Long-Term-Care Utility and Late-in-Life Saving
Online Appendix*

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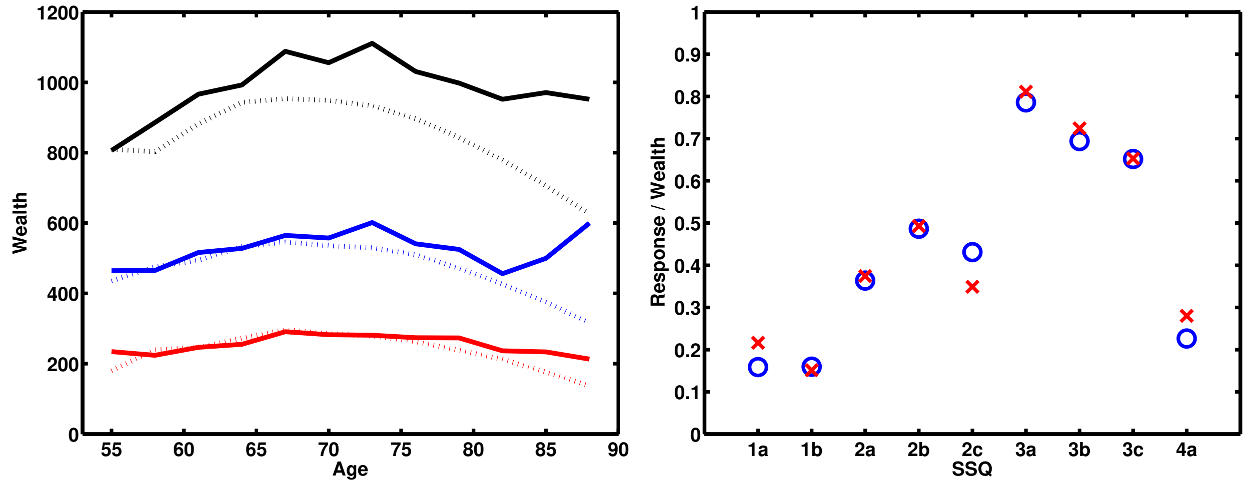
A Estimation Results Using the Optimal Weighting Matrix



(a) 75p, 50p, and 25p Wealth Moments in Model (dashed) and Data (solid)

(b) SSQ Means in Model (blue circle) and Data (red x)

Figure A.1: Model Fit When Jointly Targeting Wealth and SSQ Moments Using the Optimal Weighting Matrix

Table A.1: Estimated Parameters: Alternative Weighting Matrices

Joint Estimation: Baseline Model					
γ	θ_{ADL}	κ_{ADL}	θ_{beq}	κ_{beq}	ψ_G
5.27	0.67	-37.44	1.09	7.83	77.43
(0.05)	(0.37)	(0.22)	(0.25)	(0.48)	(9.49)

Joint Estimation: Optimal Weighting Matrix					
γ	θ_{ADL}	κ_{ADL}	θ_{beq}	κ_{beq}	ψ_G
4.69	1.00	-38.71	1.08	15.32	78.54
(0.05)	(0.08)	(0.98)	(0.02)	(0.24)	(2.21)

This table presents parameter estimates for the estimation targeting jointly both sets of moments for the cases in which we use the baseline weighting matrix and the optimal weighting matrix. Standard errors are reported in parentheses.

B Estimation Technicalities

The technicalities presented here, especially those related to the wealth moments, are adapted from the detailed appendix provided in De Nardi, French, and Jones (2010). The conditional moment conditions are defined as

$$\mathbb{E}\left[\mathbb{I}_{\{a_i<a_x^p(\Xi,\Theta,X)\}} - p|x_i \in x\right] = 0 \tag{B.1}$$

and

$$\mathbb{E}\left[s_m(\Theta) - z_{i,m}|x_i = x\right] = 0. \tag{B.2}$$

Converting the above conditional moment into an unconditional moment for estimation requires an application of the law of iterated expectations. Following Chamberlain (1992) the unconditional moment conditions are written using indicator functions. These moment conditions are presented here as the expectations

$$\mathbb{E}\left[\left(\mathbb{I}_{\{a_i<a_x^p(\Xi,\Theta,X)\}} - p\right) \times \mathbb{I}(x_i \in x)\right] = 0 \tag{B.3}$$

and

$$\mathbb{E}\left[(s_m(\Theta) - z_{i,m}) \times \mathbb{I}(x_i \in x)\right] = 0, \tag{B.4}$$

which can be feasibly be implemented as

$$\frac{1}{N}\sum_{i=1}^{N}\left[\left(\mathbb{I}_{\{a_i<a_x^p(\Xi,\Theta,X)\}} - p\right) \times \mathbb{I}(x_i \in x)\right] = 0 \tag{B.5}$$

and

$$\frac{1}{N}\sum_{i=1}^{N}\left[(s_m(\Theta) - z_{i,m}) \times \mathbb{I}(x_i \in x)\right] = 0. \tag{B.6}$$

This set of the sample analogue moment conditions is denoted by $g(\hat{\Xi}, \Theta, X)$. There are I individuals in the survey and we draw N individuals (with replacement) to compute the simulated moments. Define $\tau = \frac{I}{N}$. Let Ω denote the covariance matrix of these moment conditions, with $\hat{\Omega}$ denoting the empirical covariance matrix. Define the optimal weighting matrix $\hat{W} = \hat{\Omega}^{-1}$, and define

$$\hat{\Theta} = \arg\min_{\Theta} \frac{I}{1+\tau} g(\hat{\Xi}, \Theta, X)\hat{W}g(\hat{\Xi}, \Theta, X).$$

For matrix $W = plim_{N\to\infty}\hat{W}$, we know that

$$\sqrt{N}(\hat{\Theta} - \Theta_0) \to \mathbb{N}(0, \Psi),$$

where

$$\Psi = (1+\tau)(D'WD)^{-1}(D'W\Omega WD)(D'WD)^{-1}.$$

D is the gradient:

$$D = \left.\frac{\partial g(\hat{\Xi},\Theta,X)}{\partial \Theta'}\right|_{\Theta=\Theta_0}.$$

To obtain an expression for D, let f_a denote the density function of the empirical asset distribution and let f_z denote the density function of the empirical survey response distributions. f_a and f_z are both estimated using kernel density estimation. Then, following Pakes and Pollard (1989), Newey and McFadden (1994), and Powell (1994) the unconditional wealth moments can be represented as

$$\mathbb{P}(x_i \in x) \times \left[\int_{-\infty}^{a_x^p(\hat{\Xi},\Theta,X)} f_a(a_i|x)da_i - p\right] \tag{B.7}$$

and the unconditional strategic survey question moments can be represented as

$$\mathbb{P}(x_i \in x) \times \left[\int (s(\Theta) - z_{i,m}) \times f_z(z_{i,m}|x)dz_{i,m}\right]. \tag{B.8}$$

Then, the rows of the derivative matrix can be expressed as

$$D_V = \mathbb{P}(x_i \in x) \times f(a_x^p(\hat{\Xi},\Theta,X))|x) \times \left.\frac{\partial a_x^p(\hat{\Xi},\Theta,X)}{\partial \Theta'}\right|_{\Theta=\Theta_0} \tag{B.9}$$

and

$$D_S = \mathbb{P}(x_i \in x) \times \int \left.\frac{\partial s(\Theta)}{\partial \Theta'}\right|_{\Theta=\Theta_0} \times f_z(z_m^i|x)dz_m^i. \tag{B.10}$$

The above expressions are used to calculate the respective D matrices, with numerical derivatives used to calculate $\frac{\partial s(\Theta)}{\partial \Theta'}$ and $\frac{\partial a_x^p(\hat{\Xi},\Theta,X)}{\partial \Theta'}$.

Newey (1985) proves, with the following definitions,

$$Q = I - D(D'WD)^{-1}D'W \tag{B.11}$$

$$R = Q\hat{\Omega}Q, \tag{B.12}$$

that

$$\frac{I}{1+\tau} g(\hat{\Xi},\hat{\Theta},X)R^{-1}g(\hat{\Xi},\hat{\Theta},X) \rightsquigarrow \chi^2_{J-M}. \tag{B.13}$$

Finally, noting that asymptotically $\hat{W} \to \Omega^{-1}$ then $W = \hat{\Omega}^{-1}$, $V = (D'\hat{\Omega}^{-1}D)^{-1}$ and $R = \hat{\Omega}$.

In practice, repeating the calculation of the estimated covariance matrix with the parameter set $\hat{\Theta}$ allows us to calculate standard errors using the above asymptotic distribution. In this expression, we ignore the error in the first stage estimates by treating those as fixed numbers.

B.1 Details of Simulation

This section details step-by-step the simulation procedure that is used to implement the MSM procedure.

Before beginning the process, aggregate the survey data to create the initial empirical distribution of state variables. These consist of age, health, income, wealth, and SSQ responses. In addition, estimate the first stage parameters, $\hat{\Xi}$, and treat them as constants equal to the point estimate.

1. Set the second stage preference parameters that will be used in this simulation: $\hat{\Theta} = \Theta$
2. Sample a large number of individuals (N=10,000) from the initial distribution
3. Compute optimal policies using structural model for the specified parameter set Θ
   (a) Solve for the optimal policy functions for the life cycle savings model (as detailed in VRI Technical Report: Long-term Care Model)
   (b) Solve for the optimal strategic survey question responses (as detailed in VRI Technical Report: Long-term Care SSQs)
4. For each simulated individual $i \in \{1, 2, \ldots, N\}$, using $\hat{\Xi}$, simulate a series of health, health cost, and mortality shocks that dictate their life histories
5. Conditional on each individual's life history of exogenous idiosyncratic shocks, simulate each individual's choices using the optimal policy functions, yielding a life-cycle wealth profile for each individual
6. Conditional on $\hat{\Theta}$, simulate the individual strategic survey responses according to the optimal response policies, yielding a set of strategic survey responses for each individual
7. Construct moments:
   (a) Aggregate the individual life-cycle wealth profiles to construct the wealth moments
   (b) Aggregate the individual strategic survey question answers to construct the SSQ moments
   (c) Concatenate these two sets of moments for the baseline case. Otherwise, just use the relevant moment set
8. Compare these simulated moments to their empirical counterparts and update $\hat{\Theta} = \Theta'$
9. Repeat steps 1 to 8 with the updated parameters until the minimum of the MSM objective function is located

Steps 8 and 9 require the specification of an optimization algorithm. We optimize in two parts. First, we conduct a global search for our initial point Θ by repeating the algorithm minus steps 8 and 9 for 5,000 points defined by a Sobol sequence over our feasible parameter space. We then implement the algorithm above starting from the point associated with the minimum of the objective function, using a parallelized pattern search algorithm. At each iteration, we evaluate the function at M new points drawn from a scaled basis set of the parameter space. We then define the new parameter values, Θ' to be the $\arg\min$ of the objective function evaluated at these M points and the previous $\arg\min$ Θ. If Θ remains the $\arg\min$, then we scale the basis set downwards to shrink the search region. If a new $\arg\min$ is found, the basis set is scaled upwards to expand the search region. This process repeats iteratively until convergence.

C HRS Analysis

To implement comparison to a representative population and explore how general are findings from the Vanguard Research Initiative (VRI), in this appendix we explore the implications of our findings using the Health and Retirement Study (HRS). The HRS is a longitudinal survey of a representative sample of older (age 50+) Americans. Beginning in 1992, participants in this study provide information regarding their health, labor market activity, and finances biennially. Importantly, the HRS collects health, income, and wealth measures that are needed to construct the state variables, transition probabilities, and non-SSQ moments that are used in our study of the VRI. This allows us to explore the behavioral implications of LTC and bequest motives using the life-cycle saving model presented in this paper in a sample that is very different from the VRI.

Since the HRS does not have the SSQs necessary for sharp identification of the model's parameters, we use parameters estimated from the VRI to simulate the model and compare the predicted saving profiles with those actually observed in the HRS. Our main conclusion from this analysis is that our model and estimated saving motives perform extremely well at predicting non-targeted HRS wealth moments in nearly all analyses we conduct, and the only case where our baseline model does not perform well can easily be explained by weak identification of the normal health consumption floor. This provides strong indication that our main finding that strong LTC motives are significant drivers of late in life saving is a general finding that holds outside of the VRI sample we focus on in this study.

C.1 Data and Estimated Inputs to Model

C.1.1 HRS Data

The Health and Retirement Study (HRS) is a longitudinal study of Americans aged 50+ administered and maintained by the Survey Research Center (SCR) at the Institute for Social Research (ISR) at the University of Michigan since 1992. The study interviews approximately 20,000 respondents biennially and records information on health, income, labor market activity, and finances. The HRS is composed of 7 distinct cohorts, four of which are used in this appendix's analysis. We provide an overview of each here, including date of birth, and survey years in Table C.1.

Differences in income profiles, inability to identify time/age/cohort effects, and notable differences in wealth profiles lead us to conclude (as do other studies that use the HRS as a primary data source) that it is preferable to analyze cohorts separately. We thus focus on the original HRS, AHEAD, and only pool the most recent cohorts (WB/EBB) when analyzing saving motives.

To maintain comparability to the results in the paper, we restrict our analysis to individuals who are over age 50, are not married or partnered in any observed waves, and for which we observe both wealth and income at the time of entry into the sample. Table C.2 presents summary statistics by cohorts after imposing these sample restrictions.

C.1.2 Estimated Inputs to Model

Health and Survival Transition Probabilities & Health Costs. Health transitions, survival probabilities, and health costs are estimated using the procedure described in Section 5.1 using the HRS data

Table C.1: Summary of Cohort Structure

	Cohort			
	Original HRS **(HRS)**	**Asset and Health Dynamics Among the Oldest Old (AHEAD)**	**War Babies (WB)**	**Early Baby Boomers (EBB)**
Birth year:	1931-1941	pre-1924	1942-1947	1948-1953
First Surveyed:	1992	1993	1998	2004

without imposing VRI-eligible screens.

Income. To esitmate HRS income profiles, we follow De Nardi, French, and Jones (2010). Specifically, we first define permanent income as the cohort-specific percentile of average income observed across survey waves. We then sort individuals into income quintiles and model income (y_t) as a function of age, sex, permanent income

$$\ln y_{i,t} = \beta X_{i,t} + \Gamma_i + \eta_{i,t}$$

where $X_{i,t}$ includes age, sex, permanent income quintile, and polynomial and interaction terms, Γ_i is an individual fixed effect, and $\eta_{i,t}$ is an error term. For the AHEAD sample respondents are observed at very old ages, and income can thus be estimated at all ages using the AHEAD sample alone. For the HRS and WB/EBB samples, income is not observed at older ages. Thus, when studying these cohorts, income profiles are estimated by combining these samples with the AHEAD sample and including a cohort indicator variable in $X_{i,t}$. The resulting income profiles that are used in our model are presented in Table C.3.

Table C.2: Summary of State Variables

	HRS ($N = 2258$)					
	Mean	**10p**	**25p**	**50p**	**75p**	**90p**
Wealth:	128,105	1,000	1,000	35,737	127,729	323,246
Income:	29,633	2,511	9,437	22,439	40,798	64,335
					Health	
Avg. Age		**Male**		**Good**	**Poor**	**LTC**
55.3		32%		67%	33%	0%

	AHEAD ($N = 3765$)					
	Mean	**10p**	**25p**	**50p**	**75p**	**90p**
Wealth:	143,526	1,000	4,743	61,957	164,526	340,382
Income:	16,073	7,364	9,302	12,806	18,623	27,445
					Health	
Avg. Age		**Male**		**Good**	**Poor**	**LTC**
79.		21%		56%	28%	16%

	WB/EBB ($N = 1373$)					
	Mean	**10p**	**25p**	**50p**	**75p**	**90p**
Wealth:	143,871	1,000	1,000	32,415	149,186	395,585
Income:	34,044	234	9,244	25,478	49,144	76,056
					Health	
Avg. Age		**Male**		**Good**	**Poor**	**LTC**
53.0		36%		65%	29%	5%

Note: Values are measured in 2010 dollars during the year the individual entered the HRS.

Table C.3: Income Profiles by Cohort and Quintile

	WB/EBB					HRS				
Quintile	*const.*	*Age*	Age^2	*Male*	$Male \times Age$	*const.*	*Age*	Age^2	*Male*	$Male \times Age$
1	-12.125	.518	-.003	1.570	-.017	-28.599	.972	-.006	1.620	-.019
2	2.300	.164	-9.6e-4	1.126	-.013	-7.161	.419	-.003	1.883	-.024
3	8.404	.026	-1.5e-4	.955	-.011	1.186	.213	-.001	1.981	-.026
4	11.9713	-.042	2.0e-4	.314	-.004	8.689	.033	-2.03e-4	.906	-.012
5	11.890	-.022	3.9e-5	.152	-.003	8.578	.066	-5.3e-3	.427	-.006

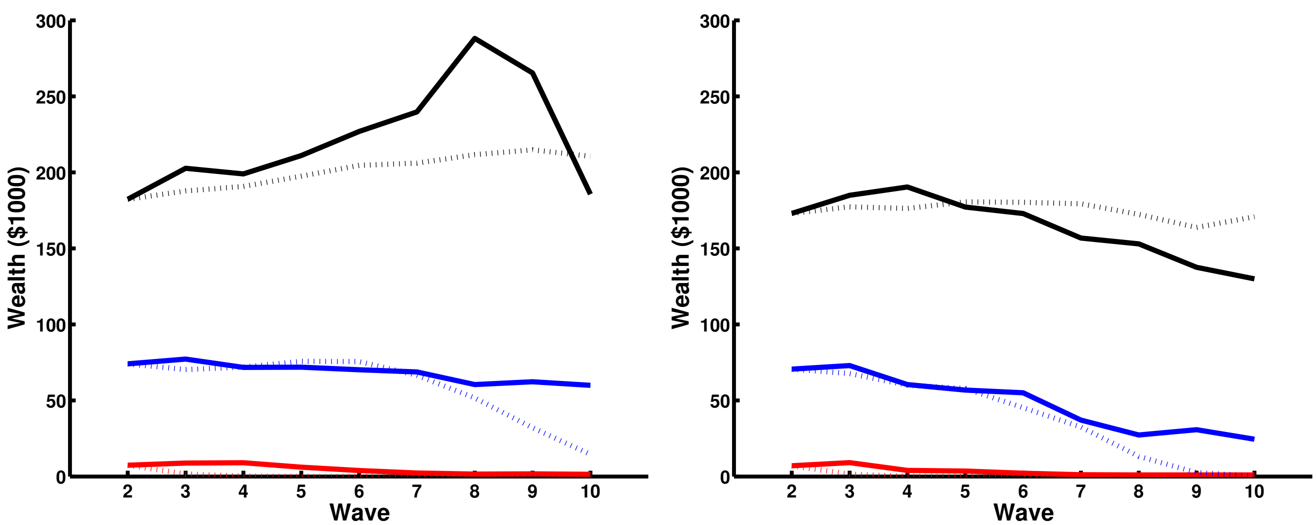
	WB/EBB				
Quintile	*const.*	*Age*	Age^2	*Male*	$Male \times Age$
1	11.030	-.062	4.0e-4	.937	-.010
2	7.500	.046	-2.9e-4	.038	8.5e-5
3	8.197	.036	-2.4e-4	.051	-2.0e-4
4	7.630	.062	-4.1e-4	-.189	.002
5	8.578	.066	-5.3e-4	.427	-.006

C.2 Model-implied Wealth Profiles for the HRS Population

Our simulation procedure is as follows: We initialize each individual in our sample at the same state variables presented in Table C.2. We then calculate the saving policy function implied by our model, simulate a set of shocks for each individual, and simulate their asset holdings forward for as long as they remain alive. We then calculate the $25/50/75^{th}$ percentile of the aggregate asset holdings in each wave that the cohort is observed. We plot these simulated asset percentiles against the actual wealth percentile observed in the wave indicated. We thus do not seek to match the plotted empirical wealth profiles after the initial wave in which we start our simulation.

C.2.1 AHEAD

De Nardi, French, and Jones (2010) estimate the saving dynamics of the AHEAD sample. To evaluate our model's performance in a familiar cohort and setting we benchmark our results by conducting the analysis presented above in the AHEAD sample. Following De Nardi, French, and Jones (2010) we split the sample into five birth-year cohorts to ensure we are comparing similar individuals. The resulting fit for the first two cohorts is presented in Figure C.1.



70-74 75-79

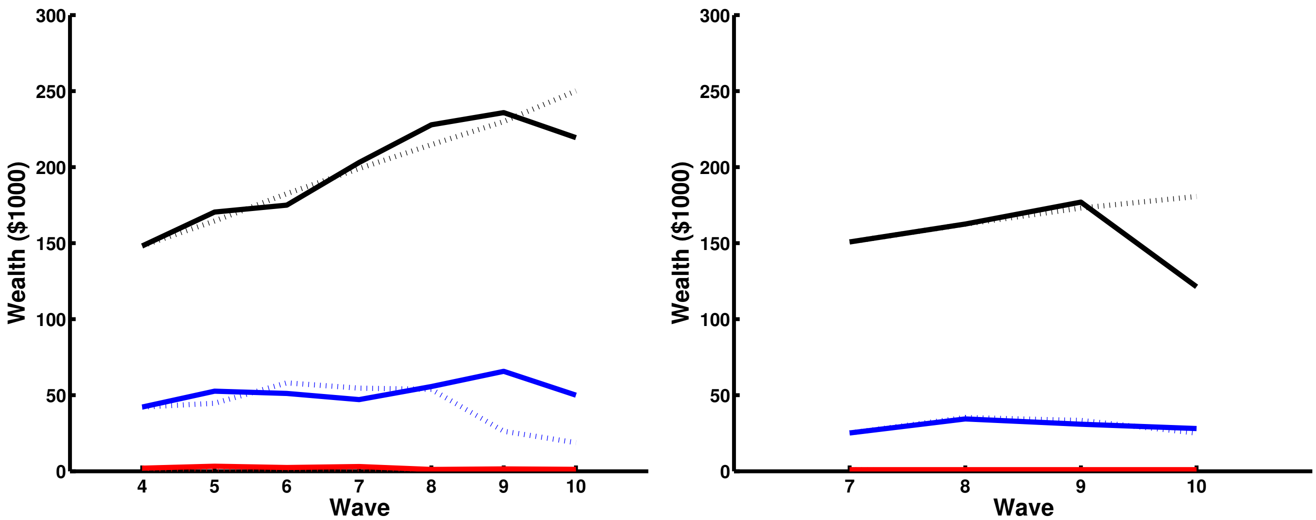
Figure C.1: Model results for AHEAD cohort

Among poorer households (25^{th} percentile) we successfully predict that households will not accumulate any wealth. Our model also performs reasonably well at matching the saving decisions of wealthier households, although our model predicted 75^{th} percentile is slightly more flat than that which we observe in the data. Our model does slightly slightly over-predict the spend down of the median wealth level in both figures, but the difference between model and data is only pronounced during the last 3 waves.

C.2.2 WB/EBB

We next examine the model's prediction for the War Babies and Early Baby Boomers. We isolate these cohorts because they are the most recent and thus make saving decisions at a similar time and environment to the VRI. The resulting fit is presented in Figure C.2.

Again, the model does a reasonably good job matching the data. It does underpredict wealth holdings in Wave 10 (2010), which could potentially be explained by our not modeling equities and therefore missing capital gain losses experienced during the experience the financial crisis.



WB EBB

Figure C.2: Model results for WB/EBB cohort

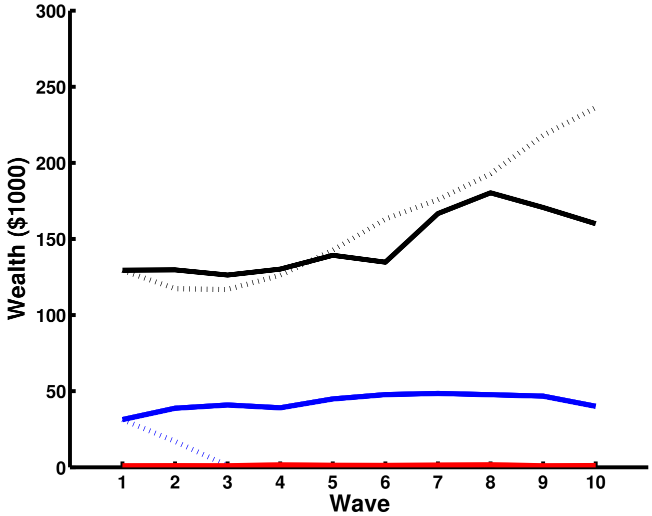
C.2.3 HRS

Finally we turn to the longest observed cohort, the original HRS. Because we match initial wealth percentiles by design, considering saving patterns over the longest possible time-frame provides the largest opportunity for us to reject our model as a reasonable approximation of the data generating process. We consider two cohorts: HRS respondents between ages 50-54 and 55-59 at the time of entry into the HRS. The results of this exercise are presented in Figure C.3.

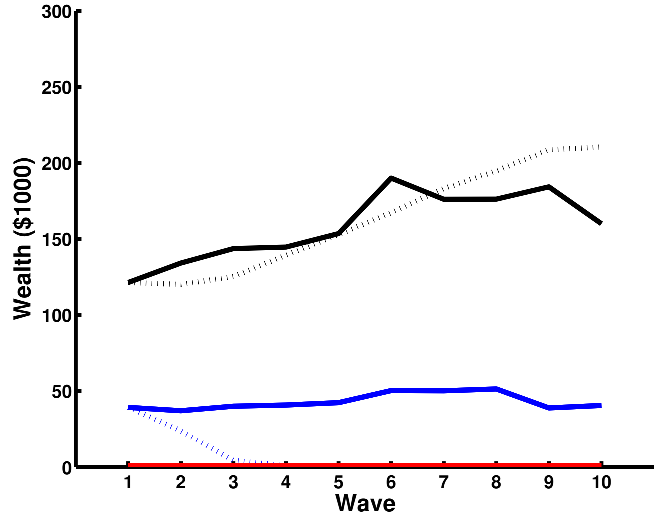
In this exercise the model fits relatively poorly. While we again match the lack of wealth accumulation among poorer households, our model significantly overpredicts the spend down of wealth for the median households. Furthermore, at longer horizons we overpredict saving at the 75^{th} percentile, which again is likely partly explained by our failure to capture capital losses during the financial crisis.

In our main analysis our normal health consumption floor was very weakly identified as this parameter had no effect on the saving decisions of households in the VRI. Because of this weak identification, and because this consumption floor is likely relevant for households in the HRS, we next explore whether we can improve the empirical fit of our model by choosing this parameter differently. As an extreme, we set the normal health consumption floor equal to $1,000, effectively making government care a non-attractive options for non-ADL state individuals, and redo our simulations. These results are presented in Figure C.4.

Here we see that our model fit is much improved, and the median wealth level predicted by our model is flat and very close to its empirical counterparts at all waves. Although our model still overpredicts wealth accumulation of wealthy households, this again can be attributed to unmodeled capital losses. Thus, if we allow ourselves the freedom to freely calibrate the one parameter that is unidentified in the VRI sample and

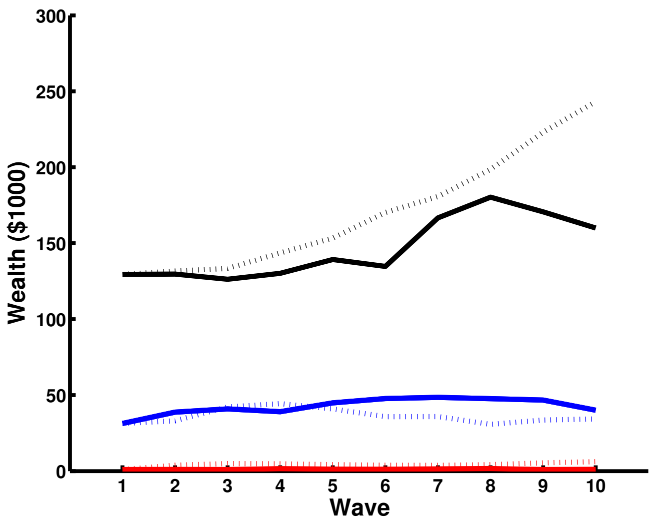


HRS: 50–54 in Wave 1

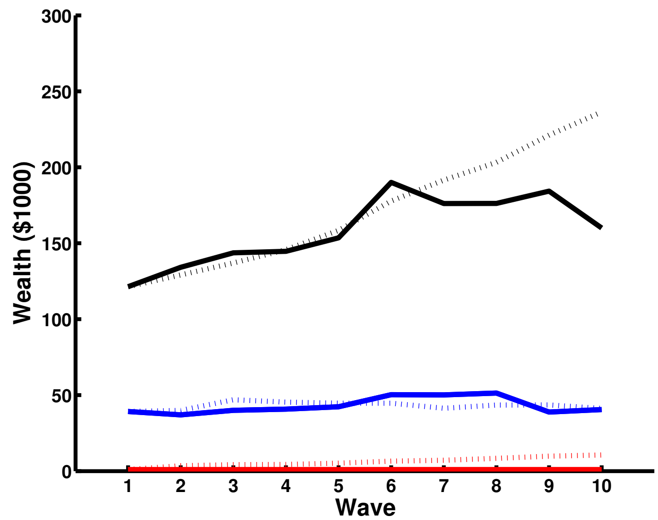


HRS: 55–59 in Wave 1

Figure C.3: Model results for original HRS cohort



HRS: 50-54 in Wave 1



HRS: 55-59 in Wave 1

Figure C.4: Model results for original HRS cohort with consumption floor (ω_G) equal to $1,000.

acknowledge we do not capture large capital losses between 2008 and 2010 HRS waves, we again conclude that our estimated saving motives predict wealth holdings in line with those observed in the HRS.

C.3 Conclusion

The above analysis provides suggestive evidence that the saving motives estimated in the VRI are capable of explaining the wealth accumulation patterns we observe in the HRS, a representative US sample. In addition, this analysis suggests that the normal-health consumption floor parameter that we estimate to be inconsequential for the VRI is potentially important for households with less significant financial resources.

References

Chamberlain, G. (1992): "Comment: Sequential Moment Restrictions in Panel Data," *Journal of Business and Economic Statistics*, 10, 20–26.

De Nardi, M., E. French, and J. B. Jones (2010): "Differential Mortality, Uncertain Medical Expenses, and the Saving of Elderly Singles," *Journal of Political Economy*, 118, 49–75.

Newey, W. (1985): "Generalized Method of Moments Specification Testing," *Journal of Econometrics*, 29, 229–256.

Newey, W. K., and D. L. McFadden (1994): "Large Sample Estimation and Hypothesis Testing," *Handbook of Econometrics*, 4, 1027–1057.

Pakes, A., and D. Pollard (1989): "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 57, 1027–1057.

Powell, J. (1994): "Estimation of Semi-Parametric Models," *Handbook of Econometrics*.