}}{\partial \mathbb{E}_t^\theta(\omega_{t+1}) \partial \rho} = - \frac{1}{\beta \mu^2(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))} \frac{\partial^2 \mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial \mathbb{E}_t^\theta(\omega_{t+1}) \partial \rho}$$

Now, from $\mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1})) = \frac{1}{\sigma\sqrt{2\pi}} \int_\rho^{+\infty} e^{-\frac{1}{2\sigma^2}[(\omega - \mathbb{E}_t^\theta(\omega_{t+1}))^2]} d\omega$ it follows that

$$\begin{aligned} \frac{1}{\mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))} \frac{\partial \mu_t^\theta(\rho)}{\partial \mathbb{E}_t^\theta(\omega_{t+1})} &= \frac{1}{\sigma^2} \int_\rho^{+\infty} \omega e^{-\frac{1}{2\sigma^2}[(\omega - \mathbb{E}_t^\theta(\omega_{t+1}))^2]} \frac{d\omega}{\mu(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))} - \frac{\mathbb{E}_t^\theta(\omega_{t+1})}{\sigma^2} \\ &= [\mathbb{E}_t^\theta(\omega_{t+1} | \omega_{t+1} \geq \rho) - \mathbb{E}_t^\theta(\omega_{t+1})] \frac{1}{\sigma^2} \end{aligned}$$

This term is strictly positive, and increasing in ρ . This implies that $\frac{\partial^2 S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial \mathbb{E}_t^\theta(\omega_{t+1}) \partial \rho} < 0$

and hence $\frac{\partial^2 S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))}{\partial S_t \partial \rho} > 0$.

To see point ii), use Equation (12) to write:

$$\frac{\partial k_{t+1}(\rho_1)}{\partial S_t k_{t+1}(\rho_2)} = \frac{\partial}{\partial S_t} \left[\frac{1 + \beta S(\rho_2, \mathbb{E}_t^\theta(\omega_{t+1}))}{1 + \beta S(\rho_1, \mathbb{E}_t^\theta(\omega_{t+1}))} \right]^{\frac{1}{1-\alpha}}$$

From point i) we know that $\frac{\partial}{\partial S_t} S(\rho, \mathbb{E}_t^\theta(\omega_{t+1}))$ is increasing in ρ . Therefore,

$$\frac{\partial}{\partial S_t} \frac{1 + \beta S(\rho_2, \mathbb{E}_t^\theta(\omega_{t+1}))}{1 + \beta S(\rho_1, \mathbb{E}_t^\theta(\omega_{t+1}))} < 0$$

for any $\rho_1 > \rho_2$, from where the result follows.

■

Proposition 3. From Equation (14) we have:

$$\begin{aligned} S_t &= \sigma_0 - \sigma_1 [b\omega_t + (1-b)\bar{\omega} + \theta_1 b\epsilon_t] \\ &= \sigma_0 - \sigma_1 [b(b\omega_{t-1} + (1-b)\bar{\omega} + \epsilon_t) + (1-b)\bar{\omega} + \theta_1 b\epsilon_t] \end{aligned}$$

where we used the AR(1) condition $\omega_t = b\omega_{t-1} + (1-b)\bar{\omega} + \epsilon_t$. Note that, rearranging the first line (valid for all t) we find:

$$\sigma_1 [b\omega_{t-1} + (1-b)\bar{\omega}] = \sigma_0 - S_{t-1} - \sigma_1 \theta_1 b\epsilon_{t-1}$$

Inserting above, we then get:

$$S_t = (1 - b)(\sigma_0 - \sigma_1 \bar{\omega}) + b \cdot S_{t-1} - (1 + \theta_1)b\sigma_1\epsilon_t + \theta_1 b^2 \sigma_1 \epsilon_{t-1}$$

Spreads thus follow an ARMA(1,1) process.

■

Lemma 1 For notational convenience, rewrite the stochastic process driving credit spreads as $S_t = a + b \cdot S_{t-1} - c\epsilon_t + d\epsilon_{t-1}$, with $a = (1 - b)(\sigma_0 - \sigma_1 \bar{\omega})$, $c = (1 + \theta)b\sigma_1$ and $d = \theta b^2 \sigma_1$. The T periods ahead diagnostic forecast of the spread is given by:

$$\mathbb{E}_t^\theta(S_{t+T}) = \mathbb{E}_t^\theta(a + b \cdot S_{t+T-1} - c\epsilon_{t+T} + d\epsilon_{t+T-1})$$

Note that $\mathbb{E}_t^\theta(\epsilon_{t+s}) = 0$ for any $s > 0$, because rational expectations of future shocks are always zero. Thus, for $T > 1$, $\mathbb{E}_t^\theta(S_{t+T})$ becomes

$$\mathbb{E}_t^\theta(S_{t+T}) = a + \mathbb{E}_t^\theta(b \cdot S_{t+T-1}) = a \sum_{s=0}^{T-2} b^s + b^{T-1} \mathbb{E}_t^\theta(S_{t+1}) = a \sum_{s=0}^{T-1} b^s + b^T S_t$$

Inserting the coefficients we get :

$$\mathbb{E}_t^\theta(S_{t+T}) = (1 - b^T)(\sigma_0 - \sigma_1 \bar{\omega}) + b^T S_t$$

Consider now the case $T = 1$. Using (14) and the law of iterated expectations for diagnostic expectations, write $\mathbb{E}_t^\theta(S_{t+1}) = \sigma_0 - \sigma_1 \mathbb{E}_t^\theta(\omega_{t+2})$. Inserting $\omega_{t+2} = b\omega_{t+1} + \epsilon_{t+2}$ and $\mathbb{E}_t^\theta(\epsilon_{t+2}) = 0$, we obtain the result.

■

Proposition 4. The forecast error at $t + 1$ is $\mathbb{E}_t[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})]$. The first result follows immediately from Lemma 1. Alternatively, write:

$$S_{t+1} - \mathbb{E}_t^\theta(S_{t+1}) = -\sigma_1 \left[\mathbb{E}_{t+1}^\theta(\omega_{t+2}) - \mathbb{E}_t^\theta \left(\mathbb{E}_{t+1}^\theta(\omega_{t+2}) \right) \right]$$

The first term is $\mathbb{E}_{t+1}^\theta(\omega_{t+2}) = \mathbb{E}_{t+1}(\omega_{t+2}) + \theta b \epsilon_{t+1}$, while the second term is

$\mathbb{E}_t^\theta \left(\mathbb{E}_{t+1}^\theta(\omega_{t+2}) \right) = \mathbb{E}_t(\omega_{t+2}) + \theta b^2 \epsilon_t$. Taking expectations on the difference, we find

$\mathbb{E}_t[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})] = \sigma_1 \theta b^2 \epsilon_t$. Thus, positive news today narrow the spread today and the predicted spread tomorrow, but the realized spread tomorrow is systematically larger than predicted.

Similarly, we can write

$$\mathbb{E}_t[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})] = -\sigma_1 \mathbb{E}_t \left[\mathbb{E}_{t+s}^\theta \left(\mathbb{E}_{t+T}^\theta(\omega_{t+T+1}) \right) - \mathbb{E}_t^\theta \left(\mathbb{E}_{t+T}^\theta(\omega_{t+T+1}) \right) \right]$$

Using the representation of Corollary 1, this becomes

$$\mathbb{E}_t[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})] = \sigma_1 \theta b^{T+1} \epsilon_t$$

Again, positive news today compress expected spreads in the future, and these expectations systematically widen going forward.

■

Corollary 2. Defining S_t^r as the credit spread that obtains under rational expectations, where $\theta = 0$, it follows immediately from Equation (14) that

$$S_t - S_t^r = -\theta b \sigma_1 \epsilon_t$$

Moreover,

$$\text{Var}[S_t | \omega_{t-1}] = \text{Var}[-(1 + \theta)b\sigma_1\epsilon_t | \omega_{t-1}] = (1 + \theta)^2 \text{Var}[S_t^r | \omega_{t-1}].$$

■

Proposition 5. Assume that at $t - 1$ spreads are low due to recent good news, $\epsilon_{t-1} > 0$. It follows from the ARMA(1,1) structure for spreads derived in Proposition 3 that the expected future path of spreads is:

$$\mathbb{E}_{t-1}[S_t] = (1 - b)(\sigma_0 - \sigma_1 \bar{\omega}) + b \cdot S_{t-1} + \theta b^2 \sigma_1 \epsilon_{t-1}$$

from which the result follows.

Aggregate investment at t and aggregate production at $t + 1$ are strictly decreasing functions of the average credit spread S_t . It follows from point i) that, under

the assumptions of the Proposition and controlling for fundamentals at $t - 1$, there is a predictable drop in these quantities from the perspective of $t - 1$.

■

Appendix

We briefly consider two alternative specifications of the reference group $-G$ used to define representativeness.

Lagged Diagnostic Expectations as Reference

We start by specifying $-G$ in terms of diagnostic expectations $\mathbb{E}_{t-1}^\theta(\omega_t)$. Assume that the agent compares the current distribution with the one implied by his past diagnostic expectation of Ω_t , namely $-G \equiv \{\Omega_t = \mathbb{E}_{t-1}^\theta(\omega_t)\}$. Diagnostic expectations at time t are then given by:

$$\mathbb{E}_t^\theta(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - b\mathbb{E}_{t-1}^\theta(\omega_t)]. \quad (17)$$

The agent is overly optimistic when news point to an outcome that is sufficiently good as compared with his past expectations, $\mathbb{E}_t(\omega_{t+1}) > b\mathbb{E}_{t-1}^\theta(\omega_t)$, and overly pessimistic otherwise. By iterating Equation (17) backwards, for $\theta b < 1$ we obtain:

$$\mathbb{E}_t^\theta(\omega_{t+1}) = (1 + \theta) \sum_{j \geq 0} (-\theta b)^j \mathbb{E}_{t-j}(\omega_{t-j+1}). \quad (18)$$

Diagnostic expectations are a weighted average of current and past one-period-ahead rational expectations, with weights that depend on θ . Again, when $\theta = 0$, expectations are rational. In Equation (18) the signs on rational expectations obtained in odd and even past periods alternate. This is an intuitive consequence of (17) and implies that news exert a non-monotonic effect in future expectations. Agents over-react on impact, but this over-reaction implies reference expectations are higher the next period, causing a reversal to pessimism (which in turn generates future optimism and so on). Specifying $-G$ in terms of diagnostic expectations thus preserves the two key properties of our basic model: expectations display over-reaction to news on impact but also reversal in the future.

Slow Moving – G

In our main specification, context – G is the immediate past. This assumption starkly illustrates our results and buys significant tractability. It is however possible that remote but remarkable memories influence the agent’s background context. Our model can be easily enriched to capture this feature by defining representativeness in terms of a mixture of current and past likelihood ratios:

$$\left(\frac{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)}{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = b\omega_{t-1})} \right)^{\alpha_1} \left(\frac{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = b\omega_{t-1})}{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = b^2\omega_{t-2})} \right)^{\alpha_2},$$

where $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ capture the weights attached to present and past representativeness, respectively. The coefficients α_i capture limited memory (past news are completely forgotten if $\alpha_2 = 0$, as in the main text), and $\alpha_1 > \alpha_2$ captures recency effects. In this case we have that:

$$\begin{aligned} \mathbb{E}_t^\theta(\omega_{t+1}) = & \mathbb{E}_t(\omega_{t+1}) + \theta\alpha_1[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})] + \\ & + \theta\alpha_2[\mathbb{E}_{t-1}(\omega_{t+1}) - \mathbb{E}_{t-2}(\omega_{t+1})]. \end{aligned} \quad (22)$$

The agent can remain too optimistic even after minor bad news, $\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1}) < 0$, provided he experienced major good news in the past $\mathbb{E}_{t-1}(\omega_{t+1}) - \mathbb{E}_{t-2}(\omega_{t+1}) \gg 0$. This feature can yield under-reaction to early warnings of crises (see Gennaioli et al. 2015 for a related formulation). At the same time, the main properties of over-reaction and reversal continue to hold in this specification with respect to repeated news in the same direction, which are plausible in the case of credit cycles.

In general, the robust predictions of the model remain over-reaction and reversals. Different specifications of – G yield different ancillary predictions that may make it possible to uncover the structure of – G in the data. This is an important avenue for future work.