Online Appendix for "Growth and Trade with Frictions: A Structural Estimation Framework"

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B Derivation of the Policy Functions of the 'Upper Level'

Our 'upper level' specification is very similar to Hercowitz and Sampson (1991) (we omit the country indexes in order to economize on the notational burden):

$$
\begin{aligned}
U_t &= \sum_{t=0}^{\infty} \beta^t \ln(C_t), && \text{(A1)}\\
K_{t+1} &= K_t^{1-\delta}\Omega_t^{\delta}, && \text{(A2)}\\
y_t &= p_t A_t L_t^{1-\alpha} K_t^{\alpha}, && \text{(A3)}\\
y_t &= P_t C_t + P_t \Omega_t, && \text{(A4)}\\
K_0 & \quad \textit{given.} && \text{(A5)}
\end{aligned}
$$

As discussed in detail in Heer and Maußner (2009, chapter 1), this specific set-up with logarithmic utility and log-linear adjustment costs has the advantage of obtaining an analytical solution. To solve for the policy functions of capital and consumption, we iterate over the value function. For ease of notation, we skip indices for current periods and denote next period variables by $'$. Further, we define $\phi = 1/\delta$. The value of the value function at step 0, v^0, is equal to 0. The value of the value function at step 1, v^1, is given by:

$$
\begin{aligned}
v' &= \max_{K'} \ln C = \max_{K'} \ln\left(y/P - \Omega\right)\\
&= \max_{K'} \ln\left(pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})\right).
\end{aligned}
$$

The associated first order condition is:

$$
\frac{1}{pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}} = 0.
$$

It follows that $K' = 0$. Hence, $v' = \ln\left(pAL^{1-\alpha}K^{\alpha}/P\right)$ and, in the next step, we have to solve:

$$
v^2 = \max_{K'} \ln\left(pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})\right) + \beta \ln\left(pAL^{1-\alpha}K'^{\alpha}/P\right).
$$

The first order condition becomes:

$$
\begin{aligned}
&\frac{1}{pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}} + \frac{\alpha\beta}{K'} = 0,\\
&\frac{\alpha\beta}{\phi}\left(pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})\right) = \frac{K'^{\phi}}{K^{\phi-1}},\\
&\frac{\alpha\beta}{\phi}\left(pAL^{1-\alpha}K^{\alpha}/P\right) = \left(\frac{\alpha\beta}{\phi}+1\right)\frac{K'^{\phi}}{K^{\phi-1}},\\
&\frac{\alpha\beta}{\alpha\beta+\phi}pAL^{1-\alpha}K^{\alpha+\phi-1}/P = K'^{\phi},\\
&\left(\frac{\alpha\beta}{\alpha\beta+\phi}pAL^{1-\alpha}/P\right)^{\frac{1}{\phi}} K^{(\alpha+\phi-1)/\phi} = K'. && \text{(A6)}
\end{aligned}
$$

Plugging in the expression for K' given in equation (A6), we end up with:

$$\begin{aligned}
v^2 &= \ln\left(pAL^{1-\alpha}K^{\alpha}/P - \left(\left(\frac{\alpha\beta}{\alpha\beta+\phi}pAL^{1-\alpha}/P\right)^{\frac{1}{\phi}} K^{(\alpha+\phi-1)/\phi}\right)^{\phi} /K^{\phi-1}\right) \\
&\quad +\beta\ln\left(pAL^{1-\alpha}\left(\left(\frac{\alpha\beta}{\alpha\beta+\phi}pAL^{1-\alpha}/P\right)^{\frac{1}{\phi}} K^{(\alpha+\phi-1)/\phi}\right)^{\alpha} /P\right), \\
&= \ln\left(\left(pAL^{1-\alpha}/P - \frac{\alpha\beta}{\alpha\beta+\phi}pAL^{1-\alpha}/P\right)K^{\alpha}\right) \\
&\quad +\beta\ln\left(pAL^{1-\alpha}\left(\frac{\alpha\beta}{\alpha\beta+\phi}pAL^{1-\alpha}/P\right)^{\frac{\alpha}{\phi}} K^{(\alpha+\phi-1)\alpha/\phi}/P\right), \\
&= \alpha\ln(K) + \beta\theta\alpha\ln(K) + const,
\end{aligned}$$

where $\theta \equiv (\alpha+\phi-1)/\phi$ and *const* collects all terms not depending on K. The next step is:

$$\begin{aligned}
v^3 &= \max_{K'} \ln\left(pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})\right) + \alpha\beta\ln(K') + \beta^2\theta\alpha\ln(K') \\
&\quad +\beta const.
\end{aligned}$$

The first order condition is given by:

$$\begin{aligned}
&\frac{1}{pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}} + \frac{\alpha\beta}{K'} + \frac{\alpha\theta\beta^2}{K'} = 0, \\
&\frac{\alpha\beta}{\phi}(1+\beta\theta)\left(pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})\right) = \frac{K'^{\phi}}{K^{\phi-1}}, \\
&\frac{\alpha\beta}{\phi}(1+\beta\theta)pAL^{1-\alpha}K^{\alpha}/P = \left(\frac{\alpha\beta}{\phi}(1+\beta\theta)+1\right)\frac{K'^{\phi}}{K^{\phi-1}}, \\
&K' = \left(\frac{\frac{\alpha\beta}{\phi}(1+\beta\theta)pAL^{1-\alpha}/P}{\frac{\alpha\beta}{\phi}(1+\beta\theta)+1}\right)^{\frac{1}{\phi}} K^{\theta}. \qquad \text{(A7)}
\end{aligned}$$

Plug in the solution of K' given in equation (A7) to obtain:

$$v^3 = \alpha\ln(K) + \alpha\beta\theta\ln(K) + \beta^2\theta^2\alpha\ln(K) + \beta const.$$

The resulting value of the value function is:

$$\begin{aligned}
v^4 &= \max_{K'} \ln\left(pAL^{1-\alpha}K^{\alpha}/P - (K'^{\phi}/K^{\phi-1})\right) + \alpha\beta\ln(K')\left[1+\beta\theta+\beta^2\theta^2\right] \\
&\quad +\beta const,
\end{aligned}$$

and the accompanying first order condition becomes:

$$\frac{1}{pAL^{1-\alpha}K^{\alpha}/P-(K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}}+\frac{\alpha\beta\left[1+\beta\theta+\beta^2\theta^2\right]}{K'}=0,$$
$$\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)\left(pAL^{1-\alpha}K^{\alpha}/P-(K'^{\phi}/K^{\phi-1})\right)=\frac{K'^{\phi}}{K^{\phi-1}},$$
$$\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)pAL^{1-\alpha}K^{\alpha}/P=\left(\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)+1\right)\frac{K'^{\phi}}{K^{\phi-1}},$$
$$K'=\left(\frac{\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)pAL^{1-\alpha}/P}{\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)+1}\right)^{\frac{1}{\phi}}K^{\theta}.$$

Note now that the general pattern can be described as:

$$v^m\Rightarrow K'=\left[\frac{\frac{\alpha\beta}{\phi}\left(pAL^{1-\alpha}/P\right)\sum_{i=0}^{m}(\beta\theta)^i}{1+\frac{\alpha\beta}{\phi}\sum_{i=0}^{m}(\beta\theta)^i}\right]^{\frac{1}{\phi}}K^{\theta},$$

where m denotes the mth-step. When $m\to\infty$, we end up with:

$$\left[\frac{\frac{\alpha\beta}{\phi}\left(pAL^{1-\alpha}/P\right)\sum_{i=0}^{m}(\beta\theta)^i}{1+\frac{\alpha\beta}{\phi}\sum_{i=0}^{m}(\beta\theta)^i}\right]^{\frac{1}{\phi}}=\left[\frac{\frac{\alpha\beta}{\phi}\left(pAL^{1-\alpha}/P\right)\frac{1}{1-\beta\theta}}{1+\frac{\alpha\beta}{\phi}\frac{1}{1-\beta\theta}}\right]^{\frac{1}{\phi}}.$$

Replace $\theta\equiv(\alpha+\phi-1)/\phi$ in the above expression to obtain:

$$\left[\frac{\frac{\alpha\beta}{\phi}\left(pAL^{1-\alpha}/P\right)\frac{1}{1-\beta(\alpha+\phi-1)/\phi}}{1+\frac{\alpha\beta}{\phi}\frac{1}{1-\beta(\alpha+\phi-1)/\phi}}\right]^{\frac{1}{\phi}}=\left[\frac{\left(pAL^{1-\alpha}/P\right)\frac{\alpha\beta}{\phi-\beta(\alpha+\phi-1)}}{1+\frac{\alpha\beta}{\phi-\beta(\alpha+\phi-1)}}\right]^{\frac{1}{\phi}}=$$
$$\left[\frac{\left(pAL^{1-\alpha}/P\right)\frac{\alpha\beta}{\phi-\beta(\alpha+\phi-1)}}{\frac{\phi-\beta\phi+\beta}{\phi-\beta(\alpha+\phi-1)}}\right]^{\frac{1}{\phi}}=\left[\frac{\left(pAL^{1-\alpha}/P\right)\alpha\beta}{\phi-\beta\phi+\beta}\right]^{\frac{1}{\phi}}.$$

Apply the definition of $\phi=1/\delta$:

$$\left[\frac{\left(pAL^{1-\alpha}/P\right)\alpha\beta}{1/\delta-\beta/\delta+\beta}\right]^{\delta}=\left[\frac{\left(pAL^{1-\alpha}/P\right)\alpha\beta\delta}{1-\beta+\beta\delta}\right]^{\delta}.$$

The resulting expression is our policy function for the capital stock in the next period, K':

$$K'=\left[\frac{\alpha\beta\delta pAL^{1-\alpha}}{(1-\beta+\beta\delta)\,P}\right]^{\delta}K^{\alpha\delta+1-\delta}. \tag{A8}$$

Intuitively, (A8) reveals that, alongside parameters, capital accumulation depends on current capital stock, labor endowments, technology, factory-gate prices, and the aggregate price index. A higher labor endowment, a higher current capital stock and a higher technology level translate into higher next-period capital stocks. The relationship between capital stock and factory-gate prices is also positive. As noted in the main text, the intuition is that an increase in the factory-gate price leads to an increase in the value of marginal product of capital and, therefore, investment. The relationship between investment and the aggregate price index is inverse. The intuition is that a higher price of investment and a higher price of consumption increase the direct cost and the opportunity cost of investment. A higher current goods price means that output today is more valuable or that more output can be produced today. Hence, consumers are willing to transfer part of their wealth to the next period by capital accumulation. On the other hand, if the current price index is high, consumption is expensive today. Therefore, a higher share of income will be spend on consumption today and less will be saved and transferred for future consumption via capital accumulation.

Note that with K' and K at hand, one can determine the level of investment as:

$$\Omega = \left(\frac{K'}{K^{1-\delta}}\right)^{\frac{1}{\delta}} = \left(\frac{\left[\frac{pAL^{1-\alpha}\alpha\beta\delta}{P(1-\beta+\beta\delta)}\right]^{\delta} K^{\alpha\delta+1-\delta}}{K^{1-\delta}}\right)^{\frac{1}{\delta}} = \left[\frac{\alpha\beta\delta pAL^{1-\alpha}}{(1-\beta+\beta\delta)\,P}\right] K^{\alpha}.$$

In addition, the optimal level of current consumption can be obtained by using the policy function for capital and reformulating $y = PC + P\Omega$, i.e.,

$$\begin{aligned} C &= \frac{y}{P} - \Omega = \frac{pAL^{1-\alpha}K^{\alpha}}{P} - \left[\frac{\alpha\beta\delta pAL^{1-\alpha}}{(1-\beta+\beta\delta)\,P}\right] K^{\alpha} \\ &= \left[1 - \frac{\alpha\beta\delta}{1-\beta+\beta\delta}\right]\frac{pAL^{1-\alpha}K^{\alpha}}{P} \\ &= \left[\frac{1-\beta+\beta\delta-\alpha\beta\delta}{1-\beta+\beta\delta}\right]\frac{pAL^{1-\alpha}K^{\alpha}}{P}. \end{aligned} \tag{A9}$$

C Transition

An important contribution of our paper is that we do not only focus on the steady-state, but we also characterize the transition path. In fact, as emphasized in the main text, all growth effects in our framework are transitional, and there is no steady-state growth. A nice feature of the theoretical framework is that the assumptions of an intertemporal log-utility function and the log-linear transition function for capital enable us to obtain a closed-form solution for the transition path in the model. In order to do that, we first calculate the policy function for capital by value function iteration as described in Online Appendix B, where consumers take the variety price p_t and the consumer price P_t as given. It should be noted that p_t and P_t are both general equilibrium indexes that consistently aggregate the decisions of all countries in the world, which are transmitted through changes in trade costs. See discussion in main text for further details. Thus, our policy function gives the optimal decision of consumers for the capital stock tomorrow as a function of prices and the capital stock today, and it is consistent with rational expectations as long as we can determine current prices and have an initial capital stock.

We take the following steps in order to characterize the transition path analytically.

First, we calculate the initial capital stock by assuming that we are in a steady-state. In particular, we solve our equation system given by equations (19)-(24) simultaneously for all N-countries at steady-state. By construction, the steady-state is consistent with all prices and steady-state capital stocks for all countries. We take this steady-state as our baseline values at time 0. Then, we consider a non-anticipated and permanent change, e.g. a change in bilateral trade costs among Canada, Mexico and the United States due to the formation of NAFTA. Given the current capital stock (which was determined yesterday), we use equations (20)-(23) to solve for new current prices and current GDPs for the new vector of bilateral trade costs. As soon as we have these prices and GDPs, we can calculate the optimal choice of consumption and investment by using the policy function (24). With a new capital stock in the next period, we can again use equations (20)-(23) to solve for next periods prices and GDPs. We then iterate until convergence, i.e. until we reach the new steady-state.

It is important to note that equations (20)-(23) solve for prices and income simultaneously for all N-countries in our model. In order to ensure that our calculations are correct, we take two steps. First, we compare the steady-state from the iterative procedure with a new steady-state that we obtain in one shot, ignoring transition, by simply solving our theoretical system directly with the new vector of trade costs. The two steady-states are identical. This is encouraging, but tells us nothing about the transition path. In order to validate the correctness of the transition path calculations, we set-up a system of first-order conditions which we then solve using Dynare. Specifically, we use our utility function (we skip country indices without loss of generality):

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(C_t),$$

and combine the budget constraint with the production function:

$$P_t C_t + P_t \Omega_t = p_t A_t L_t^{1-\alpha} K_t^{\alpha}.$$

In order to end up with only one constraint, we also use the definition of Ω_t:

$$\Omega_t = \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}},$$

leading to the following budget constraint:

$$P_t C_t + P_t \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} = p_t A_t L_t^{1-\alpha} K_t^{\alpha}.$$

The corresponding expression for the Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) + \lambda_t \left(p_t A_t L_t^{1-\alpha} K_t^{\alpha} - P_t C_t - P_t \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \right) \right].$$

Take derivatives with respect to C_t, K_{t+1} and λ_t to obtain the following set of first-order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= \frac{\beta^t}{C_t} - \beta^t \lambda_t P_t \stackrel{!}{=} 0 \quad \text{for all } t. \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \beta^{t+1} \lambda_{t+1} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1} - \beta^t \lambda_t P_t \left(\frac{1}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \frac{1}{\delta} K_{t+1}^{\frac{1}{\delta}-1} \\
&\quad - \beta^{t+1} \lambda_{t+1} P_{t+1} K_{t+2}^{\frac{1}{\delta}} \frac{\delta - 1}{\delta} K_{t+1}^{-\frac{1}{\delta}} \stackrel{!}{=} 0 \quad \text{for all } t. \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &= p_t A_t L_t^{1-\alpha} K_t^{\alpha} - P_t C_t - P_t \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \stackrel{!}{=} 0 \quad \text{for all } t.
\end{aligned}$$

Use the first-order condition for consumption to express λ_t as:

$$\lambda_t = \frac{1}{C_t P_t}.$$

Replace this in the first-order condition for capital:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \beta^{t+1} \frac{1}{C_{t+1} P_{t+1}} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1} - \beta^t \frac{1}{C_t} \left(\frac{1}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \frac{1}{\delta} K_{t+1}^{\frac{1}{\delta}-1} \\
&\quad - \beta^{t+1} \frac{1}{C_{t+1}} K_{t+2}^{\frac{1}{\delta}} \frac{\delta - 1}{\delta} K_{t+1}^{-\frac{1}{\delta}} \stackrel{!}{=} 0 \quad \text{for all } t.
\end{aligned}$$

Simplify and re-arrange to obtain:

$$\frac{\beta p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1}}{C_{t+1} P_{t+1}} = \frac{1}{C_t} \left(\frac{1}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \frac{1}{\delta} K_{t+1}^{\frac{1}{\delta}-1} + \frac{(\delta - 1)\beta}{\delta C_{t+1}} K_{t+2}^{\frac{1}{\delta}} K_{t+1}^{-\frac{1}{\delta}} \quad \text{for all } t.$$

Use the definition of y_t to re-write the left-hand side of the above expression as:

$$\frac{\alpha \beta y_{t+1}}{K_{t+1} C_{t+1} P_{t+1}} = \frac{1}{\delta C_t} \frac{K_{t+1}^{\frac{1}{\delta}-1}}{K_t^{\frac{1-\delta}{\delta}}} + \frac{\beta(\delta - 1)}{\delta C_{t+1}} \left(\frac{K_{t+2}}{K_{t+1}} \right)^{\frac{1}{\delta}} \quad \text{for all } t.$$

As expected, we end up with the standard consumption Euler-equation. Note that we have four forward-looking variables for each country: y_t, K_t, C_t, and P_t, i.e. we have $4N$ forward-looking variables in our system. These are the endogenous variables we have to solve for. In order to do that, we feed the following set of equations into Dynare:

$$
\begin{aligned}
y_{j,t} &= \frac{(y_{j,t}/y_t)^{\frac{1}{1-\sigma}}}{\gamma_j P_{j,t}} A_{j,t} L_{j,t}^{1-\alpha} K_{j,t}^{\alpha} \quad \text{for all } j \text{ and } t, && \text{(A10)}\\
y_t &= \sum_j y_{j,t} \quad \text{for all } t, && \text{(A11)}\\
y_{j,t} &= P_{j,t} C_{j,t} + P_{j,t} \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \quad \text{for all } j \text{ and } t, && \text{(A12)}\\
P_{j,t} &= \left[\sum_i \left(\frac{t_{ij,t}}{P_{i,t}} \right)^{1-\sigma} \frac{y_{i,t}}{y_t} \right]^{\frac{1}{1-\sigma}} \quad \text{for all } j \text{ and } t, && \text{(A13)}\\
\frac{\alpha \beta y_{j,t+1}}{K_{j,t+1} C_{j,t+1} P_{j,t+1}} &= \frac{1}{\delta C_{j,t}} \frac{K_{j,t+1}^{\frac{1}{\delta}-1}}{K_{j,t}^{\frac{1-\delta}{\delta}}} + \frac{\beta (\delta - 1)}{\delta C_{j,t+1}} \left(\frac{K_{j,t+2}}{K_{j,t+1}} \right)^{\frac{1}{\delta}} \quad \text{for all } j \text{ and } t. && \text{(A14)}
\end{aligned}
$$

We then take as initial and end values the baseline and the counterfactual steady-state and we let Dynare solve for the transition of our deterministic model assuming perfect foresight. The algorithm for our case is described in Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011) in Section 4.12. Comparison between the transition path from Dynare and the transition path that we solved for analytically reveals that those are identical.

D ACR formula

This section obtains the ACR-equivalent formula in our dynamic setting. Before we start, we note that in ACR the formula is based on real income, which is equivalent to welfare in their setting. However, this is no-longer the case in our framework as not all of the income is used for consumption because part of it is used to build up capital. Accordingly, our welfare measure should be based on consumption. In order to derive the ACR equivalent, we start with consumption as given by equation (A9) and use the production function $y_j = p_j A_j L_j^{1-\alpha} K_j^{\alpha}$ as given in equation (A3) to express welfare as (we skip time indices without loss of generality):

$$
W_j \equiv C_j = \left(\frac{1 - \beta + \beta\delta - \alpha\beta\delta}{1 - \beta + \beta\delta} \right) \frac{y_j}{P_j}.
$$

Taking the log-derivative leads to:

$$
d \ln W_j = d \ln y_j - d \ln P_j.
$$

Taking A_j and L_j as constant, we can express $d \ln y_j$ as:

$$
d \ln y_j = d \ln p_j + \alpha d \ln K_j. \tag{A15}
$$

Note that P_j is given by:

$$P_j = \left[\sum_{i=1}^{N} (\gamma_i p_i t_{ij})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Then:

$$\begin{aligned}
d\ln P_j &= \frac{1}{P_j} dP_j, \\
&= \frac{1}{P_j}\frac{1}{1-\sigma}\left[\sum_{i=1}^{N} (\gamma_i p_i t_{ij})^{1-\sigma}\right]^{\frac{1}{1-\sigma}-1} \\
&\quad \times \sum_{i=1}^{N}\left((1-\sigma)\gamma_i^{1-\sigma} p_i^{-\sigma} t_{ij}^{1-\sigma} dp_i + (1-\sigma)\gamma_i^{1-\sigma} p_i^{1-\sigma} t_{ij}^{-\sigma} dt_{ij}\right) \\
&= \left[\sum_{i=1}^{N} (\gamma_i p_i t_{ij})^{1-\sigma}\right]^{-\frac{1}{1-\sigma}} \left[\sum_{i=1}^{N} (\gamma_i p_i t_{ij})^{1-\sigma}\right]^{\frac{1}{1-\sigma}-1} \\
&\quad \times \sum_{i=1}^{N}\left(\gamma_i^{1-\sigma} p_i^{-\sigma} t_{ij}^{1-\sigma} dp_i + \gamma_i^{1-\sigma} p_i^{1-\sigma} t_{ij}^{-\sigma} dt_{ij}\right) \\
&= P_j^{-(1-\sigma)} \sum_{i=1}^{N}\left(\gamma_i^{1-\sigma} p_i^{-\sigma} t_{ij}^{1-\sigma} dp_i + \gamma_i^{1-\sigma} p_i^{1-\sigma} t_{ij}^{-\sigma} dt_{ij}\right) \\
&= \sum_{i=1}^{N}\left(\left(\frac{\gamma_i p_i t_{ij}}{P_j}\right)^{1-\sigma} d\ln p_i + \left(\frac{\gamma_i p_i t_{ij}}{P_j}\right)^{1-\sigma} d\ln t_{ij}\right).
\end{aligned}$$

Use $x_{ij} = \left(\frac{\gamma_i p_i t_{ij}}{P_j}\right)^{1-\sigma} y_j$ and define $\lambda_{ij} = x_{ij}/y_j = \left(\frac{\gamma_i p_i t_{ij}}{P_j}\right)^{1-\sigma}$, to simplify:

$$d\ln P_j = \sum_{i=1}^{N} \lambda_{ij}\left(d\ln p_i + d\ln t_{ij}\right). \tag{A16}$$

Combine terms:

$$d\ln W_j = d\ln y_j - d\ln P_j = d\ln p_j + \alpha d\ln K_j - \sum_{i=1}^{N} \lambda_{ij}\left(d\ln p_i + d\ln t_{ij}\right).$$

Take the ratio of λ_{ij} and λ_{jj}:

$$\frac{\lambda_{ij}}{\lambda_{jj}} = \left(\frac{\gamma_i p_i t_{ij}}{\gamma_j p_j t_{jj}}\right)^{1-\sigma}.$$

Consider a foreign shock that leaves the ability to serve the own market, t_{jj}, unchanged as in ACR. The change of this ratio is given by:

$$
\begin{aligned}
d\left(\frac{\lambda_{ij}}{\lambda_{jj}}\right) &= \frac{1-\sigma}{\left(\gamma_j p_j t_{jj}\right)^{1-\sigma}}\left(\gamma_i p_i t_{ij}\right)^{-\sigma}\left(\gamma_i p_i dt_{ij}+\gamma_i t_{ij} dp_i\right) \\
&\quad -\frac{1-\sigma}{\left(\gamma_j p_j t_{jj}\right)^{2-\sigma}}\left(\gamma_i p_i t_{ij}\right)^{1-\sigma}\gamma_j t_{jj} dp_j .
\end{aligned}
$$

Express as log-change:

$$
\frac{d\left(\frac{\lambda_{ij}}{\lambda_{jj}}\right)}{\frac{\lambda_{ij}}{\lambda_{jj}}} = d\ln\left(\frac{\lambda_{ij}}{\lambda_{jj}}\right) = d\ln\lambda_{ij} - d\ln\lambda_{jj} = (1-\sigma)\left(d\ln t_{ij} + d\ln p_i - d\ln p_j\right).
$$

Use this expression with equation (A16):

$$
\begin{aligned}
d\ln P_j &= \sum_{i=1}^{N}\lambda_{ij}\left(d\ln p_i + d\ln t_{ij}\right) \\
&= \sum_{i=1}^{N}\lambda_{ij}\left(\frac{1}{1-\sigma}\left(d\ln\lambda_{ij}-d\ln\lambda_{jj}\right)+d\ln p_j\right) \\
&= \frac{1}{1-\sigma}\left(\sum_{i=1}^{N}\lambda_{ij} d\ln\lambda_{ij} - d\ln\lambda_{jj}\sum_{i=1}^{N}\lambda_{ij}\right) + d\ln p_j\sum_{i=1}^{N}\lambda_{ij}.
\end{aligned}
$$

Assuming balanced trade, as in ACR, implies $y_j = \sum_{i=1}^{N} x_{ij}$. Hence, $\sum_{i=1}^{N}\lambda_{ij} = 1$ and $d\sum_{i=1}^{N}\lambda_{ij} = \sum_{i=1}^{N} d\lambda_{ij} = 0$. Further, $\sum_{i=1}^{N}\lambda_{ij} d\ln\lambda_{ij} = \sum_{i=1}^{N} d\lambda_{ij} = 0$. Using these facts, the above expression simplifies to:

$$
\begin{aligned}
d\ln P_j &= \frac{1}{1-\sigma}\left(\sum_{i=1}^{N}\lambda_{ij} d\ln\lambda_{ij} - d\ln\lambda_{jj}\sum_{i=1}^{N}\lambda_{ij}\right) + d\ln p_j \\
&= -\frac{1}{1-\sigma} d\ln\lambda_{jj} + d\ln p_j . \qquad \text{(A17)}
\end{aligned}
$$

Using this relationship in the welfare change expression leads to:

$$
\begin{aligned}
d\ln W_j &= d\ln y_j - d\ln P_j = d\ln p_j + \alpha d\ln K_j + \frac{1}{1-\sigma} d\ln\lambda_{jj} - d\ln p_j \\
&= \alpha d\ln K_j + \frac{1}{1-\sigma} d\ln\lambda_{jj}.
\end{aligned}
$$

Integrate between an initial situation (base case) and a counterfactual situation (counterfactual):

$$
\begin{aligned}
\int_{W^b}^{W^c} d\ln W_j &= \int_{K_j^b}^{K_j^c} \alpha d\ln K_j + \int_{\lambda_{jj}^b}^{\lambda_{jj}^c} \frac{1}{1-\sigma} d\ln\lambda_{jj}, \\
\ln W_j + C_1\Big|_{W^b}^{W^c} &= \alpha\ln K_j + C_2\Big|_{K_j^b}^{K_j^c} + \frac{1}{1-\sigma}\ln\lambda_{jj} + C_3\Bigg|_{\lambda_{jj}^b}^{\lambda_{jj}^c}, \\
\ln W_j^c + C_1 - \ln W_j^b - C_1 &= \alpha\ln K_j^c + C_2 - \alpha\ln K_j^b - C_2 + \frac{1}{1-\sigma}\ln\lambda_{jj}^c + C_3 \\
& \quad -\frac{1}{1-\sigma}\ln\lambda_{jj}^b - C_3.
\end{aligned}
$$

Use 'hat' to denote the ratio of any counterfactual and base case variable, i.e. $\widehat{v} = v^c/v^b$:

$$
\ln\widehat{W}_j = \alpha\ln\widehat{K}_j + \frac{1}{1-\sigma}\ln\widehat{\lambda}_{jj}.
$$

Take the exponent on the left- and right-hand side:

$$
\widehat{W}_j = \widehat{K}_j^{\alpha}\widehat{\lambda}_{jj}^{\frac{1}{1-\sigma}}. \tag{A18}
$$

Note that this expression for welfare holds in and out-of steady-state.

D.1 ACR Formula In Steady-State

In steady-state, we can use the expression for K_j as given in equation (25) as starting point. First, we replace y_j by the expression given in equation (23):

$$
K_j = \frac{\alpha\beta\delta p_j A_j L_j^{1-\alpha} K_j^{\alpha}}{(1-\beta+\beta\delta)P_j}.
$$

Solve for K_j:

$$
K_j = \left[\frac{\alpha\beta\delta p_j A_j L_j^{1-\alpha}}{(1-\beta+\beta\delta)P_j}\right]^{\frac{1}{(1-\alpha)}}.
$$

Next, calculate the change of K_j. To do so, first calculate the log-derivative of the left- and right-hand side:

$$
d\ln K_j = \frac{1}{1-\alpha}\left(d\ln p_j - d\ln P_j\right).
$$

Replace $d\ln P_j$ by $-\frac{1}{1-\sigma}d\ln\lambda_{jj} + d\ln p_j$:

$$
d\ln K_j = \frac{1}{(1-\alpha)}\frac{1}{(1-\sigma)}d\ln\lambda_{jj}.
$$

Note that $d \ln p_j$ cancels out. Integrating the left- and right-hand side between the baseline and the counterfactual and denoting K'/K with hats, where K' denotes the change from the baseline to the counterfactual, leads to:

$$\ln \widehat{K}_j = \frac{1}{(1-\alpha)} \frac{1}{(1-\sigma)} \ln \widehat{\lambda}_{jj}.$$

Take the exponent on the left- and right-hand to obtain:

$$\widehat{K}_j = \widehat{\lambda}_{jj}^{\frac{1}{(1-\alpha)(1-\sigma)}}.$$

Plug this expression into equation (A18):

$$\widehat{W_j} = \widehat{\lambda}_{jj}^{\frac{\alpha}{(1-\alpha)(1-\sigma)}} \widehat{\lambda}_{jj}^{\frac{1}{1-\sigma}} = \widehat{\lambda}_{jj}^{\frac{1}{(1-\alpha)(1-\sigma)}}.$$

Note that this expression is very similar to the ACR formula for intermediates with perfect competition, which also just adds the share of intermediates in production to the exponent (see page 115 in ACR). Hence, in steady-state, capital accumulation acts pretty much as adding intermediates. The key difference between our setting and a model with intermediates is the dynamics and the transition path. We characterize in Section C the transition path, and discuss the extension to allow for intermediates in Section A.4.

D.2 ACR Formula Out-of Steady-State

In Subsection D.1 we assume that we are in a steady-state. In this section, we investigate the properties of our model with respect to ACR out of steady-state. To do this, we go back and depart from equation (A18), which holds in and out-of steady-state:

$$\widehat{W_{j,t}} = \widehat{K}_{j,t}^{\alpha} \widehat{\lambda}_{jj,t}^{\frac{1}{1-\sigma}}.$$

Starting with this expression, we have to determine $\widehat{K}_{j,t}$. Take the capital equation as given by equation (24) and replace $p_{j,t}A_{j,t}$ using equation (22):

$$K_{j,t+1} = \left[\frac{y_{j,t}\beta\alpha\delta}{P_{j,t}\left(1-\beta+\delta\beta\right)}\right]^{\delta} K_{j,t}^{1-\delta}.$$

Let us next write this equation in log-derivatives:

$$d \ln K_{j,t+1} = \delta(d \ln y_{j,t} - d \ln P_{j,t}) + (1-\delta) d \ln K_{j,t}.$$

Using equation (A15)

$$d \ln y_{j,t} = d \ln p_{j,t} + \alpha d \ln K_{j,t},$$

and (A17)

$$d \ln P_{j,t} = -\frac{1}{1-\sigma} d \ln \lambda_{jj,t} + d \ln p_{j,t},$$

we end up with:

$$d \ln K_{j,t+1} = \delta(\alpha d \ln K_{j,t} + \frac{1}{1-\sigma} d \ln \lambda_{jj,t}) + (1-\delta) d \ln K_{j,t} \Rightarrow$$
$$d \ln K_{j,t+1} = \frac{1}{1-\sigma} d \ln \lambda_{jj,t} + (1-\delta(1+\alpha)) d \ln K_{j,t}.$$

Integrate between an initial situation (base case) and a counterfactual situation (counterfactual):

$$\begin{aligned}
\int_{K^b_{j,t+1}}^{K^c_{j,t+1}} d \ln K_{j,t+1} &= \int_{\lambda^b_{jj}}^{\lambda^c_{jj}} \frac{1}{1-\sigma} d \ln \lambda_{jj,t} + \int_{K^b_{j,t}}^{K^c_{j,t}} (1-\delta(1+\alpha)) d \ln K_{j,t}, \\
(\ln K_{j,t+1} + C_1) \Big|_{K^b_{j,t+1}}^{K^c_{j,t+1}} &= \left(\frac{1}{1-\sigma} \ln \lambda_{jj,t} + C_2 \right) \Big|_{\lambda^b_{jj}}^{\lambda^c_{jj}} \\
&\quad + ((1-\delta(1+\alpha)) \ln K_{j,t} + C_3) \Big|_{K^b_{j,t}}^{K^c_{j,t}}, \\
\left(\ln K^c_{j,t+1} + C_1 - \ln K^b_{j,t+1} - C_1 \right) &= \left(\frac{1}{1-\sigma} \ln \lambda^c_{jj,t} + C_2 - \frac{1}{1-\sigma} \ln \lambda^b_{jj,t} - C_2 \right) \\
&\quad + \big((1-\delta(1+\alpha)) \ln K^c_{j,t} + C_3 \\
&\quad - (1-\delta(1+\alpha)) \ln K^b_{j,t} - C_3 \big).
\end{aligned}$$

Use 'hat' to denote the ratio of any counterfactual and base case variable, i.e. $\widehat{v} = v^c/v^b$:

$$\ln \widehat{K}_{j,t+1} = \frac{1}{1-\sigma} \ln \widehat{\lambda}_{jj,t} + (1-\delta(1+\alpha)) \ln \widehat{K}_{j,t}.$$

Take the exponent on the left- and right-hand side:

$$\widehat{K}_{j,t+1} = \widehat{K}_{j,t}^{1-\delta(1+\alpha)} \widehat{\lambda}_{jj,t}^{\frac{1}{1-\sigma}}.$$

In combination with $\widehat{W}_j = \widehat{K}_j^{\alpha} \widehat{\lambda}_{jj}^{\frac{1}{1-\sigma}}$ and noting that in period zero $\widehat{K}_{j,0} = 1$, we can express welfare as an iterative formula which only depends on $\widehat{\lambda}_{jj,t}$ and changes of the capital stock:

$$\begin{aligned}
\widehat{W}_{j,t} &= \widehat{K}_{j,t}^{\alpha} \widehat{\lambda}_{jj,t}^{\frac{1}{1-\sigma}}, \\
\widehat{K}_{j,t+1} &= \widehat{K}_{j,t}^{1-\delta(1+\alpha)} \widehat{\lambda}_{jj,t}^{\frac{1}{1-\sigma}}, \\
\widehat{K}_{j,0} &= 1.
\end{aligned}$$

To show that welfare can be expressed as function of $\widehat{\lambda}_{jj}$ and parameters alone, we iteratively plug in $\widehat{K}_{j,t+1}$. In period 0 we have:

$$\begin{aligned}
\widehat{W}_{j,0} &= \widehat{\lambda}_{jj,0}^{\frac{1}{1-\sigma}}, \\
\widehat{K}_{j,1} &= \widehat{\lambda}_{jj,0}^{\frac{1}{1-\sigma}}.
\end{aligned}$$

In period 1 we have:

$$\widehat{W}_{j,1} = \widehat{\lambda}_{jj,0}^{\frac{1}{1-\sigma}} \widehat{\lambda}_{jj,1}^{\frac{1}{1-\sigma}},$$
$$\widehat{K}_{j,2} = \widehat{\lambda}_{jj,0}^{\frac{1-\delta(1+\alpha)}{1-\sigma}} \widehat{\lambda}_{jj,1}^{\frac{1}{1-\sigma}}.$$

In period 2 we have:

$$\widehat{W}_{j,2} = \widehat{\lambda}_{jj,0}^{\frac{1-\delta(1+\alpha)}{1-\sigma}} \widehat{\lambda}_{jj,1}^{\frac{1}{1-\sigma}} \widehat{\lambda}_{jj,2}^{\frac{1}{1-\sigma}},$$
$$\widehat{K}_{j,3} = \widehat{\lambda}_{jj,0}^{\frac{(1-\delta(1+\alpha))^2}{1-\sigma}} \widehat{\lambda}_{jj,1}^{\frac{1-\delta(1+\alpha)}{1-\sigma}} \widehat{\lambda}_{jj,2}^{\frac{1}{1-\sigma}}.$$

Hence, in period n we have:

$$\widehat{W}_{j,n} = \widehat{\lambda}_{jj,n}^{\frac{1}{1-\sigma}} \prod_{i=0}^{n-1} \widehat{\lambda}_{jj,i}^{\frac{(1-\delta(1+\alpha))^{n-1-i}}{1-\sigma}},$$
$$\widehat{K}_{j,n+1} = \prod_{i=0}^{n} \widehat{\lambda}_{jj,i}^{\frac{(1-\delta(1+\alpha))^{n-i}}{1-\sigma}},$$

which are both functions of $\widehat{\lambda}_{jj}$ and parameters only.

So far the out-of steady-state formulae give welfare when not taking into account the discounting. Note that $\widehat{W}_{j,t} = \widehat{C}_{j,t}$. Hence, we can calculate welfare with discounting by using equation (47):

$$\begin{aligned}
\lambda &= \left(\exp\left[(1-\beta)\left(\sum_{t=0}^{\infty}\beta^t \ln\left(C_{j,t,c}\right) - \sum_{t=0}^{\infty}\beta^t \ln\left(C_{j,t}\right)\right)\right] - 1\right) \times 100 \\
&= \left(\exp\left[(1-\beta)\left(\sum_{t=0}^{\infty}\beta^t \ln\left(\widehat{C}_{j,t}\right)\right)\right] - 1\right) \times 100 \\
&= \left(\exp\left[(1-\beta)\left(\sum_{t=0}^{\infty}\beta^t \ln\left(\widehat{K}_{j,t}^{\alpha}\widehat{\lambda}_{jj,t}^{\frac{1}{1-\sigma}}\right)\right)\right] - 1\right) \times 100 \qquad \text{(A19)}
\end{aligned}$$

Hence, out-of steady-state, welfare can also be expressed as a function of the changes in $\lambda_{jj,t}$. However, we have to trace the change of λ_{jj} only driven by the counterfactual change over the transition. As we will typically not be able to observe these changes, this expression is more for gaining theoretical insights into the working of the system than for practical use.

E Counterfactual Procedure

The counterfactuals are performed in four steps.

Step 1: Obtain trade cost estimates by estimating equations (29) and (30). Then calculate bilateral trade costs for the baseline setting:

$$\left(\widehat{t}_{ij,t}^{RTA}\right)^{1-\sigma} = \exp\left[\widehat{\eta}_1 RTA_{ij,t} + \sum_{m=2}^{5}\widehat{\eta}_m \ln DIST_{ij,m-1} + \widehat{\eta}_6 BRDR_{ij} + \widehat{\eta}_7 LANG_{ij} + \widehat{\eta}_8 CLNY_{ij}\right]. \tag{A20}$$

For the counterfactual, additional trade costs may have to be calculated. For example, in the case of our NAFTA counterfactual, we set $RTA_{ij,t}$ to zero for the NAFTA countries after 1994, resulting in $RTA_{ij,t}^c$. Then we recalculate $\left(\widehat{t}_{ij,t}^{RTA}\right)^{1-\sigma}$ by replacing $RTA_{ij,t}$ with $RTA_{ij,t}^c$ in equation (A20). The differences between the values for the key variables of interest are obtained as a response to the change in the trade costs vector from $RTA_{ij,t}$ to $RTA_{ij,t}^c$.

Step 2: Using the estimates for trade costs described in Step 1, and estimates for the capital share $\widehat{\alpha}$, the elasticity of substitution $\widehat{\sigma}$, and the capital depreciation rate $\widehat{\delta}$ obtained from equations (34) and (39), a value for β taken from the literature, and data for $L_{j,t}$ and $y_{j,t}$, and assuming that we are in a steady-state, i.e., $K_{j,t+1} = K_{j,t}$, we can calculate P_j using equations (20) and (21) and recover (from equation (24)) country-specific, theory-consistent steady-state capital stocks as follows:

$$K_j^{SS} = \frac{\alpha\beta\delta y_j}{P_j\left(1-\beta+\beta\delta\right)}.$$

We use K_j^{SS} as our capital stock in period zero, i.e., $K_0 = K_j^{SS}$.

We also recover preference-adjusted technology A_j/γ_j in the baseline setting by noting that the 'lower level' can be solved without knowledge of A_j/γ_j and then using Π_j and combining (22) and (23), leading to:

$$\frac{A_j}{\gamma_j} = \frac{y_j \Pi_j}{\left(y_j/y\right)^{\frac{1}{1-\sigma}} L_j^{1-\alpha}\left(K_j^{SS}\right)^{\alpha}}.$$

As we recover K_j^{SS} and A_j/γ_j from data and estimated parameters, we ensure that our baseline setting is perfectly consistent with our GDP and employment data. However, our model allows us to perform one validation check. Specifically, we correlate our theory-consistent steady-state capital stocks and observed capital stocks as reported in the Penn World Tables 8.0. The correlation coefficient is 0.98. Figure 1 of the main text offers a visual representation of this relationship by plotting the log of the two series against each other, showing the strong log-linear correlation, which validates our estimates and gives us confidence to proceed with the policy counterfactual analysis as described in the next steps.

Step 3: Using the values obtained in Steps 1 and 2, we solve our system given by equations (19)-(24) in the baseline and in the counterfactual starting from year 0 until convergence to the new steady-state.

Step 4: After solving the model, we calculate the effects on trade, on the MRs, on welfare, and on capital accumulation. We report the results for all countries individually, as well as aggregates for the world, NAFTA, and the non-NAFTA countries (labeled "Rest Of the World", ROW).

Trade effects: Trade effects are calculated as percentage changes in overall exports for each country between the baseline and the counterfactual values:

$$\Delta x_{i,t}\% = \frac{\left(\sum_{j\neq i} x^c_{ij,t} - \sum_{j\neq i} x_{ij,t}\right)}{\sum_{j\neq i} x_{ij,t}} \times 100,$$

where $x_{ij,t}$ is calculated according to equation (19), and $x^c_{ij,t}$ are the counterfactual trade flows. Note that, in the case of NAFTA, we calculate the change of trade from the case without NAFTA to the case with NAFTA in place, as a share of trade in the case without NAFTA, even though we have to counterfactually solve for the case without NAFTA. The effects for the world as a whole are calculated by summing over all countries, i.e. $\Delta x_{\text{World},t}\% = \left(\sum_i \sum_{j\neq i} x^c_{ij,t} - \sum_i \sum_{j\neq i} x_{ij,t}\right) / \left(\sum_i \sum_{j\neq i} x_{ij,t}\right) \times 100$. For the trade effects within NAFTA, we only sum over the six within-NAFTA trade relationships (CAN-USA, CAN-MEX, MEX-CAN, MEX-USA, USA-CAN, USA-MEX). For ROW, we sum all remaining bilateral trade relationships.

MR effects: The MR effects are also calculated as the percentage change of $P_{i,t}$ and $\Pi_{i,t}$ for each country i and year t between the baseline and the counterfactual values. Note that with balanced trade and with symmetric trade costs $P_{i,t} = \Pi_{i,t}$, hence we only have to report one effect for every country in this case:

$$\Delta P_{i,t}\% = \frac{\left(P^c_{i,t} - P_{i,t}\right)}{P_{i,t}} \times 100,$$

where P_i is given by equation (20), and $P^c_{i,t}$ are the counterfactual MRs. The effects for the world are calculated as simple means over the changes for all countries, i.e. $\Delta P_{\text{World},t} = 1/N \sum_i \Delta P_{i,t}\%$. For NAFTA, we only take the mean over the three NAFTA members, while the results for ROW are calculated as the mean over the remaining 79 countries.

Welfare effects: In the 'Conditional GE' and in the 'Full Static GE' cases, welfare is given by real GDP per capita.[57] Using equation (23), $y_i = p_i A_i L_i^{1-\alpha} K_i^{\alpha}$, and equation (22), $(\gamma_i p_i \Pi_i)^{1-\sigma} = y_i/y$, to replace p_i, we can express real GDP per capita as:

$$\tilde{y}_i = \frac{y_i}{P_i L_i} = \frac{p_i A_i L_i^{1-\alpha} K_i^{\alpha}}{P_i L_i} = \frac{(y_i/y)^{1/(1-\sigma)} A_i L_i^{-\alpha} K_i^{\alpha}}{\gamma_i \Pi_i P_i},$$

[57]Note that in our setting P can also be interpreted as an ideal price index. C/P therefore corresponds to indirect utility.

and, similarly, the counterfactual real GDP per capita as:

$$\tilde{y}_i^c = \frac{y_i^c}{P_i^c L_i^c} = \frac{p_i^c A_i \left(L_i\right)^{1-\alpha} \left(K_i^c\right)^{\alpha}}{P_i^c L_i} = \frac{\left(y_i^c / y^c\right)^{1/(1-\sigma)} A_i \left(L_i\right)^{-\alpha} \left(K_i^c\right)^{\alpha}}{\gamma_i \Pi_i^c P_i^c}.$$

The change in welfare effects is then given by:

$$\Delta \tilde{y}_i \% = \frac{\left(\tilde{y}_i^c - \tilde{y}_i\right)}{\tilde{y}_i} \times 100.$$

In the 'Full Dynamic GE, SS' and 'Full Dynamic GE, trans.' scenarios, welfare is calculated according to equation (47). The results for the world are calculated as weighted sums of the welfare effects over all countries. We use GDPs as weights. Hence, the reported world welfare effects are calculated as: $\Delta \tilde{y}_{\text{World}} \% = \sum_i \left(\Delta \tilde{y}_i \% \times \frac{y_i}{\sum_j y_j} \right)$. For NAFTA, we only take the GPD weighted sum over the three NAFTA members, while the results for ROW are calculated as the GDP weighted sums over the remaining 79 countries.

Capital effects: The effects on capital are also calculated as the percentage changes between the baseline and the counterfactual values:

$$\Delta K_{i,t} \% = \frac{\left(K_{i,t}^c - K_{i,t}\right)}{K_{i,t}} \times 100,$$

where $K_{i,t}$ is given by equation (24), and $K_{i,t}^c$ are the counterfactual capital stocks in the new steady-state. The results for the world are calculated by summing over all countries, i.e. $\Delta K_{\text{World},t} \% = \left(\sum_i K_{i,t}^c - \sum_i K_{i,t} \right) / \left(\sum_i K_{i,t} \right) \times 100$. For NAFTA, we only sum capital stocks over the three NAFTA members in the baseline and counterfactual and calculate the change of this sum, while the results for ROW are calculated as the change of the sum of capital stocks for the remaining 79 countries.

F Our System in Changes

In this appendix, we derive our system in changes using the exact hat algebra as introduced by Dekle, Eaton, and Kortum (2007, 2008). We first derive the system in changes out-of steady-state followed by the system in changes in steady-state.

Denote counterfactual values with a prime ($'$), and define the change for variable z, as $\widehat{z} = z'/z$. Start with the capital equation as given by equation (24) and replace $p_{j,t} A_{j,t}$ using equation (22):

$$K_{j,t+1} = \left[\frac{y_{j,t} \beta \alpha \delta}{P_{j,t} \left(1 - \beta + \delta \beta\right)} \right]^{\delta} K_{j,t}^{1-\delta}.$$

This relationship holds in the baseline and counterfactual, so that we can write the change as:

$$\widehat{K}_{j,t+1} = \left[\frac{\widehat{y}_{j,t}}{\widehat{P}_{j,t}} \right]^{\delta} \widehat{K}_{j,t}^{1-\delta}.$$

To derive an expression for the changes of prices we use equation (22) to write:

$$\widehat{p}_{j,t} = \frac{(\widehat{y}_{j,t}/\widehat{y}_t)^{\frac{1}{1-\sigma}}}{\widehat{\Pi}_{j,t}},$$

with

$$\widehat{y}_t = \frac{\sum_i y'_{i,t}}{\sum_i y_{i,t}} \Rightarrow y_t\widehat{y}_t = \sum_i y_{i,t}\widehat{y}_{it}.$$

Next we derive an equation for $\widehat{\Pi}_{j,t}$. We use equation (21) to write:

$$\Pi_{i,t}^{1-\sigma}\widehat{\Pi}_{i,t}^{1-\sigma} = \sum_j \left(\frac{t_{ij,t}\widehat{t}_{ij,t}}{P_{j,t}\widehat{P}_{j,t}}\right)^{1-\sigma} \frac{y_{j,t}\widehat{y}_{j,t}}{y_t\widehat{y}_t}.$$

Similarly, we can write the change for $P_{j,t}$ using equation (20):

$$P_{j,t}^{1-\sigma}\widehat{P}_{j,t}^{1-\sigma} = \sum_i \left(\frac{t_{ij,t}\widehat{t}_{ij,t}}{\Pi_{i,t}\widehat{\Pi}_{i,t}}\right)^{1-\sigma} \frac{y_{i,t}\widehat{y}_{i,t}}{y_t\widehat{y}_t}.$$

The change in GDP is derived by using equation (23), and assuming that technology and labor stay constant:

$$\widehat{y}_{j,t} = \widehat{p}_{j,t}\widehat{K}_{j,t}^{\alpha}.$$

This completes our system in changes:

$$\begin{aligned}
\widehat{x}_{ij,t} &= \frac{\widehat{y}_{i,t}\widehat{y}_{j,t}}{\widehat{y}_t}\left(\frac{\widehat{t}_{ij,t}}{\widehat{\Pi}_{i,t}\widehat{P}_{j,t}}\right)^{1-\sigma}, \\
\Pi_{i,t}^{1-\sigma}\widehat{\Pi}_{i,t}^{1-\sigma} &= \sum_j \left(\frac{t_{ij,t}\widehat{t}_{ij,t}}{P_{j,t}\widehat{P}_{j,t}}\right)^{1-\sigma} \frac{y_{j,t}\widehat{y}_{j,t}}{y_t\widehat{y}_t}, \\
P_{j,t}^{1-\sigma}\widehat{P}_{j,t}^{1-\sigma} &= \sum_i \left(\frac{t_{ij,t}\widehat{t}_{ij,t}}{\Pi_{i,t}\widehat{\Pi}_{i,t}}\right)^{1-\sigma} \frac{y_{i,t}\widehat{y}_{i,t}}{y_t\widehat{y}_t}, \\
\widehat{p}_{j,t} &= \frac{(\widehat{y}_{j,t}/\widehat{y}_t)^{\frac{1}{1-\sigma}}}{\widehat{\Pi}_{j,t}}, \\
y_t\widehat{y}_t &= \sum_i y_{i,t}\widehat{y}_{it}, \\
\widehat{y}_{j,t} &= \widehat{p}_{j,t}\widehat{K}_{j,t}^{\alpha}, \\
\widehat{K}_{j,t+1} &= \left[\frac{\widehat{y}_{j,t}}{\widehat{P}_{j,t}}\right]^{\delta}\widehat{K}_{j,t}^{1-\delta}.
\end{aligned}$$

This system needs only data on GDPs ($y_{i,t}$) and trade costs ($t_{ij,t}$), and knowledge about α,

σ and δ. In other words, knowledge about $A_{j,t}$, γ_j, and β is not necessary. The change in $t_{ij,t}$, $\widehat{t}_{ij,t}$, are exogenous, i.e. they form the basis of our counterfactual NAFTA experiment. Further, with given GDPs and trade costs, we can solve for the $\Pi_{i,t}$'s and $P_{j,t}$'s. Hence, we are left with seven equations for each t in the seven unknown changes $\widehat{x}_{ij,t}$, $\widehat{y}_{i,t}$, $\widehat{y}_t$, $\widehat{\Pi}_{i,t}$, $\widehat{P}_{j,t}$, $\widehat{p}_{j,t}$, $\widehat{K}_{j,t}$.

Note that the capital equation in changes does not determine the level of capital. However, this is also not necessary. We merely have to note that $\widehat{K}_{j,0} = 1$, i.e. that there are no capital adjustments in the first iteration. Hence, we can write and solve our system in changes and solve for all counterfactual values of all endogenous variables with given K_0. The solutions are identical to the solutions of our system in levels. This shows that our reported changes from the system in levels are also invariant to the values of $A_{j,t}$, γ_j, and β. The reason is that they all enter multiplicative and are assumed to be constant between baseline and counterfactual.

In steady-state, the capital equation in changes simplifies to:

$$\widehat{K}_j = \left[\frac{\widehat{y}_j}{\widehat{P}_j}\right]^{\delta} \widehat{K}_j^{1-\delta} \Rightarrow \widehat{K}_j = \frac{\widehat{y}_j}{\widehat{P}_j}.$$

All other equations stay the same without time index.

G Additional Results for the NAFTA Counterfactual

In this appendix we provide detailed results for our NAFTA counterfactual. Specifically, we report the changes in trade, MR, welfare, and capital stocks for all countries, as well as summary statistics for the NAFTA members, the non-NAFTA members and the world as a whole. All changes are calculated as described in Online Appendix E, Step 4.

Table A1: Evaluation of NAFTA

Country	Trade effects			MR effects			Welfare effects			Capital
	Cond. GE	Full Static GE	Full Dynamic GE, trans.	Cond. GE	Full Static GE	Full Dynamic GE, trans.	Cond. GE	Full Static GE	Full Dynamic GE, trans.	Full Dynamic GE, trans.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
AGO	-0.018	-0.180	-0.290	0.293	0.327	0.377	-0.292	-0.490	-0.562	-0.655
ARG	-0.176	-0.581	-0.775	0.746	0.682	0.569	-0.741	-1.121	-1.177	-1.268
AUS	-0.115	-0.354	-0.511	0.425	0.446	0.456	-0.423	-0.702	-0.790	-0.907
AUT	-0.004	0.009	0.062	0.051	0.106	0.222	-0.051	-0.093	-0.121	-0.156
AZE	-0.007	-0.049	-0.074	0.115	0.176	0.282	-0.115	-0.218	-0.280	-0.351
BEL	-0.002	0.032	0.104	0.021	0.079	0.203	-0.021	-0.045	-0.068	-0.097
BGD	-0.076	-0.148	-0.172	0.181	0.226	0.310	-0.180	-0.309	-0.367	-0.439
BGR	-0.011	-0.070	-0.089	0.149	0.198	0.288	-0.149	-0.258	-0.307	-0.369
BLR	-0.011	-0.068	-0.096	0.140	0.195	0.291	-0.140	-0.252	-0.310	-0.380
BRA	-0.285	-0.501	-0.578	0.465	0.465	0.454	-0.463	-0.736	-0.806	-0.902
CAN	0.780	13.053	39.278	-13.363	-13.344	-13.377	15.424	29.608	44.204	60.021
CHE	-0.000	0.044	0.118	0.004	0.067	0.198	-0.004	-0.022	-0.048	-0.078
CHL	-0.043	-0.261	-0.409	0.383	0.404	0.426	-0.382	-0.628	-0.709	-0.811
CHN	-0.444	-0.473	-0.385	0.191	0.236	0.315	-0.190	-0.327	-0.385	-0.458
COL	-0.216	-0.582	-0.749	0.697	0.644	0.550	-0.692	-1.054	-1.115	-1.207
CZE	-0.007	-0.008	0.024	0.063	0.123	0.238	-0.063	-0.123	-0.163	-0.208
DEU	-0.058	-0.060	-0.021	0.066	0.126	0.241	-0.065	-0.129	-0.171	-0.218

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Table A1 – *Continued from previous page*

Country	Trade effects: Cond. GE	Trade effects: Full Static GE	Trade effects: Full Dynamic GE, trans.	MR effects: Cond. GE	MR effects: Full Static GE	MR effects: Full Dynamic GE, trans.	Welfare effects: Cond. GE	Welfare effects: Full Static GE	Welfare effects: Full Dynamic GE, trans.	Capital: Full Dynamic GE, trans.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
DNK	-0.012	-0.029	-0.013	0.087	0.144	0.253	-0.087	-0.162	-0.206	-0.257
DOM	-0.138	-0.457	-0.633	0.577	0.558	0.509	-0.574	-0.901	-0.974	-1.078
ECU	-0.050	-0.371	-0.553	0.563	0.538	0.491	-0.560	-0.866	-0.929	-1.018
EGY	-0.051	-0.126	-0.147	0.181	0.224	0.305	-0.181	-0.306	-0.358	-0.424
ESP	-0.093	-0.228	-0.283	0.283	0.312	0.358	-0.282	-0.462	-0.522	-0.595
ETH	-0.020	-0.286	-0.483	0.440	0.459	0.464	-0.438	-0.725	-0.814	-0.934
FIN	-0.012	-0.050	-0.062	0.112	0.171	0.275	-0.112	-0.209	-0.265	-0.328
FRA	-0.057	-0.103	-0.095	0.145	0.191	0.280	-0.145	-0.246	-0.287	-0.343
GBR	-0.165	-0.240	-0.240	0.204	0.246	0.320	-0.203	-0.345	-0.399	-0.471
GHA	-0.015	-0.316	-0.530	0.497	0.502	0.487	-0.495	-0.802	-0.888	-1.005
GRC	-0.034	-0.075	-0.078	0.125	0.178	0.277	-0.124	-0.223	-0.272	-0.333
GTM	-0.335	-1.021	-1.331	1.259	1.091	0.797	-1.244	-1.842	-1.893	-1.989
HKG	-0.220	-0.280	-0.269	0.180	0.230	0.315	-0.180	-0.316	-0.379	-0.457
HRV	-0.021	-0.139	-0.200	0.238	0.274	0.336	-0.237	-0.395	-0.450	-0.524
HUN	-0.019	-0.062	-0.060	0.129	0.179	0.273	-0.129	-0.223	-0.266	-0.321
IDN	-0.179	-0.276	-0.288	0.250	0.283	0.341	-0.250	-0.410	-0.467	-0.540
IND	-0.195	-0.385	-0.476	0.383	0.403	0.423	-0.382	-0.625	-0.701	-0.803
IRL	-0.003	-0.008	0.006	0.065	0.128	0.247	-0.065	-0.133	-0.181	-0.238
IRN	-0.061	-0.190	-0.251	0.266	0.296	0.350	-0.265	-0.435	-0.493	-0.569
IRQ	-0.011	-0.117	-0.173	0.218	0.257	0.326	-0.217	-0.363	-0.421	-0.493
ISR	-0.066	-0.346	-0.565	0.455	0.484	0.490	-0.453	-0.770	-0.884	-1.017
ITA	-0.095	-0.130	-0.109	0.132	0.182	0.276	-0.132	-0.229	-0.273	-0.330
JPN	-0.856	-0.812	-0.578	0.163	0.211	0.297	-0.163	-0.282	-0.334	-0.399
KAZ	-0.001	0.000	0.000	0.047	0.120	0.250	-0.047	-0.118	-0.180	-0.247
KEN	-0.038	-0.303	-0.496	0.442	0.461	0.466	-0.440	-0.729	-0.819	-0.939
KOR	-0.227	-0.279	-0.244	0.198	0.236	0.309	-0.197	-0.327	-0.375	-0.438
KWT	-0.179	-0.242	-0.238	0.181	0.229	0.313	-0.181	-0.315	-0.374	-0.449
LBN	-0.014	-0.142	-0.194	0.263	0.286	0.335	-0.262	-0.416	-0.454	-0.522
LKA	-0.054	-0.164	-0.217	0.235	0.271	0.336	-0.234	-0.390	-0.449	-0.524
LTU	-0.009	-0.081	-0.124	0.157	0.212	0.304	-0.157	-0.284	-0.348	-0.422
MAR	-0.031	-0.142	-0.194	0.230	0.267	0.331	-0.229	-0.382	-0.435	-0.508
MEX	2.646	9.812	24.343	-8.316	-8.317	-8.346	9.070	17.071	25.015	33.309
MYS	-0.015	-0.063	-0.077	0.133	0.185	0.281	-0.133	-0.234	-0.286	-0.348
NGA	-0.009	-0.305	-0.518	0.488	0.494	0.482	-0.485	-0.788	-0.874	-0.991
NLD	-0.019	-0.011	0.031	0.053	0.113	0.231	-0.053	-0.106	-0.143	-0.185
NOR	-0.013	-0.068	-0.090	0.137	0.192	0.288	-0.137	-0.247	-0.303	-0.368
NZL	-0.041	-0.313	-0.515	0.452	0.471	0.474	-0.450	-0.746	-0.841	-0.964
OMN	-0.017	-0.152	-0.237	0.256	0.294	0.354	-0.255	-0.430	-0.495	-0.580
PAK	-0.019	-0.141	-0.235	0.228	0.277	0.352	-0.228	-0.400	-0.479	-0.574
PER	-0.042	-0.297	-0.438	0.458	0.451	0.440	-0.456	-0.712	-0.773	-0.856
PHL	-0.084	-0.310	-0.462	0.400	0.423	0.440	-0.399	-0.661	-0.747	-0.858
POL	-0.020	-0.047	-0.030	0.109	0.159	0.259	-0.109	-0.189	-0.227	-0.277
PRT	-0.020	-0.068	-0.097	0.121	0.183	0.288	-0.121	-0.232	-0.298	-0.371
QAT	-0.039	-0.138	-0.193	0.207	0.253	0.328	-0.207	-0.356	-0.419	-0.499
ROM	-0.026	-0.128	-0.164	0.225	0.256	0.319	-0.224	-0.363	-0.408	-0.469
RUS	-0.244	-0.358	-0.374	0.288	0.318	0.366	-0.288	-0.474	-0.535	-0.619
SAU	-0.073	-0.189	-0.247	0.241	0.281	0.345	-0.240	-0.407	-0.470	-0.552
SDN	-0.008	-0.143	-0.220	0.261	0.293	0.348	-0.260	-0.428	-0.486	-0.562
SER	-0.013	-0.131	-0.197	0.235	0.273	0.336	-0.234	-0.392	-0.449	-0.525
SGP	-0.140	-0.224	-0.247	0.205	0.251	0.327	-0.204	-0.353	-0.416	-0.496
SVK	-0.007	-0.042	-0.035	0.117	0.167	0.265	-0.117	-0.203	-0.243	-0.295
SWE	-0.018	-0.061	-0.070	0.122	0.177	0.277	-0.122	-0.221	-0.274	-0.335
SYR	-0.003	-0.069	-0.102	0.154	0.205	0.296	-0.153	-0.271	-0.327	-0.395
THA	-0.200	-0.268	-0.255	0.210	0.248	0.320	-0.209	-0.349	-0.403	-0.472
TKM	-0.006	-0.100	-0.162	0.192	0.241	0.322	-0.192	-0.335	-0.399	-0.478
TUN	-0.016	-0.154	-0.203	0.284	0.299	0.339	-0.283	-0.440	-0.472	-0.534
TUR	-0.078	-0.174	-0.197	0.228	0.260	0.323	-0.227	-0.370	-0.417	-0.481
TZA	-0.004	-0.205	-0.352	0.345	0.374	0.409	-0.344	-0.573	-0.653	-0.756
UKR	-0.029	-0.084	-0.110	0.138	0.194	0.292	-0.138	-0.252	-0.311	-0.383
USA	3.047	4.240	6.870	-0.774	-0.893	-1.097	0.780	1.731	2.748	4.213
UZB	-0.027	-0.138	-0.206	0.222	0.265	0.337	-0.221	-0.379	-0.444	-0.526

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Table A1 – *Continued from previous page*

Country	Trade effects: Cond. GE	Trade effects: Full Static GE	Trade effects: Full Dynamic GE, trans.	MR effects: Cond. GE	MR effects: Full Static GE	MR effects: Full Dynamic GE, trans.	Welfare effects: Cond. GE	Welfare effects: Full Static GE	Welfare effects: Full Dynamic GE, trans.	Capital: Full Dynamic GE, trans.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
VEN	-0.099	-0.430	-0.611	0.591	0.563	0.508	-0.588	-0.911	-0.978	-1.072
VNM	-0.019	-0.118	-0.164	0.212	0.250	0.320	-0.212	-0.352	-0.405	-0.474
ZAF	-0.068	-0.286	-0.439	0.381	0.408	0.433	-0.379	-0.635	-0.721	-0.834
ZWE	-0.019	-0.201	-0.332	0.322	0.354	0.396	-0.321	-0.537	-0.615	-0.715
World	0.503	1.105	2.306	-0.021	0.009	0.059	0.556	1.155	1.842	2.213
NAFTA	34.848	48.764	78.183	-7.484	-7.518	-7.607	2.554	5.073	7.671	10.024
ROW	-1.413	-1.308	-0.914	0.262	0.295	0.350	-0.220	-0.368	-0.423	-0.485

Notes: This table reports results from our NAFTA counterfactual. It is based on observed data on labor endowments and GDPs for our sample of 82 countries. Further, it uses our estimated trade costs based on equation (32) and recovered theory-consistent, steady-state capital stocks according to the capital accumulation equation (24). We calculate baseline preference-adjusted technology A_j/γ_j according to the market-clearing equation (22) and the production function equation (23). Finally, the counterfactual is based on our own estimates of the elasticity of substitution $\widehat{\sigma} = 5.1$, the share of capital in the Cobb-Douglas production function $\widehat{\alpha} = 0.55$, and the capital depreciation rate $\widehat{\delta} = 0.052$. The consumers' discount factor β is set equal to 0.98. Column (1) gives the country abbreviations. Columns (2) to (4) report the percentage change in exports for the NAFTA counterfactual for each country, for the world as a whole, the NAFTA and the non-NAFTA countries (summarized as Rest Of the World, ROW) for three different scenarios. The "Conditional GE" scenario takes into account the direct and indirect trade cost changes but holds GDPs constant, the "Full Static GE" scenario additionally takes general equilibrium income effects into account, and the "Full Dynamic GE, trans." scenario adds the capital accumulation effects. For the latter, we report the results from the steady-state taking into account that gains take time to materialize. Columns (5) to (7) report the percentage change in the multilateral resistance terms for each country for the same three scenarios. Similarly, columns (8) to (10) give the welfare effects. The last column shows the percentage change in capital stocks for each country for the "Full Dynamic GE, trans." scenario. Further details to the counterfactuals can be found in Section 5 and Online Appendix E.

H Linear Capital Accumulation Function

In this appendix we investigate the consequences of the convenient log-linear capital accumulation function by deriving our system under the assumption that capital accumulation is subject to the more standard linear transition function (we skip country indices without loss of generality):

$$K_{t+1} = \Omega_t + (1-\delta)K_t.$$

The utility function is:

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(C_t).$$

Combine the budget constraint with the production function:

$$P_t C_t + P_t \Omega_t = p_t A_t L_t^{1-\alpha} K_t^{\alpha}.$$

Use the linear transition function for capital to express Ω_t as:

$$P_t C_t + P_t \left(K_{t+1} - (1-\delta)K_t\right) = p_t A_t L_t^{1-\alpha} K_t^{\alpha}.$$

Set up the Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) + \lambda_t \left(p_t A_t L_t^{1-\alpha} K_t^{\alpha} - P_t C_t - P_t \left(K_{t+1} - (1-\delta)K_t\right)\right)\right].$$

Take derivatives with respect to C_t, K_{t+1} and λ_t:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= \frac{\beta^t}{C_t} - \beta^t \lambda_t P_t \overset{!}{=} 0 \quad \text{for all } t. \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \beta^{t+1} \lambda_{t+1} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1} - \beta^t \lambda_t P_t \\
&\quad + \beta^{t+1} \lambda_{t+1} P_{t+1} (1-\delta) \overset{!}{=} 0 \quad \text{for all } t. \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &= p_t A_t L_t^{1-\alpha} K_t^{\alpha} - P_t C_t - P_t \left(K_{t+1} - (1-\delta) K_t \right) \overset{!}{=} 0 \quad \text{for all } t.
\end{aligned}$$

Use the first-order condition for consumption to express λ_t as:

$$\lambda_t = \frac{1}{C_t P_t}.$$

Replace this in the first-order condition for capital:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \beta^{t+1} \frac{1}{C_{t+1} P_{t+1}} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1} - \beta^t \frac{1}{C_t} \\
&\quad + \beta^{t+1} \frac{1}{C_{t+1}} (1-\delta) \overset{!}{=} 0 \quad \text{for all } t.
\end{aligned}$$

Simplify and re-arrange:

$$\frac{\beta p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_t^{\alpha-1}}{C_{t+1} P_{t+1}} = \frac{1}{C_t} - \frac{\beta}{C_{t+1}} (1-\delta) \quad \text{for all } t.$$

Use the definition of y_t to re-write the left-hand side of the above expression:

$$\frac{\alpha \beta y_{t+1}}{K_{t+1} C_{t+1} P_{t+1}} = \frac{1}{C_t} - \frac{\beta (1-\delta)}{C_{t+1}} \quad \text{for all } t.$$

Rearrange to obtain:

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left(\frac{\alpha y_{t+1}}{K_{t+1} P_{t+1}} + 1 - \delta \right) \quad \text{for all } t,$$

which is the familiar and standard consumption Euler-equation. Note that there are three forward-looking variables for each country in this system: y_t, C_t, and P_t (K_{t+1} is determined in t and therefore it is not a forward-looking variable). Thus, overall, we have $3N$ forward-looking variables in this system. These are also the endogenous variables we have to solve for.

Since there exists no analytical solution for this system, we feed the following set of equations into Dynare:

$$\begin{aligned}
y_{j,t} &= \frac{(y_{j,t}/y_t)^{\frac{1}{1-\sigma}}}{\gamma_j P_{j,t}} A_{j,t} L_{j,t}^{1-\alpha} K_{j,t}^{\alpha} \quad \text{for all } j \text{ and } t, \\
y_t &= \sum_j y_{j,t} \quad \text{for all } t, \\
y_{j,t} &= P_{j,t} C_{j,t} + P_{j,t} \left(K_{j,t+1} - (1-\delta) K_{j,t} \right) \quad \text{for all } j \text{ and } t, \\
P_{j,t} &= \left[\sum_i \left(\frac{t_{ij,t}}{P_{i,t}} \right)^{1-\sigma} \frac{y_{i,t}}{y_t} \right]^{\frac{1}{1-\sigma}} \quad \text{for all } j \text{ and } t, \\
\frac{1}{C_{j,t}} &= \frac{\beta}{C_{j,t+1}} \left(\frac{\alpha y_{j,t+1}}{K_{j,t+1} P_{j,t+1}} + 1 - \delta \right) \quad \text{for all } j \text{ and } t.
\end{aligned}$$

As noted in the main text, the Euler-equation is the only difference between our main system and the corresponding system obtained under linear capital accumulation (compare these equations to the ones we used in Dynare for our original system given in equations (A10)-(A14)). We also can formulate the original system for the case of a linear capital accumulation function:

$$x_{ij,t} = \frac{y_{i,t} y_{j,t}}{y_t} \left(\frac{t_{ij,t}}{\Pi_{i,t} P_{j,t}} \right)^{1-\sigma}, \tag{A21}$$

$$P_{j,t}^{1-\sigma} = \sum_i \left(\frac{t_{ij,t}}{\Pi_{i,t}} \right)^{1-\sigma} \frac{y_{i,t}}{y_t}, \tag{A22}$$

$$\Pi_{i,t}^{1-\sigma} = \sum_j \left(\frac{t_{ij,t}}{P_{j,t}} \right)^{1-\sigma} \frac{y_{j,t}}{y_t}, \tag{A23}$$

$$p_{j,t} = \frac{(y_{j,t}/y_t)^{\frac{1}{1-\sigma}}}{\gamma_j \Pi_{j,t}}, \tag{A24}$$

$$y_{j,t} = p_{j,t} A_{j,t} L_{j,t}^{1-\alpha} K_{j,t}^{\alpha}, \tag{A25}$$

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left(\frac{\alpha y_{t+1}}{K_{t+1} P_{t+1}} + 1 - \delta \right), \tag{A26}$$

$$K_0 \quad \text{given.}$$

When we compare the above equations with our original system given by equations (19)-(24), we see that the only differing equation is again the expression for capital accumulation. As noted above, equation (A26) is the consumption Euler equation, which gives an expression for the relationship that determines investment and, hence, capital stocks, but it no longer offers an analytical expression for next period capital stocks.

What does this new system imply for our results?

1. Concerning the empirical specification, we see that the trade cost estimates and the output equation estimates do not change at all. Therefore, trade costs $t_{ij}^{1-\sigma}$, the capital share α, and the elasticity of substitution σ can be estimated as in the case with the Cobb-Douglas transition function. However, as we no longer have a closed-form

solution for our policy function, we can not derive an estimable *Capital equation* and, therefore, we are no longer able to back out the depreciation rate δ and test for causal effects of trade on growth.

2. The steady-state version of equation (A26) is:

$$\begin{aligned}\frac{1}{C} &= \frac{\beta}{C}\left(\frac{\alpha y}{KP}+1-\delta\right) \Rightarrow \\ K &= \frac{\alpha y}{\left(\frac{1}{\beta}-1+\delta\right)P} = \frac{\alpha\beta y}{\left(1-\beta+\beta\delta\right)P}.\end{aligned}$$

Given this solution for the steady-state capital stock, which is again a function of parameters and y/P, all our analytical insights from Section 3.1 go through. Actually, the only difference is the missing δ in the numerator for the steady-state capital stock. However, when plugging in $y = PC + P\left(K_{t+1} - (1-\delta)K_t\right) = PC + \delta PK$, we see that δ reappears. From this equation we also can calculate steady-state consumption:

$$\begin{aligned}C &= \frac{y}{P} - \delta K = \frac{y}{P} - \frac{\alpha\beta\delta y}{\left(1-\beta+\beta\delta\right)P} = \\ &= \left[\frac{1-\beta+\beta\delta-\alpha\beta\delta}{1-\beta+\beta\delta}\right]\frac{y}{P}.\end{aligned}$$

This demonstrates that consumption is given by exactly the same function as in the case of our Cobb-Douglas transition function for capital in steady-state. Similarly, the level of investment δK is identical. With our estimated parameters of $\alpha = 0.55$, $\beta = 0.98$, $\delta = 0.05$, we end up with $\Omega P/y = 0.3943$ and $CP/y = 0.6057$. Note, however, that the capital stock as a share of GDP is now given by $\Omega P/(y\delta) = 7.886$.

3. Finally, for our counterfactuals, we have to back out A/γ. This can be done in the exact same fashion, given that we can determine the steady-state capital stock.

I Derivation of the Policy Functions of the 'Upper Level' when Accounting for Intermediates

In this appendix we extend our model to allow for intermediates. Intermediates in country j at time t, Q_{jt}, are assumed as an additional production factor in our Cobb-Dougals production function following Eaton and Kortum (2002) and Caliendo and Parro (2015). While α still denotes the capital share of production, we now introduce ξ as the labor share of produciton. The share of intermediates is then given by $1-\alpha-\xi$. We assume that intermediates are CES composites of domestic components ($q_{jj,t}$) and imported components from all other countries $i \neq j$ ($q_{ij,t}$), i.e. $Q_{j,t} = \left(\sum_i \gamma_i^{(1-\sigma)/\sigma} q_{ij,t}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$. With intermediates, the corresponding 'upper level' setting becomes (we omit the country indexes in order to economize on the notational burden):

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(C_t), \tag{A27}$$

$$K_{t+1} = K_t^{1-\delta} \Omega_t^{\delta}, \tag{A28}$$

$$y_t = p_t A_t K_t^{\alpha} L_t^{\xi} Q_t^{1-\alpha-\xi}, \tag{A29}$$

$$y_t = P_t C_t + P_t Q_t + P_t \Omega_t, \tag{A30}$$

$$K_0 \quad \textit{given}. \tag{A31}$$

As in Online Appendix B, we skip indices for current periods and denote next period variables by $'$. Further, we again define $\phi = 1/\delta$. Due to the Cobb-Douglas production function, the cost shares for all three inputs are given by their respective Cobb-Douglas coefficients. Specifically, $P_t Q_t$ is equal to $(1-\alpha-\xi)y_t$. Thus, we can rewrite (A30) as $P_t C_t = (\alpha+\xi)y_t - P_t \Omega_t$. The value of the value function at step 0, v^0, is equal to 0. In the next step, the value of the value function is given by:

$$\begin{aligned} v' &= \max_{K'} \ln C = \max_{K'} \ln\left((\alpha+\xi)y/P - \Omega\right) \\ &= \max_{K'} \ln\left((\alpha+\xi)y/P - \Omega\right) \\ &= \max_{K'} \ln\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P - (K'^{\phi}/K^{\phi-1})\right). \end{aligned}$$

The corresponding first order condition is:

$$\frac{1}{(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P - (K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}} = 0.$$

It follows that $K' = 0$.

Hence, $v' = \ln\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P\right)$. In the next step, we solve:

$$\begin{aligned} v^2 &= \max_{K'} \ln\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P - (K'^{\phi}/K^{\phi-1})\right) \\ &\quad +\beta \ln\left((\alpha+\xi)pAK'^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P\right). \end{aligned}$$

The first order condition is:

$$\frac{1}{(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}}+\frac{\alpha\beta}{K'}=0,$$
$$\frac{\alpha\beta}{\phi}\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})\right)=\frac{K'^{\phi}}{K^{\phi-1}},$$
$$\frac{\alpha\beta}{\phi}\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P\right)=\left(\frac{\alpha\beta}{\phi}+1\right)\frac{K'^{\phi}}{K^{\phi-1}},$$
$$\frac{\alpha\beta}{\phi+\alpha\beta}\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}}{P}K^{\alpha+\phi-1}=K'^{\phi},$$
$$\left(\frac{\alpha\beta}{\phi+\alpha\beta}\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}}{P}\right)^{\frac{1}{\phi}}K^{(\alpha+\phi-1)/\phi}=K'. \tag{A32}$$

Plug in the expression for K' given in equation (A32):

$$\begin{aligned}
v^2 &= \ln\Bigg((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P \\
&\quad -\left(\left(\left(\frac{\alpha\beta}{\phi+\alpha\beta}\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}}{P}\right)^{\frac{1}{\phi}}K^{(\alpha+\phi-1)/\phi}\right)^{\phi}/K^{\phi-1}\right) \\
&\quad +\beta\ln\Bigg((\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi} \\
&\quad \left(\left(\left(\frac{\alpha\beta}{\phi+\alpha\beta}\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}}{P}\right)^{\frac{1}{\phi}}K^{(\alpha+\phi-1)/\phi}\right)^{\alpha}/P\right), \\
&= \ln\left(\left((\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}/P-\frac{\alpha\beta}{\phi+\alpha\beta}\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}}{P}\right)K^{\alpha}\right) \\
&\quad +\beta\ln\Bigg((\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}\left(\frac{\alpha\beta}{\phi+\alpha\beta}\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}}{P}\right)^{\frac{\alpha}{\phi}} \\
&\quad K^{(\alpha+\phi-1)\alpha/\phi}/P\Bigg), \\
&= \alpha\ln(K)+\beta\theta\alpha\ln(K)+const,
\end{aligned}$$

where $\theta\equiv(\alpha+\phi-1)/\phi$ and *const* collects all terms not depending on K. The next step is

$$\begin{aligned}
v^3 &= \max_{K'}\ln\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})\right) \\
&\quad +\alpha\beta\ln\left(K'\right)+\beta^2\theta\alpha\ln(K')+\beta const.
\end{aligned}$$

The first order condition is given by:

$$
\begin{aligned}
&\frac{1}{(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}}\\
&+\frac{\alpha\beta}{K'}+\frac{\alpha\theta\beta^2}{K'}=0,\\
&\frac{\alpha\beta}{\phi}(1+\beta\theta)\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})\right)=\frac{K'^{\phi}}{K^{\phi-1}},\\
&\frac{\alpha\beta}{\phi}(1+\beta\theta)(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P=\left(\frac{\alpha\beta}{\phi}(1+\beta\theta)+1\right)\frac{K'^{\phi}}{K^{\phi-1}},\\
&K'=\left(\frac{\frac{\alpha\beta}{\phi}(1+\beta\theta)(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}/P}{\frac{\alpha\beta}{\phi}(1+\beta\theta)+1}\right)^{\frac{1}{\phi}}K^{\theta}. \qquad \text{(A33)}
\end{aligned}
$$

Plug in the solution of K' given in equation (A33):

$$v^3=\alpha\ln(K)+\alpha\beta\theta\ln(K)+\beta^2\theta^2\alpha\ln(K)+\beta const.$$

The next value of the value function takes the form:

$$
\begin{aligned}
v^4 \;=\; &\max_{K'}\ln\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})\right)\\
&+\alpha\beta\ln(K')\left[1+\beta\theta+\beta^2\theta^2\right]+\beta const,
\end{aligned}
$$

with the following first order condition:

$$
\begin{aligned}
&\frac{1}{(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})}(-\phi)\frac{K'^{\phi-1}}{K^{\phi-1}}\\
&+\frac{\alpha\beta\left[1+\beta\theta+\beta^2\theta^2\right]}{K'}=0,\\
&\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)\left((\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P-(K'^{\phi}/K^{\phi-1})\right)\\
&=\frac{K'^{\phi}}{K^{\phi-1}},\\
&\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}/P\\
&=\left(\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)+1\right)\frac{K'^{\phi}}{K^{\phi-1}},\\
&K'=\left(\frac{\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}/P}{\frac{\alpha\beta}{\phi}\left(1+\beta\theta+\beta^2\theta^2\right)+1}\right)^{\frac{1}{\phi}}K^{\theta}.
\end{aligned}
$$

Now we see the general pattern that can be described as:

$$v^m \Rightarrow K' = \left[\frac{\frac{\alpha\beta}{\phi}(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\sum_{i=0}^{m}(\beta\theta)^i}{1+\frac{\alpha\beta}{\phi}\sum_{i=0}^{m}(\beta\theta)^i} \right]^{\frac{1}{\phi}} K^{\theta},$$

where m denotes the mth-step. When $m \to \infty$, we end up with

$$\left[\frac{\frac{\alpha\beta}{\phi}(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\sum_{i=0}^{m}(\beta\theta)^i}{1+\frac{\alpha\beta}{\phi}\sum_{i=0}^{m}(\beta\theta)^i} \right]^{\frac{1}{\phi}} = \left[\frac{\frac{\alpha\beta}{\phi}(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\frac{1}{1-\beta\theta}}{1+\frac{\alpha\beta}{\phi}\frac{1}{1-\beta\theta}} \right]^{\frac{1}{\phi}}.$$

Replace $\theta \equiv (\alpha+\phi-1)/\phi$:

$$\left[\frac{\frac{\alpha\beta}{\phi}(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\frac{1}{1-\beta(\alpha+\phi-1)/\phi}}{1+\frac{\alpha\beta}{\phi}\frac{1}{1-\beta(\alpha+\phi-1)/\phi}} \right]^{\frac{1}{\phi}} = \left[\frac{(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\frac{\alpha\beta}{\phi-\beta(\alpha+\phi-1)}}{1+\frac{\alpha\beta}{\phi-\beta(\alpha+\phi-1)}} \right]^{\frac{1}{\phi}} = \left[\frac{(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\frac{\alpha\beta}{\phi-\beta(\alpha+\phi-1)}}{\frac{\phi-\beta\phi+\beta}{\phi-\beta(\alpha+\phi-1)}} \right]^{\frac{1}{\phi}} = \left[\frac{(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\alpha\beta}{\phi-\beta\phi+\beta} \right]^{\frac{1}{\phi}}.$$

Apply $\phi = 1/\delta$:

$$\left[\frac{(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\alpha\beta}{1/\delta-\beta/\delta+\beta} \right]^{\delta} = \left[\frac{(\alpha+\xi)\left(pAL^{\xi}Q^{1-\alpha-\xi}/P\right)\alpha\beta\delta}{1-\beta+\beta\delta} \right]^{\delta}.$$

Obtain the investment equation in the case with intermediates:

$$K' = \left[\frac{(\alpha+\xi)\alpha\beta\delta pAL^{\xi}Q^{1-\alpha-\xi}}{(1-\beta+\beta\delta)\,P} \right]^{\delta} K^{\alpha\delta+1-\delta}.$$

The main difference between this policy function for the capital stock in the next period, K', and the one in our main system is the appearance of the term for intermediates. If $(\alpha+\xi)=1$, i.e. if there are no intermediates, we end up with equation (15). As discussed in the main text, the main implications are that the effects of domestic investment in our model are magnified through this term, and that foreign capital now has an indirect impact on domestic output and investment that is also channeled through the new term for intermediates.

Finally, once we have pinned down the values for K' and K, we can determine the level of investment:

$$\begin{aligned}\Omega &= \left(\frac{K'}{K^{1-\delta}}\right)^{\frac{1}{\delta}} = \left(\frac{\left[\frac{(\alpha+\xi)\alpha\beta\delta pAL^{\xi}Q^{1-\alpha-\xi}}{(1-\beta+\beta\delta)P}\right]^{\delta}K^{\alpha\delta+1-\delta}}{K^{1-\delta}}\right)^{\frac{1}{\delta}} \\ &= \left[\frac{(\alpha+\xi)\alpha\beta\delta pAL^{\xi}Q^{1-\alpha-\xi}}{(1-\beta+\beta\delta)\,P}\right]K^{\alpha}.\end{aligned}$$

In addition, we can obtain the optimal level of current consumption by using the policy function for capital and reformulating $y = PC + P\Omega + PQ$, i.e.,

$$\begin{aligned}C &= \frac{y}{P} - \Omega - Q \\ &= \frac{pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}}{P} - \left[\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}\alpha\beta\delta}{P\,(1-\beta+\beta\delta)}\right]K^{\alpha} \\ &\quad -(1-\alpha-\xi)\frac{pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}}{P} \\ &= (\alpha+\xi)\frac{pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}}{P} - \left[\frac{(\alpha+\xi)pAL^{\xi}Q^{1-\alpha-\xi}\alpha\beta\delta}{P\,(1-\beta+\beta\delta)}\right]K^{\alpha} \\ &= \left[1-\frac{\alpha\beta\delta}{1-\beta+\beta\delta}\right]\frac{(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}}{P} \\ &= \left[\frac{(1-\beta+\beta\delta)-\alpha\beta\delta}{1-\beta+\beta\delta}\right]\frac{(\alpha+\xi)pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}}{P}.\end{aligned}$$

Note again, that:

$$\begin{aligned}Q &= (1-\alpha-\xi)\frac{pAK^{\alpha}L^{\xi}Q^{1-\alpha-\xi}}{P} \Rightarrow \\ Q &= \left[(1-\alpha-\xi)\frac{pAK^{\alpha}L^{\xi}}{P}\right]^{\frac{1}{\alpha+\xi}}.\end{aligned}$$

J Iso-Elastic Utility Function

Our log-linear utility function implies an intertemporal elasticity of substitution of 1. The macro-literature often uses a value of 0.5. Empirical studies seem to support values between 0.25 and 1, cf. Sampson (2014). In order to investigate the sensitivity of our results concerning the log-linear utility function, we generalize our utility function to an iso-elastic one (we skip country indices without loss of generality):

$$U_t = \sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\rho}-1}{1-\rho}, \quad \rho > 0,$$

where $1/\rho$ denotes the intertemporal elasticity of substitution. Note that this utility function approaches $\ln(C_t)$ for $\rho \to 1$. We retain all other assumptions of our baseline model.

Combining the budget constraint with the production function leads to:

$$P_t C_t + P_t \Omega_t = p_t A_t L_t^{1-\alpha} K_t^{\alpha}.$$

In order to end up with only one constraint, we replace Ω_t by using our capital transition function:

$$\Omega_t = \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}}.$$

Replacing Ω_t, we end up with the following constraint:

$$P_t C_t + P_t \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} = p_t A_t L_t^{1-\alpha} K_t^{\alpha}.$$

Setting up the Lagrangian leads to:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho} - 1}{1-\rho} + \lambda_t \left(p_t A_t L_t^{1-\alpha} K_t^{\alpha} - P_t C_t - P_t \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \right) \right].$$

Taking derivatives with respect to C_t, K_{t+1} and λ_t leads to the following set of first-order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= \beta^t C_t^{-\rho} - \beta^t \lambda_t P_t \stackrel{!}{=} 0 \quad \text{for all } t. \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \beta^{t+1} \lambda_{t+1} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1} - \beta^t \lambda_t P_t \left(\frac{1}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \frac{1}{\delta} K_{t+1}^{\frac{1}{\delta}-1} \\
&\quad - \beta^{t+1} \lambda_{t+1} P_{t+1} K_{t+2}^{\frac{1}{\delta}} \frac{\delta - 1}{\delta} K_{t+1}^{-\frac{1}{\delta}} \stackrel{!}{=} 0 \quad \text{for all } t. \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &= p_t A_t L_t^{1-\alpha} K_t^{\alpha} - P_t C_t - P_t \left(\frac{K_{t+1}}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \stackrel{!}{=} 0 \quad \text{for all } t.
\end{aligned}$$

Using the first-order condition for consumption, we can express λ_t as:

$$\lambda_t = \frac{C_t^{-\rho}}{P_t}.$$

Replacing this in the first-order condition for capital leads to:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \beta^{t+1} \frac{C_{t+1}^{-\rho}}{P_{t+1}} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1} - \beta^t C_t^{-\rho} \left(\frac{1}{K_t^{1-\delta}} \right)^{\frac{1}{\delta}} \frac{1}{\delta} K_{t+1}^{\frac{1}{\delta}-1} \\
&\quad - \beta^{t+1} C_{t+1}^{-\rho} K_{t+2}^{\frac{1}{\delta}} \frac{\delta - 1}{\delta} K_{t+1}^{-\frac{1}{\delta}} \stackrel{!}{=} 0 \quad \text{for all } t.
\end{aligned}$$

Simplifying a bit and re-arranging leads to:

$$\frac{\beta C_{t+1}^{-\rho} p_{t+1} A_{t+1} L_{t+1}^{1-\alpha} \alpha K_{t+1}^{\alpha-1}}{P_{t+1}} = C_t^{-\rho} \left(\frac{1}{K_t^{1-\delta}}\right)^{\frac{1}{\delta}} \frac{1}{\delta} K_{t+1}^{\frac{1}{\delta}-1} + C_{t+1}^{-\rho} \frac{(\delta-1)\beta}{\delta} K_{t+2}^{\frac{1}{\delta}} K_{t+1}^{-\frac{1}{\delta}} \quad \text{for all } t.$$

Using our definition of y_t, we can further re-write the left-hand side of this expression as:

$$\frac{\alpha\beta C_{t+1}^{-\rho} y_{t+1}}{K_{t+1} P_{t+1}} = \frac{C_t^{-\rho}}{\delta} \frac{K_{t+1}^{\frac{1}{\delta}-1}}{K_t^{\frac{1-\delta}{\delta}}} + \frac{\beta(\delta-1) C_{t+1}^{-\rho}}{\delta} \left(\frac{K_{t+2}}{K_{t+1}}\right)^{\frac{1}{\delta}} \quad \text{for all } t.$$

This is the standard consumption Euler-equation. Note that we have four forward-looking variables for each country: y_t, K_t, C_t, and P_t. Hence, overall we have $4N$ forward-looking variables in our system here. These are also the endogenous variables we have to solve for. So in Dynare, we use the following set of equations:

$$\begin{aligned}
y_{j,t} &= \frac{(y_{j,t}/y_t)^{\frac{1}{1-\sigma}}}{\gamma_j P_{j,t}} A_{j,t} L_{j,t}^{1-\alpha} K_{j,t}^{\alpha} \quad \text{for all } j \text{ and } t,\\
y_t &= \sum_j y_{j,t} \quad \text{for all } t,\\
y_{j,t} &= P_{j,t} C_{j,t} + P_{j,t} \left(\frac{K_{j,t+1}}{K_{j,t}^{1-\delta}}\right)^{\frac{1}{\delta}} \quad \text{for all } j \text{ and } t,\\
P_{j,t} &= \left[\sum_i \left(\frac{t_{ij,t}}{P_{i,t}}\right)^{1-\sigma} \frac{y_{i,t}}{y_t}\right]^{\frac{1}{1-\sigma}} \quad \text{for all } j \text{ and } t,\\
\frac{\alpha\beta C_{j,t+1}^{-\rho} y_{j,t+1}}{K_{j,t+1} P_{j,t+1}} &= \frac{C_{j,t}^{-\rho}}{\delta} \frac{K_{j,t+1}^{\frac{1}{\delta}-1}}{K_{j,t}^{\frac{1-\delta}{\delta}}} + \frac{\beta(\delta-1) C_{j,t+1}^{-\rho}}{\delta} \left(\frac{K_{j,t+2}}{K_{j,t+1}}\right)^{\frac{1}{\delta}} \quad \text{for all } j \text{ and } t. \qquad \text{(A34)}
\end{aligned}$$

Note that Equation (A34) only gives a relationship for determining the capital stocks, it is no longer an analytical expression for next period capital stocks, but rather the consumption Euler-equation.

What does this new system imply for our results:

1. Concerning the empirical specification, we see that the trade cost estimates and the output equation estimates do not change at all. Hence, trade costs, α and σ would be estimated as we did so far. However, we could not estimate a capital equation. Hence, we would not be able to back out δ's and establish a causal relationship between trade liberalization and capital accumulation.

2. Let us next study the implications for the steady-state (SS). In SS, Equation (A34)

reads as:

$$\begin{aligned}
\frac{\alpha\beta C_j^{-\rho} y_j}{K_j P_j} &= \frac{C_j^{-\rho}}{\delta} \frac{K_j^{\frac{1}{\delta}-1}}{K_j^{\frac{1-\delta}{\delta}}} + \frac{\beta(\delta-1) C_j^{-\rho}}{\delta} \left(\frac{K_j}{K_j}\right)^{\frac{1}{\delta}} \Rightarrow \\
\frac{\alpha\beta y_j}{K_j P_j} &= \frac{1}{\delta} + \frac{\beta(\delta-1)}{\delta} \Rightarrow \\
K_j &= \frac{\delta}{1+\beta(\delta-1)} \frac{\alpha\beta y_j}{P_j} \Rightarrow \\
K_j &= \frac{\alpha\beta\delta y_j}{(1-\beta+\beta\delta) P_j}.
\end{aligned}$$

Given this solution for the steady-state capital stock, which is again a function of parameters and y/P, all our analytical insights from Sections 2 and 3 of this document go through. Actually, the expression for the steady-state capital stock is identical to our expression for the steady-state capital stock in our baseline setting. Also consumption in steady-state is identical:

$$\begin{aligned}
C &= \frac{y}{P} - K = \frac{y}{P} - \frac{\alpha\beta\delta y}{(1-\beta+\beta\delta) P} = \\
&= \left[\frac{1-\beta+\beta\delta-\alpha\beta\delta}{1-\beta+\beta\delta}\right] \frac{y}{P}.
\end{aligned}$$

This shows that consumption is given by exactly the same function as in the case of our log-linear intertemporal utility function.

3. For our counterfactuals, we have to back out A/γ. This can be done in the exact same fashion, given that we can determine the steady-state capital stock.

Concerning welfare, we have to take care to use the iso-elastic utility function. Additionally, for our Lucas formula, we now have:

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \frac{C_{j,t,c}^{1-\rho} - 1}{1-\rho} &= \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(1+\frac{\lambda}{100}\right) C_{j,t}\right]^{1-\rho} - 1}{1-\rho} \Rightarrow \\
\sum_{t=0}^{\infty} \beta^t C_{j,t,c}^{1-\rho} &= \sum_{t=0}^{\infty} \beta^t \left[\left(1+\frac{\lambda}{100}\right) C_{j,t}\right]^{1-\rho} \Rightarrow \\
\left(1+\frac{\lambda}{100}\right)^{1-\rho} &= \frac{\sum_{t=0}^{\infty} \beta^t C_{j,t,c}^{1-\rho}}{\sum_{t=0}^{\infty} \beta^t C_{j,t}^{1-\rho}} \Rightarrow \\
\lambda &= \left[\left(\frac{\sum_{t=0}^{\infty} \beta^t C_{j,t,c}^{1-\rho}}{\sum_{t=0}^{\infty} \beta^t C_{j,t}^{1-\rho}}\right)^{\frac{1}{1-\rho}} - 1\right] \times 100.
\end{aligned}$$