## Online Appendix

## Outline

- Section 1 provides additional details about the data and variable construction.
- Section 2 describes the propensity score re-weighting method used to adjust for chance imbalances in baseline characteristics.
- Section 3 explains how we construct the tests for equality and first order stochastic dominance whose p-values are reported in Figure 4 of the paper.
- Section 4 presents the baseline model described in Sections 4. We start by introducing definitions and restating the assumptions made in the paper. We then prove a few intermediate lemmas and conclude with the main propositions and their proofs which support the revealed preference restrictions summarized in Section 5 of the paper. Specifically,
- Lemma 1 establishes that no woman truthfully reports earnings above the federal poverty level while on assistance. Lemma 2 characterizes optimal reporting of earnings to the welfare agency. Corollary 1 describes the implication of optimal reporting for the dependence of preferences on the policy regime.
- Lemma 3 characterizes the relative attractiveness of each state under the two policy regimes. Lemma 4 provides the main revealed preference argument regarding pairing of states under JF and AFDC.
- Propositions 1 and 2 formally establish Table 4 in the paper. Corollary 2 establishes additional disallowed responses under the special form of the utility function introduced in Section 5 of the paper.
- Section 5, specifically Lemma 5, describes the exhaustive set of testable restrictions on state probabilities implied by revealed preference, as presented in Section 6 of the paper.
- Section 6 lists the analytical expressions for the bounds on the response probabilities and explain how they were derived. An example of such bounds is reproduced in Section 6 of the paper.
- Section 7 describes the construction of the $95 \%$ confidence intervals reported in Table 6 of the paper.
- Section 8 develops an extended model that relaxes the lower bound on stigma assumed in Section 4 the paper. This model is briefly referenced in Section 8 of the paper. Specifically,
- Propositions 3 and 4 establishes the effect of this relaxation on the response margins, as summarized in Table A4. Corollary 4 establishes additional disallowed responses under the special form of the utility function introduced in Section 4 of the paper.
- Corollary 5 shows that the relaxation of the lower bound on stigma may enable exit from the labor force in response to the JF reform only in the presence of labor market constraints.
- Section 9 develops an extended model that allows for participation in the FS program and accounts for taxes, including the EITC. This model is summarized in Section 8 of the paper. Specifically,
- Lemmas 6 and 7 characterize the combined welfare and FS transfer.
- Lemma 8 establishes that no woman truthfully reports earnings above the federal poverty level while on welfare assistance. Lemma 9 characterizes optimal reporting. Corollary 6 describes the implication of optimal reporting for the dependence of preferences on the policy regime.
- Lemma 10 provides the main revealed preference argument regarding pairing of states under JF and AFDC. Lemma 11 characterizes the relative attractiveness of each state under the two policy regimes.
- Propositions 6 and 7 establish the allowed and disallowed responses, as summarized in Table A5.
- Proposition 8 derives the response matrix. Proposition 9 and Remark 11 demonstrate that integrating out FS yields a response matrix with the same zero and unitary entries as the response matrix presented in Section 6 of the paper.
- Section 10 establishes the form of the response matrix when a finer coarsening of earnings is adopted. The results of this extension are summarized in Section 8 of the paper and the marginal distributions used for inference are reported in Table A6.
- Appendix Figures and Tables are provided at the end, along with references.


## 1 Data

## From Monthly to Quarterly Data

The public use files do not report the month of randomization. However, we were able to infer it by contrasting monthly assistance payments with an MDRC constructed variable providing quarterly assistance payments. For each case, we found that a unique month of randomization leads the aggregation of the monthly payments to match the quarterly measure to within rounding error.

## Measures of AU Size

The administrative measure of AU size is missing for most cases, which is problematic because the JF notch occurs at the FPL which varies with AU size. For the Jobs First sample we are able to infer an AU size in most months from the grant amount while the women are on welfare. However if AU size changes while off welfare we are not able to detect this change. ${ }^{1}$ Moreover, in some cases the grant amount does not match any of the base grant amounts. This can result when a woman reports some unearned income or because of sanctions. In both of these situations, we use the grant amount in other months to impute AU size. For the AFDC sample, the grant amount depends on many unobserved factors, preventing us from inferring the AU size from the administrative data.

The kidcount variable described in the text records the number of children in the household at the time of random assignment and is top-coded at three children. Appendix Table A1 gives a cross-tabulation, in the JF sample, of kidcount with our more reliable AU size measure inferred from grant amounts. The tabulation suggests the kidcount variable is a reasonably accurate measure of AU size over the first 7 quarters post-random assignment conditional on the number of children at baseline being less than three. As might be expected, the kidcount variable tends to underestimate the true AU size as women may have additional children over the 7 quarters following the baseline survey. To deal with this problem we inflate the kidcount based AU size by one in order to avoid understating the location of the poverty line for most assistance units. That is, we use the following mapping from kidcount to AU size: $0 \rightarrow 3,1 \rightarrow 3,2 \rightarrow 4,3 \rightarrow 5$, which maps each kidcount value to the modal inferred AU size in Appendix Table A1 plus one. This mapping is conservative in ensuring that earnings levels below the FPL are indeed below it.

[^0]
## 2 Propensity Score Re-weighting

We use propensity score re-weighting methods to adjust for the chance imbalances in baseline characteristics between the AFDC and JF groups. Following BGH (2006) we estimate a logit of the JF assignment dummy on: quarterly earnings in each of the 8 pre-assignment quarters, separate variables representing quarterly AFDC and quarterly food stamps payments in each of the 7 pre-assignment quarters, dummies indicating whether each of these 22 variables is nonzero, and dummies indicating whether the woman was employed at all or on welfare at all in the year preceding random assignment or in the applicant sample. We also include dummies indicating each of the following baseline demographic characteristics: being white, black, or Hispanic; being never married or separated; having a high-school diploma/GED or more than a high-school education; having more than two children; being younger than 25 or age $25-34$; and dummies indicating whether baseline information is missing for education, number of children, or marital status.

Denote the predicted values from this model by $\widehat{p}_{i}$. The propensity score weights used to adjust the moments of interest are given by:

$$
\omega_{i}=\frac{\frac{1\left[T_{i}=j\right]}{\hat{p}_{i}}}{\sum_{n=1}^{N} \frac{1\left[T_{n}=j\right]}{\hat{p}_{n}}}+\frac{\frac{1-\mathbf{1}\left[T_{i}=j\right]}{1-\hat{p}_{i}}}{\sum_{n=1}^{N} \frac{1-\mathbf{1}\left[T_{n}=j\right]}{1-\hat{p}_{n}}} .
$$

where $N$ is the number of cases. These are inverse probability weights, re-normalized to sum to one within policy group. When examining subgroups we always recompute a new set of propensity score weights and re-normalize them.

## 3 Distributional Tests

## Kolmogorov-Smirnov Test for Equality of Distributions

We use a bootstrap procedure to compute the p-values for our re-weighted Kolmogorov-Smirnov (KS) tests for equality of distribution functions across treatment groups. Let $F_{n}^{t}(e)$ be the propensity score re-weighted EDF of earnings in treatment group $t$. That is,

$$
F_{n}^{t}(e) \equiv \sum_{i} \omega_{i} \mathbf{1}\left[E_{i} \leq e, T_{i}=t\right]
$$

Define the corresponding bootstrap EDF as:

$$
F_{n}^{t *}(e) \equiv \sum_{i} \omega_{i}^{*} \mathbf{1}\left[E_{i}^{*} \leq e, T_{i}^{*}=t\right]
$$

where stars refer to resampled values (we resampled at the case level in order to preserve serial correlation in the data). The K-S test statistic is given by:

$$
\widehat{K S} \equiv \sup _{e}\left|F_{n}^{j}(e)-F_{n}^{a}(e)\right| .
$$

To obtain a critical value for this statistic, we compute the bootstrap distribution of the recentered K-S statistic:

$$
K S^{*} \equiv \sup _{e}\left|F_{n}^{j *}(e)-F_{n}^{a *}(e)-\left(F_{n}^{j}(e)-F_{n}^{a}(e)\right)\right| .
$$

Recentering is necessary to impose the correct null hypothesis on the bootstrap DGP (Giné and Zinn, 1990). We compute an estimated p-value $\widehat{\alpha}_{K S}$ for the null hypothesis that the two distributions are equal as:

$$
\widehat{\alpha}_{K S} \equiv \frac{1}{1000} \sum_{b=1}^{1000} \mathbf{1}\left[K S^{*(b)}>\widehat{K S}\right]
$$

where $b$ indexes the bootstrap replication.

## Barrett-Donald test for stochastic dominance

Our test statistic for detecting violations of the null hypothesis that the JF distribution of earnings stochastically dominates the AFDC distribution is given by:

$$
\widehat{B D} \equiv \sup _{e} F_{n}^{j}(e)-F_{n}^{a}(e) .
$$

As suggested by Barrett and Donald (2003), we bootstrap the re-centered version of this statistic given by:

$$
B D^{*} \equiv \sup _{e}\left[F_{n}^{j *}(e)-F_{n}^{a *}(e)-\left(F_{n}^{j}(e)-F_{n}^{a}(e)\right)\right] .
$$

We compute an estimated p-value $\widehat{\alpha}_{B D}$ as:

$$
\widehat{\alpha}_{B D} \equiv \frac{1}{1000} \sum_{b=1}^{1000} \mathbf{1}\left[B D^{*(b)}>\widehat{B D}\right] .
$$

## 4 Baseline Model

## Notation, Definitions, and Assumptions

Notation (Policy Regimes). Throughout, we use $a$ to refer to AFDC and $j$ to refer to JF. The policy regime is denoted by $t \in\{a, j\}$.

Definition 1 (Earnings, Reported Earnings, and Program Participation). Let $D$ be an indicator for a woman participating in welfare: $D=1$ if she is on assistance and $D=0$ otherwise. Let $E$ denote a woman's earnings. Earnings are the product of hours of work, $H$, and an hourly wage rate, $W$. Let $E^{r}$ denote the earnings a woman reports to the welfare agency and let $R$ be an indicator that takes the value 1 when a welfare recipient reports zero earnings and takes the value 0 otherwise, that is, $R=R\left(D, E^{r}\right) \equiv \mathbf{1}\left[E^{r}=0\right] D$.

Definition 2 (Earning Ranges). Earnings range 0 refers to zero earnings. Earnings range 1 refers to the interval $\left(0, F P L_{i}\right.$ ] where $F P L_{i}$ is woman $i$ 's federal poverty line. Earnings range 2 refers to the interval $\left(F P L_{i}, \infty\right)$.

Definition 3 (Welfare Transfer Functions). For any reported earnings $E^{r}$, the regime dependent transfers are

$$
G_{i}^{a}\left(E^{r}\right) \equiv \max \left\{\bar{G}_{i}-\mathbf{1}\left[E^{r}>\delta_{i}\right]\left(E-\delta_{i}\right) \tau_{i}, 0\right\},
$$

and

$$
G_{i}^{j}\left(E^{r}\right) \equiv \mathbf{1}\left[E^{r} \leq F P L_{i}\right] \bar{G}_{i} .
$$

The parameter $\delta_{i} \in\{90,120\}$ gives woman $i$ 's fixed disregards and the parameter $\tau_{i} \in\{.49, .73\}$ governs her proportional disregard. $\bar{G}_{i}$, the base grant amount, and $F P L_{i}$, the federal poverty level, vary across women due to differences in AU size. Define woman $i$ 's break-even earnings level under regime $a$ as $\bar{E}_{i} \equiv \bar{G}_{i} / \tau_{i}+\delta_{i}$, this is the level at which benefits are exhausted.

Definition 4 (Consumption Equivalent). Consider the triple ( $E, D, E^{r}$ ). Under regime $t \in$ $\{a, j\}$, woman $i$ 's consumption equivalent corresponding to $\left(E, D, E^{r}\right)$ is

$$
\begin{equation*}
C_{i}^{t}\left(E, D, E^{r}\right) \equiv E+D\left(G_{i}^{t}\left(E^{r}\right)-\kappa_{i} \mathbf{1}\left[E^{r}<E\right]\right) . \tag{1}
\end{equation*}
$$

For simplicity we refer to $C_{i}^{t}=C_{i}^{t}\left(E, D, E^{r}\right)$ as consumption. Below, when the consumption associated with a triple ( $E, D, E^{r}$ ) and calculated according to (1) does not vary across regimes we omit the superscript $t$, and we omit the subscript $i$ when it does not vary across women.

Definition 5 (State). Consider the triple ( $E, D, E^{r}$ ). The state corresponding to ( $E, D, E^{r}$ ) is defined by the function:

$$
s\left(E, D, E^{r}\right)=\left\{\begin{array}{ll}
0 n & \text { if } E=0, D=0 \\
1 n & \text { if } E \text { in range } 1, D=0 \\
2 n & \text { if } E \text { in range } 2, D=0 \\
0 r & \text { if } E=0, D=1 \\
1 r & \text { if } E \text { in range } 1, D=1, E^{r}=E \\
1 u & \text { if } E \text { in range } 1, D=1, E^{r}<E \\
2 u & \text { if } E \text { in range } 2, D=1, E^{r}<E \\
2 r & \text { if } E \text { in range } 2, D=1, E^{r}=E
\end{array} .\right.
$$

Definition 6 (Job Offers). A woman's samples $K_{i}$ job offers, composed of wage and hours offer pairs: $\Theta_{i}=\left\{\left(W_{i}^{k}, H_{i}^{k}\right)\right\}_{k=1}^{K_{i}}$ where $K_{i}$ is an integer number (possibly zero), $\left(W_{i}^{k}, H_{i}^{k}\right) \in$ $(0, \infty) \times\left(0, \bar{H}_{i}\right]$ with $\bar{H}_{i}$ denoting the woman's total disposable time. The limiting case $K_{i}=\infty$ is treated as follows: for any $H \in\left(0, \bar{H}_{i}\right]$ a woman's samples a wage offer $W_{i}(H)$. When $K_{i}=\infty$ let $\Theta_{i}=W_{i}() \times.\left(0, \bar{H}_{i}\right]$.

Definition 7 (Alternative). An alternative is wage, hours of work, welfare participation indicator, and earning report tuple ( $W, H, D, E^{r}$ ).

Definition 8 (Sub-alternative). A sub-alternative is wage, hours of work, and welfare participation indicator tuple ( $W, H, D$ ).

Definition 9 (Alternative Compatible with a State). We say that alternative ( $W, H, D, E^{r}$ ) is compatible with state $s$ for woman $i$ if, letting $E \equiv W H, s=s\left(E, D, E^{r}\right)$.

Definition 10 (Alternative Compatible with a State and Available). We say that alternative ( $W, H, D, E^{r}$ ) is available and compatible with state $s$ for woman $i$ if $\left(W, H, D, E^{r}\right)$ is compatible with state $s$ and $(W, H) \in \Theta_{i} \cup(0,0)$.

Definition 11 (Dominated State). We say that state $s$ is dominated under regime $t$ if no available alternative compatible $s$ under regime $t$ is chosen by any woman.

Definition 12 (Utility Function). Define $U_{i}^{t}(H, C, D, R)$ as the utility woman $i$ derives from the tuple $(H, C, D, R)$ under regime $t \in\{a, j\}$. When the utility of a tuple ( $H, C, D, R$ ) is regimeinvariant we omit the superscript $t$.

Definition 13 (Relative attractiveness of a State). We say that state $s$ is:

1. no better under regime $j$ than under regime $a$ if, for any alternative ( $W, H, D, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \leq U_{i}^{a}\left(H, C_{i}^{a}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \text { for all } i .
$$

2. no worse under regime $j$ than under regime $a$ if, for any alternative $\left(W, H, D, E^{r}\right)$ compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \geq U_{i}^{a}\left(H, C_{i}^{a}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \text { for all } i .
$$

3. equally attractive under regime $j$ and regime $a$ if, for any alternative ( $W, H, D, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right)=U_{i}^{a}\left(H, C_{i}^{a}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \text { for all } i .
$$

Definition 14 (Collections of States). Define $\mathcal{S} \equiv\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}, \mathcal{C}_{+} \equiv\{1 r\}, \mathcal{C}_{-} \equiv$ $\{0 r\}$, and $\mathcal{C}_{0} \equiv\{0 n, 1 n, 2 n, 1 u, 2 u\}$.

Assumption 1 (Preferences). Woman $i$ 's utility functions $U_{i}^{a}(., ., .,$.$) and U_{i}^{j}(., ., .,$.$) satisfy the$ following restrictions:
A. 1 utility is strictly increasing in $C$;
A. $2 \quad U_{i}^{t}(H, C, 1,1) \leq U_{i}^{t}(H, C, 1,0)$ for all $(H, C)$ and $t \in\{a, j\}$;
A. $3 \quad U_{i}^{j}(H, C, 1,1) \leq U_{i}^{a}(H, C, 1,1)$ for all $(H, C)$;
A. $4 \quad U_{i}^{a}(H, C, 1,0)=U_{i}^{j}(H, C, 1,0)$ for all $(H, C)$ with $H>0$;
A. $5 \quad U_{i}^{a}(H, C, 0,0)=U_{i}^{j}(H, C, 0,0)$ for all $(H, C)$;
A. $6 \quad U_{i}^{a}\left(H, C_{i}^{a}(E, 1, E), 1,0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, 0, E), 0,0\right)$ for all $(H, W)$ such that $E=$ $W H \in\left(F P L_{i}, \bar{E}_{i}\right]$ whenever $\bar{E}_{i}>F P L_{i}$.

Remark 1 (Preferences: Verbalizing Assumption 1). A. 2 states that hassle does not increase utility; this "hassle disutility" can vary across alternatives. A. 3 states that regime $j$ 's hassle disutility is no smaller than regime $a$ 's; the difference in hassle disutility between two regimes may vary with the alternative. Assumption A. 4 states that the impact on utility of welfare participation does not vary with the regime whenever reported earnings are not zero. A. 5 states that the utility value of an alternative entailing no welfare recipiency is independent of the treatment. A. 6 implicitly defines a lower bound on the disutility from stigma. It says that at earning levels above $F P L_{i}$, the extra consumption due to the transfer income does not suffice to compensate the woman for the stigma disutility she incurs when being on assistance.
Remark 2 (Preferences: A Special Case). In the paper, we consider a restricted specification of the 4 -argument utility function $U_{i}^{t}(., ., .,$.$) in Assumption 1$. We do so to aid in illustrating the mechanics of the model and the implications of further restricting preferences. Specifically, we employ a 2-argument utility function $U_{i}(.,$.$) :$

$$
\begin{equation*}
U_{i}\left(H, C-\mu_{i} \mathbf{1}[E>0]-\phi_{i} D-\eta_{i}^{t} R\right), \tag{2}
\end{equation*}
$$

where $\mu_{i}$ is a fixed cost of working, $\phi_{i}$ is a stigma cost from welfare participation, and $\eta_{i}^{t}$ is a hassle cost from reporting zero earnings on assistance. The parameters $\left(\mu_{i}, \phi_{i}, \eta_{i}^{a}, \eta_{i}^{j}\right)$ are such that, for all $i, \mu_{i} \geq 0$ in accordance with A.1, the stigma cost $\phi_{i}$ is regime invariant in accordance with $\mathbf{A} .4$ in Assumption 1, $\eta_{i}^{j} \geq \eta_{i}^{a} \geq 0$ in accordance with A. 2 and A. 3 in Assumption 1, and the utility function is not indexed by regime $t$ in accordance with A. 5 in 1. A sufficient condition for A. 6 in Assumption 1 to hold, is that, $\phi_{i}>G_{i}^{a}\left(F P L_{i}\right)$ for all $i$. Furthermore, the 2-argument utility function $U_{i}(.,$.$) is strictly increasing in its second argument in accordance with A. 1$ in Assumption 1. To preview, form (2) is used below in Corollaries 2 and 4.

Remark 3 (Preferences: Another Special Case). In this Appendix, we consider a second special case of the 4 -argument utility function $U_{i}^{t}(., ., .,$.$) under Assumption 1. We do so to provide$ examples. Specifically, we let the utility that a generic woman $i$ derives under regime $t$ from alternative ( $W, H, D, E^{r}$ ) is, letting $E \equiv W H$ :

$$
\begin{align*}
U_{i}^{t}\left(H, C_{i}^{t}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right)= & -\alpha_{i} H+v\left(C_{i}^{t}\left(E, D, E^{r}\right)-\mu_{i} \mathbf{1}[E>0]\right)  \tag{3}\\
& -\phi_{i} D-\eta_{i}^{t} R\left(D, E^{r}\right)
\end{align*}
$$

where $\alpha_{i}$ is the change in utility that the woman derives from one additional unit of work, $\mu_{i}$ is a fixed cost of working, $\phi_{i}$ is a stigma cost (or benefit) from welfare participation, $\eta_{i}^{t}$ is a hassle cost from reporting zero earnings on assistance, and $v($.$) is a strictly increasing function by A.1. By A.2-A. 5$ in Assumption 1, the parameters $\left(\mu_{i}, \eta_{i}^{j}, \eta_{i}^{a}, \phi_{i}\right)$ are such that $\mu_{i} \geq 0, \eta_{i}^{j} \geq \eta_{i}^{a} \geq 0$. By A. 6 in Assumption $1 \phi_{i}$ is bounded below by $\underline{\phi}_{i} \equiv \max _{E \in\left[F P L_{i}, \bar{E}_{i}\right]}\left[v\left(E-\mu_{i}+G_{i}^{a}\left(F P L_{i}\right)\right)-v\left(E-\mu_{i}\right)\right]$.

For convenience we assume that $\alpha_{i} \geq 0$, that is, leisure is a good. We consider three forms of $v$ (.): the identity function (hence $v($.$) linear), a strictly concave function (hence the marginal utility of$ consumption is strictly decreasing in consumption), a strictly convex function (hence the marginal utility of consumption is strictly increasing in consumption). When $v($.$) is linear the lower bound$ on the stigma disutility implied by A. 6 in Assumption 1 simplifies to $\underline{\phi}_{i} \equiv G_{i}^{a}\left(F P L_{i}\right)$. To preview, form (3) is used below in the proof of Propositions 2 and 4.

Assumption 2 (Under-reporting Earning Penalty). For each woman $i, \kappa_{i}>0$.
Assumption 3 (Ineligible Earning Levels). No woman may be on welfare assistance and truthfully report earnings above $F P L_{i}$ under regime $j$ or above $\bar{E}_{i}$ under regime a.

Assumption 4 (Utility Maximization). Under regime t, woman i makes choices by solving the optimization problem:

$$
\max _{(W, H) \in \Theta_{i} \cup(0,0), D \in\{0,1\}, E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) .
$$

Assumption 5 (Breaking Indifference). Women break indifference in favor of the same alternative irrespective of the regime.

## Intermediate Lemmas

Lemma 1 (State 2r). Given Assumptions 1, 3, and 4, no woman chooses an alternative compatible with state $2 r$.

Proof. Under regime $j$ no alternative is compatible with state $2 r$ by Assumption 3. Consider now a woman with $\bar{E}_{i} \leq F P L_{i}$ under regime $a$. By Assumption 3 she may not be on assistance and truthfully report earnings above $F P L_{i}$ (range 2). Finally, consider a woman with $\bar{E}_{i}>F P L_{i}$ under regime $a$. By Assumption 3 she may not be on assistance and truthfully report earnings above $\bar{E}_{i}$. By A. 6 in Assumption 1 she will not truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ because she can attain a higher utility level by being off assistance (Assumption 4): the extra consumption due to the transfer income does not suffice to compensate the woman for the stigma disutility she incurs when being on assistance.

Lemma 2 (Optimal Reporting). Write woman $i$ 's optimization problem (Assumption 4) as a nested maximization problem:

$$
\begin{equation*}
\max _{(W, H) \in \Theta_{i} \cup(0,0), D \in\{0,1\}}\left[\max _{E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, D, E^{r}\right), D, R\left(D, E^{r}\right)\right)\right] . \tag{4}
\end{equation*}
$$

Focus on the inner maximization problem in (4) for given sub-alternative $(W, H, D)$ with $D=1$. Let $E \equiv W H$ and $E_{i}^{r, t} \equiv E_{i}^{r, t}(W, H)$ denote woman $i$ 's utility maximizing earning report conditional on ( $W, H, 1$ ). Given Assumptions 1-5:

1. under regime $j, E_{i}^{r, t}$ entails either truthful reporting, that is, $E_{i}^{r, t}=E$, or under-reporting such that $E>E_{i}^{r, t} \in\left[0, F P L_{i}\right]$; in particular, state $1 u$ is dominated;
2. under regime a, $E_{i}^{r, t}$ entails either truthful reporting, that is, $E_{i}^{r, t}=E$, or under-reporting such that $E>E_{i}^{r, t} \in\left[0, \delta_{i}\right]$.

Proof. We prove each part of the Lemma in turn. In what follows, for convenience, we let $U_{i}^{t}$ serve as be shortcut notation for $U_{i}^{t}\left(H, C_{i}^{t}\left(E, 1, E^{r}\right), 1, R\left(1, E^{r}\right)\right)$.

1. Under regime $j$, consider three mutually exclusive pairs ( $W, H$ ) spanning the range of values for $E$ :
(a) $(W, H)$ such that $E=0$

A woman cannot over-report earnings (Assumption 4). Thus, $E_{i}^{r, j}=E$.
(b) $(W, H)$ such that $E \in\left(0, F P L_{i}\right]$

Woman $i$ 's utility while on welfare depends on reported earnings $E^{r}$ as follows (A. 4 in Assumption 1):

$$
U_{i}^{j}=\left\{\begin{array}{ll}
{[1]: U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{5}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in(0, E) \\
{[3]: U_{i}\left(H, E+\bar{G}_{i}, 1,0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By Assumption 2 and A. 1 in Assumption 1, truthful reporting yields higher utility than any under-report $E^{r} \in(0, E)$ : [3] $>[2]$ in (5). By A. 2 in Assumption 1, any underreport $E^{r} \in(0, E)$ yields at least as much utility as reporting $E^{r}=0$ : [2] $\geq[1]$ in (5). Thus, truthful reporting solves the inner maximization problem in (4) hence $E_{i}^{r, j}=E$. This shows that state $1 u$ is dominated under regime $j$ because the previous arguments holds for all $E \in\left(0, F P L_{i}\right]$ and $\left(0, F P L_{i}\right]$ corresponds to range 1 (Definition 2).
(c) $(W, H)$ such that $E>F P L_{i}$

Woman $i$ must be under-reporting (Lemma 1). Her utility while on welfare depends on reported earnings $E^{r}$ as follows (A. 4 in Assumption 1):

$$
U_{i}^{j}=\left\{\begin{array}{ll}
{[1]: U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{6}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(0, F P L_{i}\right]
\end{array} .\right.
$$

By A. 2 in Assumption 1, any report $E^{r} \in\left(0, F P L_{i}\right.$ ] yields at least as much utility as reporting $E^{r}=0:[2] \geq[1]$ in (6). If $\mathbf{A .} 2$ in Assumption 1 holds as an equality then $[2]=[1]$ in (6) and woman $i$ is indifferent among reports in [0,FPLi]. In this case, any $E^{r} \in\left[0, F P L_{i}\right]$ solves the inner maximization problem in (4) thus $E_{i}^{r, j} \in\left[0, F P L_{i}\right]$. If A. 2 in Assumption 1 holds as a strict inequality then [2] $>$ [1] in (6) and woman $i$ is indifferent among (under-) reports in ( $\left.0, F P L_{i}\right]$ and prefers them to (under-) reporting $E^{r}=0$. In this case, any report $E^{r} \in\left(0, F P L_{i}\right]$ solves the inner maximization problem in (4) thus $E_{i}^{r, j} \in\left(0, F P L_{i}\right]$.
2. Under regime $a$, consider four mutually exclusive pairs ( $W, H$ ) spanning the range of values for $E$ :
(a) $(W, H)$ such that $E=0$

A woman cannot over-report earnings (Assumption 4). Thus, $E_{i}^{r, a}=E$.
(b) $(W, H)$ such that $E \in\left(0, \delta_{i}\right]$

Woman $i$ 's utility while on welfare depends on reported earnings as follows (A. 4 in Assumption 1):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{7}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in(0, E) . \\
{[3]: U_{i}\left(H, E+\bar{G}_{i}, 1,0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By Assumption $2\left(\kappa_{i}>0\right)$, A. 1 and A. 2 in Assumption 1: $[3]>[2] \geq[1]$ in (7). Thus, truthful reporting solves the inner maximization problem in (4) hence $E_{i}^{r, a}=E$.
(c) $(W, H)$ such that $E \in\left(\delta_{i}, F P L_{i}\right]$

Woman $i$ 's utility while on welfare depends on reported earnings as follows (A. 4 in Assumption 1):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{8}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, E+G_{i}^{a}\left(E^{r}\right)-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right) \\
{[4]: U_{i}\left(H, E+G_{i}^{a}\left(E^{r}\right), 1,0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By Assumption 2, A. 1 and A. 2 in Assumption 1, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)>$ $G_{i}^{a}\left(E^{r}\right)$ for all $E^{r}$ in $\left(\delta_{i}, F P L_{i}\right]:[1] \leq[2]$ and $[3]<[2]$ in (8). Thus, only truthful reports or under-reports in $\left[0, \delta_{i}\right]$ may solve the inner maximization problem in (4). Specifically, if A. 2 in Assumption 1 holds as an equality then $[1]=[2]$ in (8) and woman $i$ is indifferent among (under-) reports in $\left[0, \delta_{i}\right]$. In this case $E_{i}^{r, a}=E$ or $E_{i}^{r, a} \in\left[0, \delta_{i}\right]$ depending on whether $[4] \geq[2]$ or $[4] \leq[2]$. If A. 2 in Assumption 1 holds as a strict inequality then $[1]<[2]$ in (8) and woman $i$ is indifferent among (under-) reports in $\left(0, \delta_{i}\right]$. In this case $E_{i}^{r, a}=E$ or $E_{i}^{r, a} \in\left(0, \delta_{i}\right]$ depending on whether [4] $\geq[2]$ or $[4] \leq[2]$.
(d) $(W, H)$ such that $E>F P L_{i}$

Woman $i$ must be under-reporting (Lemma 1). Her utility while on welfare depends on reported earnings as follows (A. 4 in Assumption 1):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{9}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, E+G_{i}^{a}\left(E^{r}\right)-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(\delta_{i}, F P L_{i}\right]
\end{array} .\right.
$$

By A. 1 and A. 2 in Assumption 1, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)>G_{i}^{a}\left(E^{r}\right)$ for all $E^{r}$ in $\left(\delta_{i}, F P L_{i}\right]:[3]<[2]$ and $[1] \leq[2]$ in (9). Thus, only under-reports in $\left[0, \delta_{i}\right]$ may solve the inner maximization problem in (4). Specifically, if A. 2 in Assumption 1 holds as an equality then $[1]=[2]$ in (9)) and woman $i$ is indifferent among (under-) reports in $\left[0, \delta_{i}\right]$. In this case $E_{i}^{r, a} \in\left[0, \delta_{i}\right]$. If $\mathbf{A . 2}$ in Assumption 1 holds as a strict inequality then $[1]<[2]$ in (9)) and woman $i$ is indifferent among (under-) reports in ( $\left.0, \delta_{i}\right]$ and prefers them to reporting $E^{r}=0$. In this case $E_{i}^{r, a} \in\left(0, \delta_{i}\right]$.

Corollary 1 (Optimal Reporting and Policy Invariance). Given Assumptions 1-5, the utility associated with any alternative compatible with states $1 u$ and $2 u$ and entailing optimal reporting is regime invariant.

Proof. We examine each state in turn.

1. State $1 u$
(a) Consider a woman $i$ and any sub-alternative ( $W, H, 1$ ) such that, letting $E \equiv W H, E$ is in range 1 and $E_{i}^{r, j}(W, H)<E$. Thus alternative $\left(W, H, 1, E_{i}^{r, j}(W, H)\right)$ is compatible with state $1 u$ and entails optimal reporting under regime $j$. Let $C_{i}^{j} \equiv C_{i}^{j}\left(E, 1, E_{i}^{r, j}(W, H)\right)$ and $R_{i}^{j} \equiv R\left(1, E_{i}^{r, j}(W, H)\right)$. We next show that $U_{i}^{j}\left(H, C_{i}^{j}, 1, R_{i}^{j}\right)=U_{i}\left(H, C_{i}^{j}, 1, R_{i}^{j}\right)$. By Lemma $2, E_{i}^{r, j}(W, H) \in\left(0, F P L_{i}\right]$ or $E_{i}^{r, j}(W, H) \in\left[0, F P L_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$
which equals $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1. In the second case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ which also equals $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1 and because she is indifferent between (under-) reports in ( $\left.0, F P L_{i}\right]$ and reporting zero earnings, that is,
$U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)=U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$.
(b) Consider any sub-alternative $(W, H, 1)$ such that, letting $E \equiv W H, E$ is in range 1 and $E_{i}^{r, a}(W, H)<E$. Thus alternative $\left(W, H, 1, E_{i}^{r, a}(W, H)\right)$ is compatible with state $1 u$ and entails optimal reporting under regime $a$. Let $C_{i}^{a} \equiv C_{i}^{a}\left(E, 1, E_{i}^{r, a}(W, H)\right)$ and $R_{i}^{a} \equiv$ $R\left(1, E_{i}^{r, a}(W, H)\right)$. We next show that $U_{i}^{a}\left(H, C_{i}^{a}, 1, R_{i}^{a}\right)=U_{i}\left(H, C_{i}^{a}, 1, R_{i}^{a}\right)$. By Lemma $2, E_{i}^{r, a}(W, H) \in\left(0, \delta_{i}\right]$ or $E_{i}^{r, a}(W, H) \in\left[0, \delta_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ which equals $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1. In the second case, the utility woman $i$ enjoys is also $U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)=U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1 and because she is indifferent between (under-) reports in ( $0, \delta_{i}$ ] and reporting zero earnings, that is, $U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)=U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$.
(c) In 1.(a) and 1.(b) we have shown that any alternative compatible with state $1 u$ and entailing optimal reporting yields regime-invariant consumption $E+\bar{G}_{i}-\kappa_{i}$ and regimeinvariant utility level $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$.

## 2. State $2 u$

The proof corresponding to state $2 u$ is the same as that for state $1 u$ once we consider a sub-alternative $(W, H, 1)$ such that, letting $E \equiv W H, E$ is in range 2 (Lemma 2 ).

Remark 4 (Optimal under-Reporting and Alternatives Considered). In what follows, it is without loss of generality that we only focus on alternatives entailing optimal (under-) reporting among those compatible with states $1 u$ and $2 u$. No woman would select an alternative compatible with states $1 u$ or $2 u$ not entailing optimal (under-) reporting (Assumption 4). Additionally, it is without loss of generality that we disregard alternatives compatible with state $1 u$ under regime $j$. No woman would select an alternative compatible with state $1 u$ under regime $j$ because it is dominated (Lemma 2, part 1.(b)).

Lemma 3 (Policy Impact on Attractiveness of States). Given Assumptions 1-5:

1. the states in $\mathcal{C}_{+}$are no worse under regime $j$ than under regime $a$;
2. the states in $\mathcal{C}_{-}$are no better under regime $j$ than under regime $a$;
3. the states in $\mathcal{C}_{0}$ are equally attractive under regime $j$ and regime $a$.

Proof. We prove each statement in turn.

1. The only state in $\mathcal{C}_{+}$is $1 r$. All alternatives compatible with state $1 r$ entail $E$ in range $1, D=1$, and $E^{r}=E$. Thus, the utility function associated with each of these alternatives is invariant to the treatment (A. 4 in Assumption 1). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is not lower under regime $j$ than under regime $a$. Because $G_{i}^{a}(E) \leq \bar{G}_{i}$ for all $E$ in range $1, C_{i}^{j}(E, 1, E)=E+\bar{G}_{i} \geq E+G_{i}^{a}(E)=C_{i}^{a}(E, 1, E)$, which verifies the desired inequality.
2. The only state in $\mathcal{C}_{-}$is $0 r$. All the alternatives compatible with state $0 r$ entail $E=H=0$, $D=1$, and $E^{r}=0$. Thus, it suffices to show that $U_{i}^{a}\left(0, \bar{G}_{i}, 1,1\right) \geq U_{i}^{j}\left(0, \bar{G}_{i}, 1,1\right)$. This inequality holds by A. $\mathbf{3}$ in Assumption 1.
3. All alternatives compatible with states $\{0 n, 1 n, 2 n\}$ entail $D=0$. Thus, the utility associated with each of these alternatives is invariant to the policy regime (A. 5 in Assumption 1). Accordingly, it suffices to show that the consumption associated with any of these alternatives is the same under regime $j$ than under regime $a$. Because off assistance consumption is either zero, when $s_{i}=0 n$, or $E$, when $s_{i} \in\{1 n, 2 n\}$ consumption is unaffected by the regime. Finally consider states $\{1 u, 2 u\}$ entailing $0 \leq E^{r}<E$ and $D=1$. Given optimal reporting, the utility function associated with each of these alternatives is invariant to the policy regime (Corollary 1). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is the same under regime $j$ and under regime $a$. If $s_{i} \in\{1 u, 2 u\}$ consumption is $E+\bar{G}_{i}-\kappa_{i}$ under both regimes (see Lemma 2). Thus consumption is also policy invariant.

Lemma 4 (Revealed Preferences). Consider any pair of states $\left(s^{a}, s^{j}\right)$ obeying: I) $s^{a} \neq s^{j}$; II) state $s^{a}$ is no worse under regime $j$ than under regime $a$; III) state $s^{j}$ is no better under regime $j$ than under regime $a$. Given Assumptions 1 and 5, no woman pairs states $s^{a}$ and $s^{j}$.

Proof. The proof is by contradiction. Consider any pair of states $\left(s^{a}, s^{j}\right)$ satisfying properties I)III). Suppose that woman $i$ chooses alternative ( $W, H, D, E^{r}$ ) under regime $a$ compatible with state $s^{a}$; and alternative $\left(H^{\prime}, W^{\prime}, D^{\prime}, E^{r \prime}\right)$ under regime $j$ compatible with state $s^{j}$. Let with $E \equiv W H$, $E^{\prime} \equiv W^{\prime} H^{\prime}, C_{i}^{t}=C_{i}^{t}\left(E, D, E^{r}\right)$ and $C_{i}^{t \prime}=C_{i}^{t}\left(E^{\prime}, D^{\prime}, E^{r \prime}\right)$ all $t \in\{a, j\}, R=R\left(D, E^{r}\right)$, and $R^{\prime}=R\left(D^{\prime}, E^{r \prime}\right)$. The woman's choice under regime $a$ reveals that

$$
U_{i}^{a}\left(H, C_{i}^{a}, D, R\right) \geq U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, D^{\prime}, R^{\prime}\right) .
$$

By property II)

$$
U_{i}^{j}\left(H, C_{i}^{j}, D, R\right) \geq U_{i}^{a}\left(H, C_{i}^{a}, D, R\right) .
$$

By property III)

$$
U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, D^{\prime}, R^{\prime}\right) \geq U_{i}^{j}\left(H^{\prime}, C_{i}^{j \prime}, D^{\prime}, R^{\prime}\right)
$$

Combining the above three inequalities we have

$$
\begin{equation*}
U_{i}^{j}\left(H, C_{i}^{j}, D, R\right) \geq U_{i}^{a}\left(H, C_{i}^{a}, D, R\right) \geq U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, D^{\prime}, R^{\prime}\right) \geq U_{i}^{j}\left(H^{\prime}, C_{i}^{j \prime}, D^{\prime}, R^{\prime}\right) . \tag{10}
\end{equation*}
$$

If any of the inequalities is strict, optimality of $\left(H^{\prime}, C_{i}^{j \prime}, D^{\prime}, R^{\prime}\right)$ under regime $j$ is contradicted (Assumption 4). If no inequality is strict, we have to consider $9=3^{2}$ possible situations based on the possible values of $\left(D, R, D^{\prime}, R^{\prime}\right)$. Each of these situations leads to a contradiction based on a woman breaking indifference between two alternatives in favor of the same alternative irrespective of the policy regime (Assumption 5) and Property I. Specifically, in each of the following cases expression (10) simplifies to:

1. $(D, R)=(0,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(0,0)$ :

$$
U_{i}(H, C, 0,0)=U_{i}(H, C, 0,0)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. Woman $i$ is indifferent between ( $H, C, 0,0$ ) and ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C, 0,0$ ), this contradicts resolving indifference in favor of ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $j$.
2. $(D, R)=(0,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,0)$ :

$$
U_{i}(H, C, 0,0)=U_{i}(H, C, 0,0)=U_{i}\left(H^{\prime}, C_{i}^{a \prime}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime}, 1,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime hence $C_{i}^{a}=C_{i}^{j}=C$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between ( $H, C, 0,0$ ) and ( $H^{\prime}, C_{i}^{\prime}, 1,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C, 0,0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,0\right)$ under regime $j$.
3. $(D, R)=(0,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,1)$ :

$$
U_{i}(H, C, 0,0)=U_{i}(H, C, 0,0)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{a}=C_{i}^{j}=C$, and that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, 0,0)$ and ( $H^{\prime}, C_{i}^{\prime}, 1,1$ ) under regime $a$ and resolves indifference in favor of $(H, C, 0,0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $j$.
4. $(D, R)=(1,1)$ and $\left(D^{\prime}, R^{\prime}\right)=(0,0)$ :

$$
U_{i}^{j}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H, C_{i}, 1,1\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C^{\prime}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between ( $H, C_{i}, 1,1$ ) and ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,1\right)$, this contradicts resolving indifference in favor of ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $j$.
5. $(D, R)=(1,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(0,0)$ :

$$
U_{i}\left(H, C_{i}^{j}, 1,0\right)=U_{i}\left(H, C_{i}^{a}, 1,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C^{\prime}$. The fist equality implies $C_{i}^{j}=C_{i}^{a}=C_{i}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between ( $H, C_{i}, 1,0$ ) and ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C_{i}, 1,0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, 0,0\right)$ under regime $j$.
6. $(D, R)=(1,1)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,0)$ :

$$
U_{i}^{j}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H, C_{i}, 1,1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime}, 1,0\right),
$$

where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between ( $H, C_{i}, 1,1$ ) and ( $H^{\prime}, C_{i}^{\prime}, 1,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C_{i}, 1,1$ ), this contradicts resolving indifference in favor of ( $H^{\prime}, C_{i}^{\prime}, 1,0$ ) under regime $j$.
7. $(D, R)=(1,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,0)$ :

$$
U_{i}\left(H, C_{i}^{j}, 1,0\right)=U_{i}\left(H, C_{i}^{a}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime}, 1,0\right)
$$

The fist equality implies $C_{i}^{j}=C_{i}^{a}=C_{i}$ and the last equality implies $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between $\left(H, C_{i}, 1,0\right)$ and $\left(H^{\prime}, C_{i}^{\prime}, 1,0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,0\right)$, this contradicts her resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,0\right)$ under regime $j$.
8. $(D, R)=(1,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,1)$ :

$$
U_{i}\left(H, C_{i}^{j}, 1,0\right)=U_{i}\left(H, C_{i}^{a}, 1,0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)
$$

where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. The first equality implies $C_{i}^{j}=C_{i}^{a}=C_{i}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between $\left(H, C_{i}, 1,0\right)$ and $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $j$.
9. $(D, R)=(1,1)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,1)$ :

$$
U_{i}^{j}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)
$$

where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j}=C_{i}^{a}=C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=$ $C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i}, 1,1\right)$ and ( $H^{\prime}, C_{i}^{\prime}, 1,1$ ) under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $j$.

## Main Propositions

Proposition 1 (Restricted Pairings). Given Assumptions 1-5, the pairings of states corresponding to the"-" entries in Table 4 are disallowed and the pairings of states $(1 r, 1 r)$ and $(1 u, 1 r)$ must occur.

Proof. We begin with the pairings that are disallowed. State $1 u$ is dominated under regime $j$ (Lemma 2). Therefore no woman will pair state $s^{a}$ with state $s^{j}=1 u$ for any $s^{a} \in \mathcal{S}$. Next, by Lemmas 4 and 3 , no pairing of state $s^{a}$ with state $s^{j}$ can occur for all $\left(s^{a}, s^{j}\right)$ in the collection

$$
\begin{equation*}
\left\{\left(s^{a}, s^{j}\right): s^{a} \in \mathcal{C}_{0} \cup \mathcal{C}_{+}, s^{j} \in \mathcal{C}_{0} \cup \mathcal{C}_{-}, s^{a} \neq s^{j}\right\} \tag{11}
\end{equation*}
$$

Thus, it suffices to show that the properties I)-III) of Lemma 4 are met. Property I) holds trivially and properties II) and III) hold by Lemma 3. Therefore no woman will select any of the pairings in (11). We next turn to the responses that must occur. By Lemma 1, the allowable states are given by $\mathcal{S}=\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}$. We just argued that the pairings $\left\{\left(1 r, s^{j}\right): s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 u, 2 u\}\right\}$ are disallowed, therefore the pairing $(1 r, 1 r)$ must occur. Similarly, we just argued that the pairings $\left\{\left(1 u, s^{j}\right): s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 u, 2 u\}\right\}$ are disallowed, therefore the pairing $(1 u, 1 r)$ must occur.

Corollary 2 (Additional Restricted Pairings under Utility Specification (2)). Given Assumptions 1-5 and subject to specification (2) of the utility function, the pairing of states ( $0 r, 1 n$ ) is disallowed.

Proof. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$. The proof is by contradiction. Suppose there is a woman $i$ who selects an alternative compatible with state $0 r$ under regime $a$ and selects an alternative compatible with state $1 n$ under regime $j$ entailing earnings $E^{k} \equiv W^{k} H^{k}$. By Assumption 4, her choice under regime a reveals that

$$
\left[0 r \succsim^{a} 0 n\right]: U_{i}^{a}\left(0, \bar{G}_{i}-\phi_{i}-\eta_{i}^{a}\right) \geq U_{i}(0,0)
$$

which implies $\bar{G}_{i} \geq \phi_{i}+\eta_{i}^{a}$. Her choice under regime $j$ reveals that

$$
\left[1 n \succsim^{j} 1 r\right]: U_{i}\left(H^{k}, E^{k}-\mu_{i}\right) \geq U_{i}\left(H^{k}, E^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}\right)
$$

which implies $\bar{G}_{i} \leq \phi_{i}$. Thus, optimality implies $\phi_{i} \geq \bar{G}_{i} \geq \phi_{i}+\eta_{i}^{a}$. If the inequality in A. 2 of Assumption 1 holds as a strict inequality $\eta_{i}^{a}>0$ and a contradiction ensues. If the inequality in A. 2 of Assumption 1 holds as an equality $\eta_{i}^{a}=0$. Thus, $\phi_{i}=\bar{G}_{i}$ and woman $i$ must be indifferent between the alternative compatible with state $0 r$ and the alternative compatible with state $0 n$ under regime $a$ which means that $U_{i}(0,0) \geq U_{i}\left(H^{l}, E^{l}\right)$ for any offer $\left(w^{l}, H^{l}\right)$ entailing earnings $E^{l} \equiv W^{l} H^{l}$ in range 1 , including $E^{k}$. The choice of the alternative compatible with state $1 n$ under regime $j$ reveals that $U_{i}\left(H^{k}, E^{k}\right) \geq U_{i}(0,0)$. Thus, $U_{i}(0,0) \geq U_{i}\left(H^{l}, E^{l}\right) \geq U_{i}(0,0)$. If either inequality is strict a contradiction ensues. Otherwise $U_{i}(0,0)=U_{i}\left(H^{l}, E^{l}\right)=U_{i}(0,0)$ and the woman must be indifferent under regime $a$ between the alternative compatible with state $0 n$ and the alternative entailing earnings $E^{k}$ off assistance. If however she does not choose earnings $E^{k}$ off assistance under regime $a$ then she breaks indifference in the same way under $j$ (Assumption 5), which contradicts her choosing earnings $E^{k}$ off assistance under regime $j$.

Proposition 2 (Unrestricted Pairings). Given Assumptions 1-5, the pairings of states corresponding to the non "-" entries in Table 4 are allowed.

Proof. State pairings ( $1 r, 1 r$ ) and ( $1 u, 1 r$ ) must occur by Proposition 1. Table 4's remaining allowed state pairings can be conveniently organized in two collections:

$$
\begin{align*}
& \left\{\left(s^{a}, 1 r\right): s^{a} \in\{0 n, 1 n, 2 n, 2 u\}\right\},  \tag{12}\\
& \left\{\left(0 r, s_{j}\right): s^{j} \in\{0 n, 1 n, 2 n, 1 r, 2 u\}\right\} . \tag{13}
\end{align*}
$$

We start by considering the collection of pairs in (12). The common feature of the states in $\{0 n, 1 n, 2 n, 2 u\}$ is that they are equally attractive under regimes $a$ and $j$ (Lemma 3). Instead, state $1 r$ is no worse under regime $j$ than under regime $a$ (Lemma 3). In light of Proposition (1), to prove that the pairs in collection (12) are allowed it suffices to provide examples where two women occupy the same state $s^{a} \in\{0 n, 1 n, 2 n, 2 u\}$ under regime $a$, but the first woman occupies state $s^{j}=s^{a}$ under regime $j$ and the second woman occupies state $s^{j}=1 r$ under regime $j$. This also proves that no pairing in collection (12) is constrained to occur. We then turn to the collection of state pairs in (13). The common feature of the states in $\{0 n, 1 n, 2 n, 1 r, 2 u\}$ is that they are no worse under regime $j$ than under regime $a$ (Lemma 3). Instead, state $0 r$ is no better under regime $j$ than under regime $a$ (Lemma 3). To prove that the pairs in collection (13) are allowed it suffices to provide the example of a woman who occupies state $0 r$ under regime $a$ and state $s^{j} \in\{0 n, 1 n, 2 n, 1 r, 2 u\}$ under regime $j$. This also proves that no pairing in collection (13) is constrained to occur.

When providing examples we consider the specification of the utility function given in (3). Finally, we assume that woman $i$ receives either one or two job offers, that is, either $K_{i}=1$ or $K_{i}=2$. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$.

1. Pairings $(0 n, 1 r)$ and $(0 n, 0 n)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=1$. Assume that each woman's job offer entails earnings in range 1. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 1 . Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=0$, and $\alpha_{i} \geq W_{i}^{k}$ and

$$
\bar{G}_{i}-\phi_{i} \leq 0
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=0$, and $\alpha_{i} \geq W_{i}^{k}$ and

$$
H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)<\bar{G}_{i}-\phi_{i} \leq \min \left\{\eta_{i}^{a}, H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)+\kappa_{i}, H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)+\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)\right\}
$$

Both women chose an alternative compatible with state $0 n$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $0 n$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $0 n$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 1 r, 1 u, 1 n\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& {\left[0 n \succsim^{a} 0 r\right] \quad: \quad 0 \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{a}}  \tag{14}\\
& {\left[0 n \succsim^{a} 1 r\right] \quad: \quad 0 \geq E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k}}  \tag{15}\\
& {\left[0 n \succsim^{a} 1 u\right]} \tag{16}
\end{align*}: \quad 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k}, ~\left(0 n \succsim^{a} 1 n\right] \quad: \quad 0 \geq E_{i}^{k}-\alpha_{i} H_{i}^{k} .
$$

It is easy to verify that descriptions (1a) and (1b) are compatible with optimality under regime $a$, that is, with (14)-(17). Both women prefer state $0 n$ under regime $j$ to the available alternatives compatible with states $\{0 r, 1 n, 1 u\}$ by Proposition 1 . Woman $i=i^{\prime}$ also prefers state $0 n$ to the available alternatives compatible with state $1 r$ under regime $j$ because by description (1a) we have $\bar{G}_{i}-\phi_{i} \leq 0$ and $\alpha_{i} \geq W_{i}^{k}$ which imply (18):

$$
\begin{equation*}
\left[0 n \succsim^{j} 1 r\right]: 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{k} \tag{18}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $0 n$. Instead, woman $i=i^{\prime \prime}$ prefers an alternative available and compatible with state $1 r$ under regime $j$ to state $0 n$ because by description (1b) we have $H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)<\bar{G}_{i}-\phi_{i}$ which imply (19):

$$
\begin{equation*}
\left[1 r \succsim^{j} 0 n\right]: E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{k}>0 \tag{19}
\end{equation*}
$$

2. Pairings $(1 n, 1 r)$ and $(1 n, 1 n)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=1$. Assume that each woman's job offer entails earnings in range 1. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 1 . Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0$ and

$$
\bar{G}_{i}-\phi_{i} \leq 0
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0$ and

$$
0<\bar{G}_{i}-\phi_{i} \leq \min \left\{\kappa_{i}, E_{i}^{k}, \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)\right\} .
$$

Both women chose an alternative compatible with state $1 n$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $1 n$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $1 n$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 0 r, 1 r, 1 u, 1 n\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& \text { [1n } \left.\succsim^{a} 0 n\right]: E_{i}^{k} \geq 0,  \tag{20}\\
& {\left[1 n \succsim^{a} 0 r\right]: E_{i}^{k} \geq \bar{G}_{i}-\phi_{i},}  \tag{21}\\
& {\left[1 n \succsim^{a} 1 n\right]: E_{i}^{k} \geq E_{i}^{l} \forall E_{i}^{l} \text {, }}  \tag{22}\\
& {\left[1 n \succsim^{a} 1 r\right]: E_{i}^{k} \geq E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i},}  \tag{23}\\
& {\left[1 n \succsim^{a} 1 u\right]: \quad E_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} .} \tag{24}
\end{align*}
$$

It is easy to verify that descriptions (2a) and (2b) are compatible with optimality under regime $a$, that is, with (20)-(24). Both women prefer state $1 n$ under regime $j$ to the available alternatives compatible with states $\{0 r, 0 n, 1 u\}$, by Proposition 1. Woman $i=i^{\prime}$ also prefers state $1 n$ to the available alternatives compatible with state $1 r$ under regime $j$ because by description (2a) we have $\bar{G}_{i}-\phi_{i} \leq 0$ which implies (25):

$$
\begin{equation*}
\left[1 n \succsim^{j} 1 r\right]: E_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i} . \tag{25}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $1 n$. Instead, woman $i=i^{\prime \prime}$ prefers earning $E_{i}^{k}$ on assistance to earning the same amount off assistance under regime $j$ because by description (2b) we have $\bar{G}_{i}-\phi_{i}>0$ which implies (26):

$$
\begin{equation*}
\left[1 r \succsim^{j} 1 n\right]: E_{i}^{k}+\bar{G}_{i}-\phi_{i}>E_{i}^{k} . \tag{26}
\end{equation*}
$$

Thus, the available alternative entailing earnings $E_{i}^{k}$ on assistance is preferred under regime $j$ to the available alternatives compatible with all states but $1 r$.

## 3. Pairings $(2 n, 1 r)$ and $(2 n, 2 n)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=2$. Assume that each woman's two job offers entail earnings in range 1 and in range 2 respectively. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}, E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l}$ is in range 1 and $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 2. Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, W_{i}^{k} \geq W_{i}^{l}$ and

$$
\bar{G}_{i}-\phi_{i} \leq 0,
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, W_{i}^{k} \geq W_{i}^{l}$ and

$$
E_{i}^{k}-E_{i}^{l}<\bar{G}_{i}-\phi_{i} \leq \min \left\{\kappa_{i}, E_{i}^{k}, \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{l}\right)+E_{i}^{k}-E_{i}^{l}\right\} .
$$

Both women chose an alternative compatible with state $2 n$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $2 n$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $2 n$ under regime $a$ reveals (Assumption 4) that
this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 1 u, 2 u\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& \text { [ } \left.2 n \succsim^{a} 0 n\right]: E_{i}^{k} \geq 0,  \tag{27}\\
& {\left[2 n \succsim^{a} 0 r\right]: \quad E_{i}^{k} \geq \bar{G}_{i}-\phi_{i},}  \tag{28}\\
& {\left[2 n \succsim^{a} 1 n\right]: \quad E_{i}^{k} \geq E_{i}^{l} \text {, }}  \tag{29}\\
& {\left[2 n \succsim^{a} 1 r\right]: \quad E_{i}^{k} \geq E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i},}  \tag{30}\\
& {\left[2 n \succsim^{a} 1 u\right]: E_{i}^{k} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\kappa_{i},}  \tag{31}\\
& {\left[2 n \succsim^{a} 2 u\right]: \quad E_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} .} \tag{32}
\end{align*}
$$

It is easy to verify that descriptions (3a) and (3b) are compatible with optimality under regime $a$, that is, with (27)-(32). Both women prefer state $2 n$ under regime $j$ to the available alternatives compatible with states $\{0 r, 2 u, 0 n, 1 n, 1 u\}$, by Proposition 1. Woman $i=i^{\prime}$ also prefers state $2 n$ to the available alternatives compatible with state $1 r$ under $j$ because $E_{i}^{k} \geq E_{i}^{l}$ by (29) and by description (3a) we have $\bar{G}_{i}-\phi_{i} \leq 0$ which implies (33):

$$
\begin{equation*}
\left[2 n \succsim^{j} 1 r\right]: E_{i}^{k} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i} . \tag{33}
\end{equation*}
$$

By Assumption 5 she breaks indifference in favor of state $1 n$. Instead, woman $i=i^{\prime \prime}$ prefers earning $E_{i}^{l}$ on assistance to earning $E_{i}^{k}$ off assistance under regime $j$ because by description (3b) we have $\bar{G}_{i}-\phi_{i}>E_{i}^{k}-E_{i}^{l}$ which implies (34):

$$
\begin{equation*}
\left[1 r \succsim^{j} 2 n\right]: E_{i}^{l}+\bar{G}_{i}-\phi_{i}>E_{i}^{k} . \tag{34}
\end{equation*}
$$

## 4. Pairings $(2 u, 1 r)$ and $(2 u, 2 u)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=2$. Assume that each woman's two job offers entail earnings in range 1 and in range 2 respectively. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}, E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l}$ is in range 1 and $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 2. Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, \phi_{i}>\underline{\phi}_{i}, W_{i}^{k} \geq W_{i}^{l}$ and

$$
\kappa_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, E_{i}^{k}-E_{i}^{l}\right\}
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, \phi_{i}>\underline{\phi}_{i}, W_{i}^{k} \geq W_{i}^{l}$ and

$$
E_{i}^{k}-E_{i}^{l}<\kappa_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, E_{i}^{k}-E_{i}^{l}+\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{l}\right)\right\} .
$$

Both women chose an alternative compatible with state $2 u$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $2 u$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $2 u$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 1 u, 2 n\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& {\left[2 u \succsim^{a} 0 n\right] }:  \tag{35}\\
& {\left[2 u \succsim_{i}^{k} 0 r\right] }:  \tag{36}\\
& E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq 0,  \tag{37}\\
& {\left[2 u \succsim^{a} 1 n\right]: } E_{i}^{k}+\bar{G}_{i}-\kappa_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l},  \tag{38}\\
& {\left[2 u \succsim_{i} 1 r\right] }:  \tag{39}\\
& {\left[E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i},\right.}  \tag{40}\\
& {\left[2 u \succsim^{a} 1 u\right]: } E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\kappa_{i}, \\
& {\left[2 u \succsim^{a} 2 n\right]: } E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{k} .
\end{align*}
$$

It is easy to verify that descriptions (4a) and (4b) are compatible with optimality under regime $a$, that is, with (35)-(40). Both women prefer state $2 u$ under regime $j$ to the available alternatives compatible with states $\{0 r, 0 n, 1 n, 2 n, 1 u\}$, by Proposition 1 . Woman $i=i^{\prime}$ also prefers state $2 u$ to the available alternative compatible with state $1 r$ under regime $j$ because by description (4a) we have $\kappa_{i} \leq E_{i}^{k}-E_{i}^{l}$ which implies (41):

$$
\begin{equation*}
\left[2 u \succsim^{j} 1 r\right]: E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i} \tag{41}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $2 u$. Instead, woman $i=i^{\prime \prime}$ prefers earning and truthfully reporting $E_{i}^{l}$ on assistance to under-reporting earnings $E_{i}^{k}$ on assistance under regime $j$ because $\bar{G}_{i} \geq G_{i}^{a}\left(E_{i}^{l}\right)$ and by description (4b) we have $\kappa_{i}>E_{i}^{k}-E_{i}^{l}$ which implies (42):

$$
\begin{equation*}
\left[1 r \succsim^{j} 2 u\right]: E_{i}^{l}+\bar{G}_{i}-\phi_{i}>E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} . \tag{42}
\end{equation*}
$$

5. Pairings $\left(0 r, s^{j}\right)$ with $s^{j} \in\{0 r, 0 n, 1 n, 2 n, 1 r, 2 u\}$ are allowed.

Consider five women $\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}$ with preferences represented by $(3)$ with $v(x)=x$. Let $K_{i}=2$. Assume that each woman's two job offers entail earnings in range 1 and in range 2 respectively. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}, E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l}$ is in range 1 and $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 2. Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=0, \eta_{i}^{a}=\eta_{i}^{j}=\eta_{i}, \phi_{i}>\underline{\phi}_{i}, W_{i}^{l}=W_{i}^{k}=W_{i} \leq \alpha_{i}$, and

$$
0 \leq \eta_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, H_{i}^{l}\left(\alpha_{i}-W_{i}\right)\right\}
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=0, \eta_{i}^{a}=\eta_{i}^{j}=\eta_{i}, \phi_{i}>\underline{\phi}_{i}, W_{i}^{l}=W_{i}^{k}=W_{i} \leq \alpha_{i}$, and

$$
H_{i}^{l}\left(\alpha_{i}-W_{i}^{l}\right)<\eta_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, H_{i}^{l}\left(\alpha_{i}-W_{i}\right)+\kappa_{i}, H_{i}^{l}\left(\alpha_{i}-W_{i}\right)+\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{l}\right)\right\}
$$

(c) woman $i=i^{\prime \prime \prime}$ be such that $\mu_{i}=0, \eta_{i}^{a}<\eta_{i}^{j}, W_{i}^{l}=W_{i}^{k}=W_{i}<\alpha_{i}$ and

$$
\eta_{i}^{a} \leq \bar{G}_{i}-\phi_{i}<\min \left\{H_{i}^{l}\left(\alpha_{i}-W_{i}\right), \eta_{i}^{j}\right\}
$$

(d) woman $i=i^{I V}$ be such that $\eta_{i}^{a} \leq \mu_{i} \leq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \phi_{i}>\underline{\phi}_{i}, \eta_{i}^{a}<\eta_{i}^{j}, W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ and

$$
H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{a} \leq \bar{G}_{i}-\phi_{i}<\min \left\{H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{j}, H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \kappa_{i}\right\}
$$

(e) woman $i=i^{V}$ be such that $\eta_{i}^{a} \leq \mu_{i} \leq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \phi_{i}>\underline{\phi}_{i}, \eta_{i}^{a}<\eta_{i}^{j}, W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ and

$$
H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{a} \leq \kappa_{i} \leq \min \left\{H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{j}, H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \bar{G}_{i}-\phi_{i},\right\}
$$

All these women chose an alternative compatible with state $0 r$ under regime $a$. We now show that, under regime $j$, woman $i^{\prime}$ selects an alternative compatible with state $0 r$, woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$, woman $i^{\prime \prime \prime}$ selects an alternative compatible with state $0 n$, woman $i^{I V}$ selects an alternative compatible with state $2 n$, and woman $i^{V}$ selects an alternative compatible with state $2 u$. For all women, the choice of the alternative compatible with state $0 r$ under regime $a$ reveals (Assumption 4) that this alternative yields
as much utility as the available alternatives compatible with states $\{0 n, 1 n, 2 n, 1 r, 1 u, 2 u\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}$ :

$$
\begin{align*}
& {\left[0 r \succsim^{a} 0 n\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq 0,}  \tag{43}\\
& \text { [ } \left.0 r \succsim^{a} 1 n\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{l}-\mu_{i}-\alpha_{i} H_{i}^{l},  \tag{44}\\
& {\left[0 r \succsim^{a} 2 n\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k},}  \tag{45}\\
& {\left[0 r \succsim^{a} 1 r\right]: \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{l}-\mu_{i}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i}-\alpha_{i} H_{i}^{l} \text {, }}  \tag{46}\\
& {\left[0 r \succsim^{a} 1 u\right]: \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{l} \text {, }}  \tag{47}\\
& {\left[0 r \succsim^{a} 2 u\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .} \tag{48}
\end{align*}
$$

It is easy to verify that descriptions (5a)-(5e) are compatible with optimality under regime $a$, that is, with (43)-(48). Because $\eta_{i}^{a}=\eta_{i}^{j}$ for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$, state $0 r$ has the same utility value under both regimes hence both women prefer state $0 r$ under regime $j$ to the available alternatives compatible with states $\{0 n, 1 n, 2 n, 1 u, 2 u\}$, by Proposition 1. Woman $i=i^{\prime}$ also prefers state $0 r$ to the available alternative compatible with state $1 r$ under regime $j$ because by description (5a) we have $\eta_{i} \leq H_{i}^{l}\left(\alpha_{i}-W_{i}\right)$ and $\mu_{i}=0$ which imply (49):

$$
\begin{equation*}
\left[0 r \succsim^{j} 1 r\right]: \bar{G}_{i}-\phi_{i}-\eta_{i} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} . \tag{49}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $0 r$. Instead, woman $i=i^{\prime \prime}$ prefers earning and truthfully reporting $E_{i}^{l}$ on assistance to not working on assistance under regime $j$ because by description (5b) we have $\eta_{i}>H_{i}^{l}\left(\alpha_{i}-W_{i}\right)$ and $\mu_{i}=0$ which imply (50):

$$
\begin{equation*}
\left[1 r \succsim^{j} 0 r\right]: E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha H_{i}^{l}>\bar{G}_{i}-\phi_{i}-\eta_{i} . \tag{50}
\end{equation*}
$$

Consider now women $\left\{i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}$. None selects an alternative compatible with state $1 u$ under regime $j$ by Proposition 1. Woman $i=i^{\prime \prime \prime}$ prefers not working off assistance (state $0 n$ ) to the available alternatives compatible with states $\{0 r, 1 n, 1 r, 2 n, 2 u\}$ under regime $j$ because, by description (5c), we have $\mu_{i}=0$ and, respectively, $\bar{G}_{i}-\phi_{i}<\eta_{i}^{j}$ which implies (51); $H_{i}^{l}\left(\alpha_{i}-W_{i}\right) \geq 0$ which implies (52); $H_{i}^{k}\left(\alpha_{i}-W_{i}\right) \geq 0$ which implies (53); $H_{i}^{l}\left(\alpha_{i}-W_{i}\right) \geq$ $\bar{G}_{i}-\phi_{i}$ which implies (54); and $H_{i}^{k}\left(\alpha_{i}-W_{i}\right)+\kappa_{i} \geq \bar{G}_{i}-\phi_{i}$ which implies (55):

$$
\begin{align*}
{\left[0 n \succsim^{j} 0 r\right] } & : 0>\bar{G}_{i}-\phi_{i}-\eta_{i}^{j},  \tag{51}\\
{\left[0 n \succsim^{j} 1 n\right] } & : 0 \geq E_{i}^{l}-\alpha_{i} H_{i}^{l},  \tag{52}\\
{\left[0 n \succsim^{j} 2 n\right] } & : 0 \geq E_{i}^{k}+-\alpha H_{i i}^{k},  \tag{53}\\
{\left[0 n \succsim^{j} 1 r\right] } & : 0 \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l},  \tag{54}\\
{\left[0 n \succsim^{j} 2 u\right] } & : 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} . \tag{55}
\end{align*}
$$

Woman $i=i^{I V}$ prefers earning $E_{i}^{k}$ off assistance (state $2 n$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 2 u\}$ under regime $j$ because, by description ( 5 d ), we have $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \mu_{i}$ which implies (56); $\bar{G}_{i}-\phi_{i}<H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{j}$ which implies (57); $W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ which imply (58); $\bar{G}_{i}-\phi_{i}<H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)$ and $W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ which imply (59); $\bar{G}_{i}-\phi_{i}<\kappa_{i}$ which implies (60):

$$
\begin{array}{ll}
{\left[2 n \succsim^{j} 0 n\right]} & : \\
{\left[2 n \succsim_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k} \geq 0,\right.} \\
{\left[2 n \succsim^{j} 1 n\right]} & : \\
E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k}>\bar{G}_{i}-\phi_{i}-\eta_{i}^{j}, \\
{\left[2 n \succsim_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}-\alpha_{i} H_{i}^{l},\right.}  \tag{60}\\
{\left[2 n \succsim^{j} 2 u\right]} & : \\
E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l}, \\
& \alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{array}
$$

Woman $i=i^{V}$ prefers under-reporting earning $E_{i}^{k}$ on assistance (state $2 u$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 2 n\}$ under regime $j$ because, by description (5e), we have $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \mu_{i}$ and $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ which imply (61); $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)+\eta_{i}^{j}-\mu_{i} \geq$ $\kappa_{i}$ which implies (62); $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \mu_{i}, \bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ and $W_{i}^{l}=\alpha_{i}$ which imply (63); $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \kappa_{i}$ and $W_{i}^{l}=\alpha_{i}$ which imply (64); $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ which implies (65):

$$
\begin{align*}
& {\left[2 u \succsim^{j} 0 n\right] }:  \tag{61}\\
& E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq 0,  \tag{62}\\
& {\left[2 u \succsim^{j} 0 r\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k}>\bar{G}_{i}-\phi_{i}-\eta_{i}^{j},  \tag{63}\\
& {\left[2 u \succsim^{j} 1 n\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}-\alpha_{i} H_{i}^{l},  \tag{64}\\
& {\left[2 u \succsim^{j} 1 r\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l},  \tag{65}\\
& {\left[2 u \succsim^{j} 2 n\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k} .
\end{align*}
$$

## 6. Pairing $(0 r, 1 n)$ is allowed.

Consider a woman $i$ with preferences represented by (3) with $v($.$) strictly concave. Let$ $K_{i}=1$. Assume that her job offer entails earnings in range 1. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 1. Let
(a) woman $i$ be such that $\eta_{i}^{a}=\mu_{i}=0 \operatorname{and}^{2}$

$$
\begin{aligned}
& \max \left\{\begin{array}{c}
v\left(\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)+\alpha_{i} H_{i}^{k}-\eta_{i}^{j}, \\
v\left(E_{i}^{k}+\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)
\end{array}\right\} \quad<\phi_{i} \leq \min \left\{\begin{array}{c}
v\left(\bar{G}_{i}\right)-v(0), \\
v\left(\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)+\alpha_{i} H_{i}^{k}
\end{array}\right\}, \\
& \max \left\{\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-v\left(\bar{G}_{i}\right), \\
v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-v\left(\bar{G}_{i}\right)
\end{array}\right\} \leq \alpha_{i} H_{i}^{k} \leq v\left(E_{i}^{k}\right)-v(0) .
\end{aligned}
$$

Woman $i$ chooses an alternative compatible with state $0 r$ under $a$. We now show that, under regime $j$, she selects an alternative compatible with state $1 n$. The choice of the alternative compatible with state $0 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 1 n, 1 r, 1 u\}$. Thus:

$$
\begin{align*}
{\left[0 r \succsim^{a} 0 n\right] } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v(0),  \tag{66}\\
{\left[0 r \succsim^{a} 1 n\right]: } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k},  \tag{67}\\
{\left[0 r \succsim^{a} 1 r\right] } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k},  \tag{68}\\
{\left[0 r \succsim^{a} 1 u\right] } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} . \tag{69}
\end{align*}
$$

It is easy to verify that description (6a) is compatible with optimality under regime $a$, that is, with (66)-(69). Woman $i$ will not selected an alternative compatible with state $1 u$ under regime $j$ by Proposition 1. She prefers earning $E_{i}^{k}$ off assistance (state $1 n$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 r\}$ under $j$ because, by description (6a), we have $v\left(E_{i}^{k}\right)-v(0) \geq \alpha_{i} H_{i}^{k}$ which implies (70); $v\left(\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)+\alpha H_{i}^{k}-\eta_{i}^{j}<\phi_{i}$ which implies (71); and $v\left(E_{i}^{k}+\bar{G}_{i}\right)-v\left(E_{i}^{k}\right) \leq \phi_{i}$ which implies (72):

$$
\begin{align*}
& {\left[1 n \succsim^{j} 0 n\right]: v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k} \geq v(0),}  \tag{70}\\
& {\left[1 n \succsim^{j} 0 r\right]:}  \tag{71}\\
& {\left[1 n \succsim^{j} 1 r\right]} \tag{72}
\end{align*}: v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k}>v\left(\bar{G}_{i}^{k}\right)-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+\bar{G}_{i}\right)-\eta_{i}^{j}, \alpha_{i} H_{i}^{k} .
$$

[^1]7. We conclude the proof by remarking that, because pairings $\left(0 r, s^{j}\right)$ with $s^{j} \in\{0 r, 0 n, 1 n, 2 n, 1 r, 2 u\}$ are allowed, none of them must occur.

## 5 Testable Revealed Preference Restrictions

Lemma 5 (Revealed Preference Restrictions). Consider the system of equations:

$$
\begin{align*}
& p_{0 n}^{j}-p_{0 n}^{a}=-\pi_{0 n, 1 r} p_{0 n}^{a}+\pi_{0 r, 0 n} p_{0 p}^{a} \\
& p_{1 n}^{j}-p_{1 n}^{a}=-\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{0 r, 1 n} p_{0 p}^{a} \\
& p_{2 n}^{j}-p_{2 n}^{a}=-\pi_{2 n, 1 r} p_{2 n}^{a}+\pi_{0 r, 2 n} p_{0 p}^{a}  \tag{73}\\
& p_{0 p}^{j}-p_{0 p}^{a}=-\left(\pi_{0 r, 0 n}^{a}+\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}\right) p_{0 p}^{a} \\
& p_{2 p}^{j}-p_{2 p}^{a}=\pi_{0 r, 2 u} p_{0 p}^{a}-\pi_{2 u, 1 r} p_{2 p}^{a}
\end{align*}
$$

System (73) implies 16 inequality restrictions on $\mathbf{p}^{j}-\mathbf{p}^{a}$ :

$$
\begin{align*}
\left(p_{0 p}^{a}-p_{0 p}^{j}\right) & \geq 0  \tag{74}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right) & \geq 0  \tag{75}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{76}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right) & \geq 0  \tag{77}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{78}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{79}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right) & \geq 0  \tag{80}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{81}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{82}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{83}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{84}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{85}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{86}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{87}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{88}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0 \tag{89}
\end{align*}
$$

Proof. Restrictions (74-89) are obtained by using the fact that, by definition, $0 \leq \pi_{s^{a}, s^{j}} \leq 1$ all $s^{a}, s^{j} \in \mathcal{S}$ and $\sum_{s^{j} \in \mathcal{S}} \pi_{s^{a}, s^{j}}=1$ all $s^{a} \in \mathcal{S}$. The response margins $\left(\pi_{0 n, 1 r}, \pi_{1 n, 1 r}, \pi_{2 n, 1 r}, \pi_{2 u, 1 r}\right)$ may each take value 0 or 1 . The response margins ( $\pi_{0 r, 0 n}, \pi_{0 r, 1 n}, \pi_{0 r, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 2 u}$ ) may each take value 0 or 1 but if one of them takes the value 1 the others are constrained to take the value 0 . Thus, there are $2^{4}+1+5=22$ viable ordered arrangements of 9 elements each taking the boundary value 0 or 1 . Each arrangement implies restrictions on $\mathbf{p}^{j}-\mathbf{p}^{a}$ through system (73). 16 restrictions are non redundant: they are inequalities (74-89). For instance, consider the fourth equation in system (73). Letting $\pi_{0 r, 0 n}+\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}=0$, this equation implies (74). As another example, sum the first and the fourth equations in system (73) to obtain $\left(p_{0 n}^{j}-p_{0 n}^{a}\right)+\left(p_{0 p}^{j}-p_{0 p}^{a}\right)=-\pi_{0 n, 1 r} p_{0 n}^{a}-\left(\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}\right) p_{0 p}^{a}$. Letting $\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}=0$ and $\pi_{0 n, 1 r}=0$, this equation implies (75).

Remark 5 (Easy to Describe Testable Restrictions). In the paper we explicitly refer to five of the inequalities in (74-89). They are: inequality (74), inequality (75) which rewrites as $p_{0}^{a}-p_{0}^{j} \geq 0$ where $p_{0}^{t} \equiv p_{0 n}^{t}+p_{0 p}^{t}$ for $t \in\{a, j\}$; inequality (85) which rewrites as $p_{1+, p}^{a}-p_{1+, p}^{j} \leq 0$ where $p_{1+, p}^{t} \equiv p_{1 p}^{t}+p_{2 p}^{t}$ for $t \in\{a, j\} ;$ inequality (87) which rewrites as $p_{1}^{a}-p_{1}^{j} \leq 0$ where $p_{1}^{t} \equiv p_{1 n}^{t}+p_{1 p}^{t}$ for $t \in\{a, j\}$; and inequality (89) which rewrites as $p_{1 p}^{a}-p_{1 p}^{j} \leq 0$.

Corollary 3 (Additional Testable Restrictions under a Special Form of Preferences). Subject to specification (2) of the utility function, revealed preference imply a testable restriction in addition to (74-89):

$$
\begin{equation*}
p_{1 n}^{a}-p_{1 n}^{j} \geq 0 . \tag{90}
\end{equation*}
$$

Subject to (90), inequalities (76), (79), (82), (83), (85), (86), (88), and (89) are redundant.
Proof. Subject to specification (2) of the utility function, $\pi_{0 r, 1 n}=0$ by Corollary 2. System (73) simplifies accordingly. In particular, the second equation writes $p_{1 n}^{a}-p_{1 n}^{j}=\pi_{1 n, 1 r} p_{1 n}^{a}$. Letting $\pi_{1 n, 1 r}=0$ we obtain restriction (90). Redundancy of inequalities (76), (79), (82), (83), (85), (86), (88), and (89) is easily verified. For instance, inequality (76) is implied by (74) and (90).

## 6 Bounds on the Response Margins

## Derivation of Bounds

A solution to any linear programming problem has to occur at one of the vertices of the problem's constraint space (see Murty, 1983). Recall that the linear constraints are as per system (73). To obtain the set of possible solutions to the linear programming problem

$$
\max _{\pi} \boldsymbol{\pi}^{\prime} \boldsymbol{\lambda} \text { subject to (73) and } \boldsymbol{\pi} \in[0,1]^{9},
$$

we enumerated all vertices of the convex polytope defined by the intersection of the hyperplane defined by the equations in (73) with the hypercube defined by the unit constraints on the parameters. In practice, this amounted to setting all possible choices of four of the nine parameters in (73) to 0 or 1 and solving for the remaining five parameters. There were $\binom{9}{4}=126$ different possible choices of four parameters and $2^{4}=16$ different binary arrangements those parameters could take, yielding 2016 possible vertices. However we were able to use the structure of our problem to rule out the existence of solutions at certain vertices - e.g., $\pi_{2 n, 1 r}$ and $\pi_{0 r, 2 n}$ cannot both be set arbitrarily because this would lead to a violation of the third equation in (73). Such restrictions reduced the problem to solving the system at a manageable number of vertices. We then enumerated the set of minima and maxima each parameter could achieve across the relevant solutions. After eliminating dominated solutions, we arrived at the stated bounds.

## Lists of Bounds

The analytical expressions for the bounds on the response probabilities are presented below. The symbol $\left({ }^{*}\right)$ is placed next to a solution, or a term, that is redundant subject to the specification of the utility function given in (2).

## Simple Response Margins

$$
\max \left\{0, \frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 n}^{a}}\right\} \leq \pi_{2 n, 1 r} \leq \min \left\{\begin{array}{c}
1, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{2 n}^{a},}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}},(*) \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}},(*) \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}},(*) \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}}
\end{array},(*),\right.
$$

$$
\begin{aligned}
& \max \left\{0, \frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 n}^{a}}\right\} \leq \pi_{0 n, 1 r} \leq \min \{ \\
& \left.\frac{\left(p_{0 n}-p_{0 n}^{j}\right)+\left(p_{0 p}-p_{0 p}^{j}\right)+\left(p_{1 n}-p_{1 n}^{j}\right)+\left(p_{2 n}-p_{2 n}^{j}\right)+\left(p_{2 p}-p_{2 p}^{j}\right)}{p_{0 n}^{a}}(*)\right) \\
& \left.\begin{array}{c}
1, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 p}^{a}},(*) \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 p}^{a}},(*) \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}},(*) \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}}(*)
\end{array}\right\} . \\
& \max \left\{0, \frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 p}^{a}}\right\} \leq \pi_{2 u, 1 r} \leq \min \{ \\
& \max \left\{0, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)-p_{0 n}^{j}-p_{2 p}^{j}-p_{2 n}^{j}-p_{1 n}^{j}(*)}{p_{0 p}^{a}}\right\} \leq \pi_{0 r, 1 r} \leq \min \{ \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \left.\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}(*)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left\{\begin{array}{c}
\frac{p_{1 n}^{j}}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}
\end{array}\right\} . \\
& \left\{\begin{array}{c}
1, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{\left.p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)} \\
p_{1 n}^{a}
\end{array},\right. \\
& \max \left\{0, \frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{1 n}^{a}}\right\} \leq \pi_{1 n, 1 r} \leq \min \{
\end{aligned}
$$

## Composite Response Margins

$$
\begin{aligned}
& \pi_{0 r, n} \geq \max \left\{0,-\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)(*)}{p_{0 p}^{a}}\right\}, \\
& \pi_{0 r, n} \leq \min \left\{\frac{p_{0 n}^{j}+p_{2 n}^{j}+p_{1 n}^{j}(*)}{p_{0 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}\right\} . \\
& \pi_{p, n} \geq \max \left\{0,-\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)(*)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}\right\}, \\
& \pi_{p, n} \leq \min \left\{\frac{p_{0 n}^{j}+p_{2 n}^{j}+p_{1 n}^{j}(*)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}\right\} . \\
& \pi_{n, p} \geq \max \left\{0, \frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}}\right\}, \\
& \pi_{n, p} \leq \min \left\{\begin{array}{c}
1, \\
\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}}, \\
\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}}
\end{array}\right\} . \\
& \pi_{0,1+}=\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}+p_{0 n}^{a}} .
\end{aligned}
$$

## 7 Inference on Bounds

We begin with a description of the upper limit of our confidence interval. For each response probability $\pi$ we have a set of possible upper bound solutions $\left\{u b_{1}, u b_{2}, \ldots, u b_{K}\right\}$. We know that:

$$
\begin{aligned}
\pi & \leq \bar{\pi} \equiv \min \{\underline{u b}, 1\} \\
\underline{u b} & \equiv \min \left\{u b_{1}, u b_{2}, \ldots, u b_{K}\right\} .
\end{aligned}
$$

A consistent estimate of the least upper bound $\underline{u b}$ can be had by plugging in consistent sample moments $\widehat{u b}_{k} \xrightarrow{p} u b_{k}$ and using $\underline{\widehat{u b}} \equiv \min \left\{\widehat{u b}_{1}, \widehat{u b}_{2}, \ldots, \widehat{u b}_{K}\right\}$ as an estimate of $\underline{u b}$. This estimator is consistent by continuity of probability limits. We can then form a corresponding consistent estimator $\widehat{\bar{\pi}} \equiv \min \{\underline{\widehat{u b}}, 1\}$ of $\bar{\pi}$.

To conduct inference on $\pi$, we seek a critical value $r$ that obeys:

$$
\begin{equation*}
P(\underline{u b} \leq \underline{\widehat{u b}}+r)=0.95, \tag{91}
\end{equation*}
$$

as such an $r$ implies:

$$
\begin{aligned}
P(\pi \leq \min \{\underline{\widehat{u b}}+r, 1\}) \geq & P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\}) \\
= & P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\} \mid \underline{u b} \leq \underline{\widehat{u b}}+r) 0.95 \\
& +P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\} \mid \underline{u b}>\underline{\widehat{u b}}+r) 0.05 \\
\geq & P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\} \mid \underline{u b} \leq \underline{\widehat{u b}}+r) 0.95 \\
= & 0.95
\end{aligned}
$$

with the first inequality binding when $\pi=\bar{\pi}$. The last line follows because $\underline{u b} \leq \underline{\widehat{u b}}+r$ implies $\min \{\underline{u b}+r, 1\} \leq \min \{\underline{\widehat{u b}}+r, 1\}$.

We can rewrite (91) as:

$$
P\left(-\min \left\{\widehat{u b}_{1}-\underline{u b}, \widehat{u b}_{2}-\underline{u b}, \ldots, \widehat{u b}_{K}-\underline{u b}\right\} \leq r\right)=0.95,
$$

or equivalently

$$
P\left(\max \left\{\underline{u b}-\widehat{u b}_{1}, \underline{u b}-\widehat{u b}_{2}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\} \leq r\right)=0.95 .
$$

It is well known that the limiting distribution of $\max \left\{\underline{u b}-\widehat{u b}_{1}, \underline{u b}-\widehat{u b}_{2}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\}$ depends on which and how many of the upper bound constraints bind. Several approaches to this problem have been proposed which involve conducting pre-tests for which constraints are binding (e.g. Andrews and Barwick, 2012).

We take an alternative approach to inference that is simple to implement and consistent regardless of the constraints that bind. Our approach is predicated on the observation that:

$$
\begin{equation*}
P\left(\max \left\{u b_{1}-\widehat{u b}_{1}, \ldots, u b_{K}-\widehat{u b}_{K}\right\} \leq r\right) \leq P\left(\max \left\{\underline{u b}-\widehat{u b}_{1}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\} \leq r\right), \tag{92}
\end{equation*}
$$

with equality holding in the case where all of the upper bound solutions are identical. We seek an $r^{\prime}$ such that:

$$
\begin{equation*}
P\left(\max \left\{u b_{1}-\widehat{u b}_{1}, \ldots, u b_{K}-\widehat{u b}_{K}\right\} \leq r^{\prime}\right)=.95 \tag{93}
\end{equation*}
$$

From (92),

$$
P\left(\max \left\{\underline{u b}-\widehat{u b}_{1}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\} \leq r^{\prime}\right) \geq .95
$$

with equality holding when all bounds are identical.
A bootstrap estimate $r^{*} \xrightarrow{p} r^{\prime}$ of the necessary critical value can be had by considering the bootstrap analog of condition (93) (see Proposition 10.7 of Kosorok, 2008). That is, by computing the 95 th percentile of:

$$
\max \left\{\widehat{u b}_{1}-\widehat{u b}_{1}^{*}, \ldots, \widehat{u b}_{K}-\widehat{u b}_{K}^{*}\right\}
$$

across bootstrap replications, where stars refer to bootstrap quantities. An upper limit $U$ of the confidence region for $\pi$ can then be formed as:

$$
U=\min \left\{\underline{\widehat{u b}}+r^{*}, 1\right\} .
$$

Note that this procedure is essentially an unstudentized version of the inference method of Chernozhukov et al. (2013) where the set of relevant upper bounds ( $\mathcal{V}_{0}$ in their notation) is taken here to be the set of all upper bounds, thus yielding conservative inference.

We turn now to the lower limit of our confidence interval. Our greatest lower bounds are all of the form:

$$
\pi \geq \underline{\pi} \equiv \max \{l b, 0\} .
$$

We have the plugin lower bound estimator $\widehat{l b} \xrightarrow{p} l b$. By the same arguments as above we want to search for an $r^{\prime \prime}$ such that

$$
P\left(l b \geq \widehat{l b}-r^{\prime \prime}\right)=0.95 .
$$

Since $\widehat{l b}$ is just a scalar sample mean, we can choose $r^{\prime \prime}=1.65 \sigma_{l b}$ where $\sigma_{l b}$ is the asymptotic standard error of $\widehat{l b}$ in order to guarantee the above condition holds asymptotically. To account for the propensity score re-weighting, we use a bootstrap standard error estimator $\widehat{\sigma}_{l b}$ of $\sigma_{l b}$ which is consistent via the usual arguments. Thus, our "conservative" $95 \%$ confidence interval for $\pi$ is:

$$
\left[\max \left\{0, \widehat{l b}-1.65 \widehat{\sigma}_{l b}\right\}, \min \left\{\underline{\widehat{u b}}+r^{*}, 1\right\}\right] .
$$

This confidence interval covers the parameter $\pi$ with asymptotic probability of at least $95 \%$.

## 8 Relaxation of Lower Bound on the Stigma Disutility

## The Issue

In the paper we restrict a woman's preferences when $F P L_{i}<\bar{E}_{i}$. Specifically, A. $\mathbf{6}$ in Assumption 1 states that for all offers $(W, H)$ such that $E \equiv W H \in\left(F P L_{i}, \bar{E}_{i}\right]$ :

$$
U_{i}^{a}\left(H, C^{a}(E, D, E), D, 0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, 0, E), 0,0\right) .
$$

A. 6 in Assumption 1 implicitly establishes a lower bound on the stigma disutility and it guarantees that woman $i$ does not report earnings above $F P L_{i}$ while on welfare under regime $a$ when $F P L_{i}<\bar{E}_{i}$. That is, state $2 r$ is dominated under regime $a$ subject to A. 6 in Assumption 1 and Assumption 3. Without A. 6 in Assumption 1, participation in welfare may decrease or increase utility (other things equal). The number of observations in our control sample corresponding to alternatives compatible with state $2 r$ is tiny. Nevertheless, it is of pedagogical interest to consider what additional responses emerge if we do not rule out such choices a priori, that is, when we do not impose A. 6 in Assumption 1.

## A Roadmap of the Results: Table A4 and Figure A1

Table A4 catalogs the allowed and disallowed responses when A. 6 in Assumption 1 is not imposed. The possible states are $\mathcal{S} \cup\{2 r\}$. Accordingly, all but the last row and last column of Table A4 appear also in Table 4. The last row of Table A4 corresponds to the responses of a woman who under regime $a$ has earnings in the range ( $\left.F P L_{i}, \bar{E}_{i}\right]$, is on assistance, and truthfully reports her earnings to the welfare agency (state $2 r$ ).

The presentation of the results is organized as follows. Proposition 3 pertains to the disallowed pairings of states in Table A4. Corollary 4 derives additional restricted pairings when the utility function is of the special form given in (2). Proposition 4 pertains to the allowed pairings of states in Table A4. Interestingly, dispensing with A. 6 in Assumption 1 enables the emergence of flows out of the labor force, which were absent in the model of Section 4 of the paper. Corollary 5 shows that labor market constraints on hours are essential to the emergence of these flows. Figure A1 illustrates this point. To ease the graphical representation, we use the special form of the utility function in (2). Figure A1 portrays a woman who receives two job offers entailing earnings $\left(E^{1}, E^{2}\right)$ that are both in range 2 and obey $\left(E^{1}, E^{2}\right) \in\left(F P L_{i}, \bar{E}_{i}\right]$. Her welfare stigma is zero. For convenience, her fixed cost of work is also zero and her cost of under-reporting is sufficiently large that under-reporting earnings to the welfare agency is always a dominated choice. Under AFDC, the woman earns $E^{1}$, is on assistance, and truthfully reports her earnings. Observe that she would make the same choice even if earning constraints were absent. Under JF, the woman does not work and is off assistance. However, if earning constraints were absent she would be better off by earning below the FPL on assistance and truthfully reporting her earnings.

## Propositions

With reference to Section 4 in this Appendix, all Lemmas and Corollaries hold but for Lemma 1 which hinges on A. 6 in Assumption 1. Proposition 1, Corollary 2, and Proposition 2 in Section 4 are superseded by the following propositions and corollary.

Proposition 3 (Restricted Pairings). Given Assumption 1 but for A.6, and Assumptions 2-5, the pairings of states corresponding to the "-" entries in Table $A_{4}$ cannot occur and the pairings of states $(1 r, 1 r)$ and $(1 u, 1 r)$ must occur.

Proof. We proved the entries in the first 7 rows and 7 columns of Table A4 in Propositions 1 and Proposition 2. State $2 r$ is not defined under regime $j$ (Assumption 3) which proves the "-" entries in Table A4 rows 1 through 7 and column 8. We are thus left to prove the disallowed pairings in row 8 and columns 1 through 7 of Table A4. No woman pairs state $2 r$ under regime $a$ with state $1 u$ under regime $j$ because $1 u$ is dominated by state $1 r$ under $j$ (Lemma 2).

Corollary 4 (Additional Restricted Pairings under Utility Specification (2)). Given Assumption 1 but for A.6, and Assumptions 2-5, and subject to specification (2) of the utility function, the pairings of states $(0 r, 1 n)$ and $(2 r, 1 n)$ are disallowed.

Proof. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$. The proof that the pairing of states $(0 r, 1 n)$ is disallowed is contained in Corollary 2. The proof that the pairings of state $(2 r, 1 n)$ is disallowed is by contradiction. Suppose there is a woman $i$ who selects an alternative compatible with state $2 r$ under regime $a$ entailing earnings $E^{k} \equiv W^{k} H^{k}$ and selects an alternative compatible with state $1 n$ under regime $j$ entailing earnings $E^{l} \equiv W^{l} H^{l}$. By Assumption 4, her choice under regime $a$ reveals that

$$
\left[2 r \succsim^{a} 2 n\right]: U_{i}\left(H^{k}, E^{k}-\mu_{i}+G_{i}^{a}\left(E^{k}\right)-\phi_{i}\right) \geq U_{i}\left(H^{k}, E^{k}-\mu_{i}\right),
$$

which implies $G_{i}^{a}\left(E^{k}\right) \geq \phi_{i}$. Her choice under regime $j$ reveals that

$$
\left[1 n \succsim^{j} 1 r\right]: U_{i}\left(H^{l}, E^{l}-\mu_{i}\right) \geq U_{i}\left(H^{l}, E^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}\right),
$$

which implies $\bar{G}_{i} \leq \phi_{i}$. Thus, optimality implies $\bar{G}_{i} \leq \phi_{i} \leq G_{i}^{a}\left(E^{k}\right)$ which yields a contradiction because $G_{i}^{a}(E)<\bar{G}_{i}$ for all $E \in\left(F P L_{i}, \bar{E}_{i}\right]$ including $E^{k}$.

Proposition 4 (Unrestricted Pairings). Given Assumption 1 but for A.6, and Assumptions 2-5, the non "-" entries in Table A4 correspond to pairings of states that are allowed.

Proof. The entries in the first 7 rows and 7 columns Table 4A were proven in Propositions 1 and Proposition 2. We are left to prove the allowed pairings in row 8 and columns 1 through 7 of Table A4. To prove that the pairs in collection

$$
\begin{equation*}
\left\{\left(2 r, s^{j}\right) \mid s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 r, 2 u\}\right\} \tag{94}
\end{equation*}
$$

are allowed it suffices to provide examples where six women occupy the same state $s^{a}=2 r$ under regime $a$ but occupy state $s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 r, 2 u\}$ under regime $j$. This also proves that no pairing in collection (94) is constrained to occur. When providing these examples we consider the specification of the utility function given in (3). Finally, we assume that woman $i$ receives either one or two job offers, that is, either $K_{i}=1$ or $K_{i}=2$. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$.

1. Pairings $(2 r, 0 n),(2 r, 0 r)$, and $(2 r, 2 u)$ are allowed.

Consider three women $i^{\prime}, i^{\prime \prime}$, and $i^{\prime \prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=1$ for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}\right\}$. Assume that all three women's job offer entails earnings in $\in\left(F P L_{i}, \bar{E}_{i}\right]$. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}\right\}$. Let
(a) woman $i=i^{\prime}$ be such that $\alpha_{i}=W_{i}^{k}, \mu_{i}=0$, and

$$
\eta_{i}^{a} \geq \kappa_{i} \geq \bar{G}_{i}-\phi_{i} \geq \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)
$$

(b) woman $i=i^{\prime \prime}$ be such that $\alpha_{i}=W_{i}^{k}, \mu_{i}=0$, and

$$
\kappa_{i} \geq \bar{G}_{i}-\phi_{i} \geq \eta_{i}^{j} \geq \eta_{i}^{a} \geq \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)
$$

(c) woman $i=i^{\prime \prime \prime}$ be such that $\alpha_{i}=W_{i}^{k}, \mu_{i}=0$, and

$$
\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right) \leq \eta_{i}^{a} \leq \kappa_{i} \leq \min \left\{\eta_{i}^{j}, \bar{G}_{i}-\phi_{i}\right\} .
$$

All women choose to earn and truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ on assistance under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $0 n$ under regime $j$, woman $i^{\prime \prime}$ chooses an alternative compatible with state $0 r$ under regime $j$, and woman $i^{\prime \prime \prime}$ chooses an alternative compatible with state $2 u$ under regime $j$. For all women, the choice of the alternative compatible with state $2 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 0 n, 2 n, 2 u\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}\right\}$ :

$$
\begin{align*}
& {\left[2 r \succsim^{a} 0 r\right] }:  \tag{95}\\
& {\left[2 r \succsim_{i}^{k} 0 n\right] }:  \tag{96}\\
& {\left[G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{a},\right.}  \tag{97}\\
& {\left[2 r \succsim^{a} 2 n\right] }:  \tag{98}\\
& {\left[\phi_{i}^{k}+G_{i}^{a}\left(E_{i}^{k} H_{i}^{k} \geq 0,\right.\right.} \\
& {\left[2 r \succsim^{a} 2 u\right] }: \\
& E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i} H_{i}^{k} \geq \alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\alpha_{i} H_{i}^{k}, \\
& \sigma_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{align*}
$$

It is easy to verify that descriptions (1a), (1b), and (1c) are compatible with optimality under regime $a$ for woman $i^{\prime}, i^{\prime \prime}$, and $i^{\prime \prime \prime}$ respectively, that is, with (95)-(98). No woman selects an alternative compatible with state $2 r$ under regime $j$ because it is not defined.
Woman $i^{\prime}$ prefers not working off assistance (state $0 n$ ) to the available alternatives compatible with states $\{0 r, 2 n, 2 u\}$ under regime $j$ because, by description (1a), we have $\eta_{i}^{j} \geq \bar{G}_{i}-\phi_{i}$ which implies (99); $\alpha_{i}=W_{i}^{k}$ which implies (100); and $\kappa_{i} \geq \bar{G}_{i}-\phi_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (101):

$$
\begin{align*}
& {\left[0 n \succsim^{j} 0 r\right]: 0 \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{j} \text {, }}  \tag{99}\\
& \text { [ } \left.0 n \succsim^{j} 2 n\right]: 0 \geq E_{i}^{k}-\alpha_{i} H_{i}^{k} \text {, }  \tag{100}\\
& {\left[0 n \succsim^{j} 2 u\right]: 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .} \tag{101}
\end{align*}
$$

Woman $i^{\prime \prime}$ prefers not working on assistance (state $0 r$ ) to the available alternatives compatible with states $\{0 n, 2 n, 2 u\}$ under regime $j$ because, by description (1b), we have $\bar{G}_{i}-\phi_{i} \geq \eta_{i}^{j}$ which implies (102); $\bar{G}_{i}-\phi_{i} \geq \eta_{i}^{j}$ and $\alpha_{i}=W_{i}^{k}$ which imply (103); and $\kappa_{i} \geq \eta_{i}^{j}$ which implies (104):

$$
\begin{array}{ll}
{\left[0 r \succsim^{j} 0 n\right]} & : \\
{\left[0 r \succsim^{j} 2 n\right]} & : \bar{G}_{i}-\phi_{i}-\eta_{i}^{j} \geq 0, \\
{\left[0 r \succsim^{j} 2 u\right]} & : \bar{G}_{i}-\phi_{i}-\eta_{i}^{j} \geq E_{i}^{j} \geq E_{i}^{k}+\alpha_{i} H_{i}^{k},  \tag{104}\\
& -\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{array}
$$

Woman $i^{\prime \prime \prime}$ prefers earning $E_{i}^{k}$ on assistance and under-report (state $2 u$ ) to the available alternatives compatible with states $\{0 n, 0 r, 2 n\}$ under regime $j$ because, by description (1c),
we have $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (105); $\eta_{i}^{j} \geq \kappa_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (106); and $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (107):

$$
\begin{array}{ll}
{\left[2 u \succsim^{j} 0 n\right]:} & E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq 0, \\
{\left[2 u \succsim^{j} 0 r\right]:} & E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{j}, \\
{\left[2 u \succsim^{j} 2 n\right]:} & E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\alpha_{i} H_{i}^{k} . \tag{107}
\end{array}
$$

## 2. Pairing $(2 r, 1 r)$ is allowed.

Consider woman $i$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=2$. Assume that her first job offer entails earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ and her second job offer entails earnings in range 1. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ and $E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l} \in\left(0, F P L_{i}\right]$. Let
(a) woman $i$ be such that $W_{i}^{k}>\alpha_{i}=W_{i}^{l}, \mu_{i}=0$, and

$$
\max \left\{\begin{array}{c}
\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)-\eta_{i}^{a}, \\
G_{i}^{a}\left(E_{i}^{l}\right)-G_{i}^{a}\left(E_{i}^{k}\right)
\end{array}\right\} \leq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \leq \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right) \leq \min \left\{\bar{G}_{i}-\phi_{i}, \kappa_{i}\right\} .
$$

Woman $i$ chooses to earn and truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ on assistance under regime $a$. We now show that she chooses an alternative compatible with state $1 n$ under regime $j$. The choice of the alternative compatible with state $2 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 0 n, 1 n, 1 r, 1 u, 2 n, 2 u\}$. Thus:

$$
\begin{align*}
& {\left[2 r \succsim^{a} 0 r\right] }:  \tag{108}\\
& E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{a},  \tag{109}\\
& {\left[2 r \succsim^{a} 0 n\right] }:  \tag{110}\\
& E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq 0,  \tag{111}\\
& {\left[2 r \succsim^{a} 1 n\right] }:  \tag{112}\\
& E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\alpha_{i} H_{i}^{l},  \tag{113}\\
& {\left[2 r \succsim^{a} 1 r\right] }:  \tag{114}\\
& {\left[2 r \succsim_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i}-\alpha_{i} H_{i}^{l},\right.} \\
& {[2 r} E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{l}, \\
& {\left[2 r \succsim^{a} 2 n\right]: } E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\alpha_{i}-H_{i}^{k}, \\
& {\left[2 r \succsim^{a} 2 u\right] }: \\
& E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{align*}
$$

It is easy to verify that description (2a) is compatible with optimality under regime $a$ for woman $i$, that is, with (108)-(114). Woman $i$ does not select an alternative compatible with state $2 r$ under regime $j$ because it is not defined; she does not selects an alternative compatible with state $1 u$ under regime $j$ because it is dominated. Woman $i$ prefers earning and truthfully report $E_{i}^{l}$ on assistance (state $1 r$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 n, 2 n, 2 u\}$ under regime $j$ because, by description (2a), we have $\bar{G}_{i}-\phi_{i} \geq 0$ and $W_{i}^{l}=\alpha_{i}$ which imply (115); $\eta_{i}^{j} \geq 0$ and $W_{i}^{l}=\alpha_{i}$ which imply (116); $\bar{G}_{i}-\phi_{i} \geq 0$ which implies (117); $\bar{G}_{i}-\phi_{i} \geq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)$ and $W_{i}^{l}=\alpha_{i}$ which imply (118); and $\kappa_{i} \geq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)$ and $W_{i}^{l}=\alpha_{i}$ which imply (119):

$$
\begin{align*}
& {\left[1 r \succsim^{j} 0 n\right] }:  \tag{115}\\
& E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq 0,  \tag{116}\\
& {\left[1 r \succsim^{j} 0 r\right] }:  \tag{117}\\
& E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{j},  \tag{118}\\
& {\left[1 r \succsim^{j} 1 n\right]: } E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq E_{i}^{l}-\alpha_{i} H_{i}^{l},  \tag{119}\\
& {\left[1 r \succsim^{j} 2 n\right]: } E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq E_{i}^{k}-\alpha_{i} H_{i}^{k}, \\
& {\left[1 r \succsim^{j} 2 u\right]: } E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{align*}
$$

## 3. Pairing $(2 r, 1 n)$ is allowed.

Consider woman $i$ with preferences represented by (3) with $v(x)$ convex. Let $K_{i}=2$. Assume that her first job offer entails earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ and her second job offer entails earnings in range 1. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ and $E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l} \in\left(0, F P L_{i}\right]$. Let
(a) woman $i$ be such that $\alpha_{i}>0, \mu_{i}=0$ and $^{3}$

$$
\left.\begin{array}{rl}
\max \left\{\begin{array}{c}
v\left(E_{i}^{l}+\bar{G}_{i}\right)-v\left(E_{i}^{l}\right), \\
\left.\left[\begin{array}{c}
v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-v\left(E_{i}^{l}\right) \\
-\alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right)
\end{array}\right]\right\}
\end{array} \leq \phi_{i} \leq \min \left\{\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-v\left(E_{i}^{k}\right) \\
{\left[\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-v\left(E_{i}^{l}\right) \\
-\alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right)
\end{array}\right]}
\end{array}\right\},\right. \\
v\left(E_{i}^{k}\right)-v\left(E_{i}^{l}\right) & \leq \alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right) \leq \min \left\{\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right) \\
-v\left(E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)\right)
\end{array}\right], \\
{\left[\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right) \\
-v\left(E_{i}^{l}+\bar{G}_{i}-\kappa_{i}\right)
\end{array}\right]}
\end{array}\right\},
$$

Woman $i$ chooses to earn and truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ on assistance under regime $a$. We now show that she chooses an alternative compatible with state $1 n$ under regime $j$. The choice of the alternative compatible with state $2 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 0 n, 1 n, 1 r, 1 u, 2 n, 2 u\}$. Thus:

$$
\begin{align*}
& {\left[2 r \succsim^{a} 0 r\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(\bar{G}_{i}\right)-\phi_{i}-\eta_{i}^{a},}  \tag{120}\\
& {\left[2 r \succsim^{a} 0 n\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v(0) \text {, }}  \tag{121}\\
& {\left[2 r \succsim^{a} 1 n\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{l}\right)-\alpha_{i} H_{i}^{l} \text {, }}  \tag{122}\\
& {\left[2 r \succsim^{a} 1 r\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{l} \text {, }}  \tag{123}\\
& {\left[2 r \succsim^{a} 1 u\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{l}+\bar{G}_{i}-\kappa_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{l},}  \tag{124}\\
& {\left[2 r \succsim^{a} 2 n\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k} \text {, }}  \tag{125}\\
& {\left[2 r \succsim^{a} 2 u\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} .} \tag{126}
\end{align*}
$$

It is easy to verify that description (3a) is compatible with optimality under regime $a$ for woman $i$, that is, with (120)-(126). Woman $i$ does not selects an alternative compatible with state $2 r$ under regime $j$ because it is not defined; she does not selects an alternative compatible with state $1 u$ under regime $j$ because it is dominated. Woman $i$ prefers earning $E_{i}^{l}$ off assistance (state $1 n$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 r, 2 n, 2 u\}$ under regime $j$ because, by description (3a), we have $\alpha_{i} H_{i}^{l} \leq v\left(E_{i}^{l}\right)-v(0)$ which implies (127); $\alpha_{i} H_{i}^{l} \leq v\left(E_{i}^{l}\right)-v(0)$ which by convexity, and since $\eta_{i}^{j} \geq 0$, implies $\alpha_{i} H_{i}^{l} \leq v\left(E_{i}^{l}+\bar{G}_{i}\right)-$ $v\left(\bar{G}_{i}\right)+\eta_{i}^{j}$ which along with $\phi_{i} \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-v\left(E_{i}^{l}\right)$ imply (128); $\phi_{i} \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-$ $v\left(E_{i}^{l}\right)$ which implies (129); $v\left(E_{i}^{k}\right)-v\left(E_{i}^{l}\right) \leq \alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right)$ which implies (130); and $\phi_{i} \geq$

[^2]$$
v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-v\left(E_{i}^{l}\right)-\alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right) \text { which implies (131): }
$$
\[

$$
\begin{align*}
& {\left[1 n \succsim^{j} 0 n\right]: v\left(E_{i}^{l}\right)-\alpha_{i} H_{i}^{l} \geq v(0),}  \tag{127}\\
& {\left[1 n \succsim^{j} 0 r\right]: v\left(E_{i}^{l}-\alpha_{i} H_{i}^{l} \geq v\left(\bar{G}_{i}\right)-\phi_{i}-\eta_{i}^{j},\right.}  \tag{128}\\
& {\left[1 n \succsim^{j} 1 r\right]:}  \tag{129}\\
& {\left[1 n \succsim^{j} 2 n\right]:}  \tag{130}\\
& {\left[1 n \succsim^{j} 2 u\right]-\alpha_{i} H_{i}^{l} \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{l},}  \tag{131}\\
& :
\end{align*}
$$, \alpha_{i} H_{i}^{l} \geq v\left(E_{i}^{l}\right)-\alpha_{i} H_{i}^{l} \geq v\left(E_{i}^{k}+\bar{G}_{i}-H_{i}^{k}, \phi_{i}\right)-\phi_{i} H_{i}^{k} .
\]

Corollary 5 (Additional Restricted Pairings in the absence of Labor Market Constraints). Suppose that there are no hours constraints, that is, let $\Theta_{i}=\left\{\left(W_{i}(H), H\right) \mid H \in\left(0, \bar{H}_{i}\right]\right\}$ in Assumption 4 and suppose that wages are continuous and weakly increasing in hours worked and utility is a weakly decreasing function of hours worked. Then, given Assumption 1 but for A.6, and Assumptions 2-5, no woman pairs state $2 r$ under regime a with states $\{0 n, 0 r\}$ under regime $j$.

Proof. We show that no woman pairs state $2 r$ under regime $a$ with state $0 n$ under regime $j$; the proof that no woman pairs state $2 r$ under regime a with state $0 r$ under regime $j$ is similar. The proof is by contradiction. Suppose that there is a woman $i$ who selects an alternative compatible with state $2 r$ under regime $a$, entailing earnings $E^{k} \equiv W\left(H^{k}\right) H^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ and selects an alternative compatible with state $0 n$ under $j$. By Assumption 4, her choice under regime $a$ reveals that

$$
\begin{equation*}
\left[2 r \succsim^{a} 0 n\right]: U_{i}\left(H^{k}, E^{k}+G_{i}^{a}\left(E^{k}\right), 1,0\right) \geq U_{i}(0,0,0,0) \tag{132}
\end{equation*}
$$

Because there are no hour constraints and because program rules are such that $\bar{E}_{i}<F P L_{i}+$ $\bar{G}_{i}$, there exists a job offer $\left(W\left(H^{l}\right), H^{l}\right)$ such that $E^{l} \equiv W\left(H^{l}\right) H^{l}$ is in range 1 and $E^{l}+$ $\bar{G}_{i}=E^{k}+G_{i}^{a}\left(E^{k}\right)$. Hence, $H^{k} \geq H^{l}$ because wages are weakly increasing in hours. Thus, $U_{i}\left(H^{l}, E^{l}+\bar{G}_{i}, 1,0\right) \geq U_{i}\left(H^{k}, E^{k}+G_{i}^{a}\left(E^{k}\right), 1,0\right)$ because utility is weakly decreasing in hours worked for given $(C, D, R)$ by Assumption 1. Together with (132), this means that

$$
\begin{equation*}
U_{i}\left(H^{l}, E^{l}+\bar{G}_{i}, 1,0\right) \geq U_{i}(0,0,0,0) \tag{133}
\end{equation*}
$$

If inequality (133) holds strictly, a contradiction ensures because this shows that no alternative compatible with state $0 n$ can be optimal under regime $j$ (it is dominated by an alternative compatible with state $1 r$ ). If inequality (133) holds as an equality, woman $i$ is indifferent between earning (and truthfully reporting) $E^{k}$ and not working off assistance under regime $a$. By Assumption 5, if the woman resolved an indifference situation against not working off assistance under regime $a$, she will also resolve an indifference situation against not working off assistance under regime $j$. This contradicts her selecting not to work off assistance over earning (and truthfully reporting) $E^{l}$ on assistance under $j$.
Proposition 5. Define $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$. Given Assumption 1 but for A. $\boldsymbol{6}$, and Assumptions 2-5, the system of equations describing the impact of the JF reform on observable state probabilities is:

$$
\begin{align*}
& p_{0 n}^{j}-p_{0 n}^{a}=-\pi_{0 n, 1 r} p_{0 n}^{a}+\pi_{0 r, 0 n} p_{0 p}^{a}+\pi_{2 r, 0 n} q_{2 r}^{a} \\
& p_{1 n}^{\prime}-p_{1 n}^{a}=-\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{0 r, 1 n} p_{0 p}^{a}+\pi_{2 r, 1 n} q_{2 r}^{r} \\
& p_{2 n}^{j}-p_{2 n}^{a}=-\pi_{2 n, 1 r} p_{2 n}^{a}+\pi_{0 r, 2 n} p_{0 p}^{a}+\pi_{2 r, 2 n} q_{2 r}^{a}  \tag{134}\\
& p_{0 p}^{j}-p_{0 p}^{a}=-\left(\pi_{0 r, 0 n}+\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}\right) p_{0 p}^{a}+\pi_{2 r, 0 r} q_{2 r}^{a} \\
& p_{2 p}^{j}-p_{2 p}^{a}=\pi_{0 r, 2 u} p_{0 p}^{a}-\pi_{2 u, 1 r} p_{2 p}^{a}-\left(\pi_{2 r, 0 n}+\pi_{2 r, 1 n}+\pi_{2 r, 2 n}+\pi_{2 r, 0 r}+\pi_{2 r, 1 r}-\pi_{2 u, 1 r}\right) q_{2 r}^{a}
\end{align*}
$$

Proof. By definition $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$, Table A4, and a simple application of the law of total probability.

Remark 6. Given Assumption 1 but for A.6, and Assumptions 2-5, bounds on the response probabilities
$\pi^{\prime} \equiv\left[\pi_{0 n, 1 r}, \pi_{0 r, 0 n}, \pi_{2 n, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 1 r}, \pi_{0 r, 1 n}, \pi_{1 n, 1 r}, \pi_{0 r, 2 u}, \pi_{2 u, 1 r}, \pi_{2 r, 0 n}, \pi_{2 r, 1 n}, \pi_{2 r, 0 n}, \pi_{2 r, 02 n}, \pi_{2 r, 0 r}, \pi_{2 r, 1 r}\right]^{\prime}$.
are implied by system (134) and $0 \leq q_{2 r}^{a} \leq \frac{3}{14,784}$. Because $\frac{3}{14,784} \approx 0$, the numerical bounds on $\boldsymbol{\pi} \equiv\left[\pi_{0 n, 1 r}, \pi_{0 r, 0 n}, \pi_{2 n, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 1 r}, \pi_{0 r, 1 n}, \pi_{1 n, 1 r}, \pi_{0 r, 2 u}, \pi_{2 u, 1 r}\right]^{\prime}$ are indistinguishable from those obtained when A. 6 is maintained.

## 9 Extended Model with FS and Taxes

We begin with some additional notation and definitions that supersede those from Section 4 in this Appendix. All lemmas, corollaries, and propositions supersede those from Section 4 in this Appendix.

## Notation, Definitions, and Assumptions

Notation 1 (Policy Regimes). Throughout, we use $a$ to refer to the JF reform's control welfare and FS policy and $j$ to refer to JF reform's experimental welfare and FS policy. The policy regime is denoted by $t \in\{a, j\}$. The assistance program mix is denoted by $m \in\{w, f, w f\}$ where " $w$ " refers to welfare only, " $f$ " refers to FS only, and " $w f$ " refers to welfare joint with FS.

Definition 15 (Program Participation, Earnings and Reported Earnings). Let $D^{f}, D^{w}$, and $D^{w f}$ be indicators for a woman participating in, respectively, FS only, welfare only, and both FS and welfare; $D^{f}, D^{w}$, and $D^{w f}$ take values in $\{0,1\}$. These program participation alternatives are mutually exclusive: $D^{f}+D^{w}+D^{w f} \in\{0,1\}$. Let $\mathbf{D} \equiv\left(D^{w}, D^{f}, D^{w f}\right)$. Let $E$ denote a woman's earnings. Earnings are the product of hours worked, $H$, and an hourly wage rate $W$. Let $E^{r}$ denote earnings reported to the relevant assistance agency. Let $R \equiv R\left(\mathbf{D}, E^{r}\right)=\mathbf{1}\left[E^{r}=0\right]\left(D^{w}+D^{w f}\right)$ be an indicator for zero reported earnings by a welfare recipient.

Definition 16 (Transfer and Tax Functions). Throughout, we use $G^{t}(),. F^{t}($.$) , and T($.$) to$ refer to, respectively, the welfare transfer function, the FS transfer function, and the federal income tax function (inclusive of the EITC). These functions are defined as follows.

1. Welfare Transfer Functions. For any reported earning level $E^{r}$, the regime-dependent welfare transfers are

$$
\begin{align*}
G_{i}^{a}\left(E^{r}\right) & =\max \left\{\bar{G}_{i}-\mathbf{1}\left[E^{r}>\delta_{i}\right]\left(E^{r}-\delta_{i}\right) \tau_{i}, 0\right\},  \tag{136}\\
G_{i}^{j}\left(E^{r}\right) & =\mathbf{1}\left[E^{r} \leq F P L_{i}\right] \bar{G}_{i} . \tag{137}
\end{align*}
$$

The parameter $\delta_{i} \in\{90,120\}$ gives woman $i$ 's fixed disregard and the parameter $\tau_{i} \in\{.49, .73\}$ governs her proportional disregard. $\bar{G}_{i}$ and $F P L_{i}$ vary across women due to differences in AU size. Define woman $i$ 's break-even earnings level under a as $\bar{E}_{i} \equiv \bar{G}_{i} / \tau_{i}+\delta_{i}$, this is the level at which welfare benefits are exhausted.

## 2. Food Stamps (FS) Transfer Functions.

For any reported earning level $E^{r}$, the regime-dependent FS transfers are:

$$
\begin{align*}
F_{i}^{a}\left(E^{r}\right) & =F_{i}\left(E^{r}, 0\right),  \tag{138}\\
F_{i}^{j}\left(E^{r}\right) & =F_{i}\left(E^{r}, 0\right),  \tag{139}\\
F_{i}^{a, w f}\left(E^{r}\right) & =F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right) \mathbf{1}\left[G_{i}^{a}\left(E^{r}\right)>0\right],  \tag{140}\\
F_{i}^{j, w f}\left(E^{r}\right) & =F_{i}\left(0, \bar{G}_{i}\right) \mathbf{1}\left[E^{r} \leq F P L_{i}\right], \tag{141}
\end{align*}
$$

where $F_{i}(\cdot, \cdot)$ is the standard FS formula, as described next. Let $\mathbf{1}\left[\right.$ elig $\left.g_{i}\right]$ denote the eligibility for FS. Then, for any pair of reported earnings and welfare transfer, denoted ( $E^{r}, G$ ), the FS transfer is:

$$
\begin{equation*}
F_{i}\left(E^{r}, G\right)=\max \left\{\overline{\bar{F}}_{i}-\tau_{1}^{f} \chi_{i}\left(E^{r}, G\right), 0\right\} \mathbf{1}\left[e l i g_{i}\right] \tag{142}
\end{equation*}
$$

with

$$
\begin{equation*}
\chi_{i}\left(E^{r}, G\right) \equiv \max \left\{E^{r}+G-\tau_{2}^{f} \min \left\{E^{r}, F P L_{i}\right\}-\beta_{1 i}^{f}-\beta_{2 i}^{f}\left(E^{r}, G\right), 0\right\} \tag{143}
\end{equation*}
$$

where $\overline{\bar{F}}_{i}$ is the maximum FS transfer; $\tau_{1}^{f} \chi_{i}\left(E^{r}, G\right)$ is a the net income deduction; $\tau_{2}^{f}$ is the earned income deduction rate; $\beta_{1 i}^{f}$ is the sum of the per unit standard deduction, the medical deduction, the child support deduction, and the dependent care deduction; and $\beta_{2 i}^{f}\left(E^{r}, G\right)$ is the excess shelter deduction as a function of earnings plus the welfare transfer. The variation in $\left(\beta_{1 i}^{f}, \beta_{2 i}^{f}().\right)$ across women with the same earnings and welfare transfer is due to differences in actual medical, shelter, and child care expenses. The variation in $F_{i}(.,$.$) across women$ is due to differences in AU size. To simplify notation let $\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$. We remark that $\overline{\bar{F}}_{i} \equiv F_{i}(0,0)$. The eligibility indicator $\mathbf{1}\left[\right.$ elig $\left.g_{i}\right]$ reflects categorical eligibility, when FS is taken up jointly with welfare, or the FS's gross and net income tests, when FS is taken up alone:

$$
\mathbf{1}\left[e l i g_{i}\right]=\left\{\begin{array}{ll}
1 & \text { if } G>0  \tag{144}\\
\mathbf{1}\left[E^{r} \leq \tau_{3}^{f} F P L_{i}\right] \mathbf{1}\left[\chi_{i}\left(E^{r}, 0\right) \leq F P L_{i}\right] & \text { if } G=0
\end{array},\right.
$$

where $\tau_{3}^{f}$ is a multiplier factor. The parameters $\left(\tau_{1}^{f}, \tau_{2}^{f}, \tau_{3}^{f}\right)$ take values $(0.30,0.20,1.3) .{ }^{4}$

## 3. Earned Income Tax Credit (EITC) and Federal Income Tax Functions.

For any earning level $E$, earnings inclusive of one-twelfth of the total annual EITC credit, net of federal (gross) income taxes (with head of household filing status) and net of payroll and medicare taxes are given by:

$$
T_{i}(E) \equiv E-I_{i}(E)-E I T C_{i}(E)-\left(\tau^{l}+\tau^{m}\right) E .
$$

The parameters $\left(\tau^{l}, \tau^{m}\right)=(0.062,0.0145)$ give the payroll and medicare tax rates, $I_{i}(E)$ is amount of (gross) federal income taxes, and $E I T C_{i}(E)$ is the amount of the earned income tax credit. Specifically, the earned income tax function $E I T C_{i}(\cdot)$ is given by ${ }^{5}$

$$
\begin{aligned}
\operatorname{EITC}_{i}(E)= & \tau_{1 i}^{e} E 1\left[0<E \leq \bar{E}_{1 i}^{e}\right]+\tau_{1 i}^{e} \bar{E}_{1 i}^{e} 1\left[\bar{E}_{1 i}^{e}<E \leq \bar{E}_{2 i}^{e}\right]+ \\
& \left(\tau_{1 i}^{e} \bar{E}_{1 i}^{e}-\tau_{2 i}^{e}\left(E-\bar{E}_{2 i}^{e}\right)\right) 1\left[\bar{E}_{2 i}^{e}<E \leq \bar{E}_{2 i}^{e}+\frac{\tau_{1 i}^{e}}{\tau_{2 i}^{e}} \bar{E}_{1 i}^{e}\right] .
\end{aligned}
$$

The parameters $\left(\tau_{1 i}^{e}, \tau_{2 i}^{e}\right)$ give a woman $i$ 's phase-in and phase-out rates. The parameters $\left(\bar{E}_{1 i}^{e}, \bar{E}_{2 i}^{e}\right)$ give a woman $i$ 's earning thresholds defining the earnings region yielding maximum credit. Both sets of parameters vary across women due to differences in the number of children. The (gross) federal income tax function $I_{i}(\cdot)$ is given by ${ }^{6}$

$$
I_{i}(E)=\sum_{k=1}^{5} \tau_{k}^{I} \max \left\{\min \left\{Y_{i}^{I}-y_{k-1}^{I}, y_{k}^{I}-y_{k-1}^{I}\right\}, 0\right\}
$$

[^3]where $Y_{i}^{I}$ is the woman's taxable income which is given by her earnings net of the personal exemption and of the standard deduction: $Y_{i}^{I}=E-D_{1 i}^{I}-D_{2}^{I}$. The personal exemption $D_{1 i}^{I}$ varies across women due to differences in the number of children. The parameters $\left(\tau_{1}^{I}, \tau_{2}^{I}, \tau_{3}^{I}, \tau_{4}^{I}, \tau_{5}^{I}\right)$ give the marginal tax rates and the parameters $\left(y_{0}^{I}, y_{1}^{I}, y_{2}^{I}, y_{3}^{I}, y_{4}^{I}, y_{5}^{I}\right)$ give the tax brackets with $y_{0}^{I} \equiv 0$ and $y_{5}^{I} \equiv \infty$.

Definition 17 (Consumption Equivalent). Consider a tuple ( $E, \mathbf{D}, E^{r}$ ). Under regime $t$, woman $i$ 's consumption equivalent corresponding to $\left(E, \mathbf{D}, E^{r}\right)$ is

$$
\begin{align*}
C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right) \equiv & T_{i}(E)+  \tag{145}\\
& \left(G_{i}^{t}\left(E^{r}\right)+F_{i}^{t, w f}\left(E^{r}\right)-\gamma_{i} 1\left[E<E^{r}\right]\right) D^{w f}+ \\
& \left(F_{i}^{t}\left(E^{r}\right)-\omega_{i} 1\left[E<E^{r}\right]\right) D^{f}+ \\
& \left(G_{i}^{t}\left(E^{r}\right)-\kappa_{i} 1\left[E<E^{r}\right]\right) D^{w} .
\end{align*}
$$

The parameters ( $\kappa_{i}, \omega_{i}, \gamma_{i}$ ) are the costs of under-reporting earnings. For simplicity, we refer to $C_{i}^{t}=$ $C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)$ as consumption. Below, when the consumption associated with a triple ( $E, \mathbf{D}, E^{r}$ ) and calculated according to (145) does not vary across regimes we omit the superscript $t$, and we omit the subscript $i$ when it does not vary across women.

Definition 18 (State). Consider a tuple ( $E, \mathbf{D}, E^{r}$ ). The "state" corresponding to ( $E, \mathbf{D}, E^{r}$ ) is defined by the function:

$$
s\left(E, \mathbf{D}, E^{r}\right)=\left\{\begin{array}{ll}
0 n n & \text { if } E=0, \mathbf{D}=\mathbf{0}, \\
1 n n & \text { if } E \text { in range } 1, \mathbf{D}=\mathbf{0}, \\
2 n n & \text { if } E \text { in range } 2, \mathbf{D}=\mathbf{0}, \\
0 n r & \text { if } E=0, D^{f}=1, \\
1 n r & \text { if } E \text { in range } 1, D^{f}=1, E^{r}=E, \\
2 n r & \text { if } E \text { in range } 2, D^{f}=1, E^{r}=E, \\
1 n u & \text { if } E \text { in range } 1, D^{f}=1, E^{r}<E, \\
2 n u & \text { if } E \text { in range } 2, D^{f}=1, E^{r}<E, \\
0 r n & \text { if } E=0, D^{w}=1, \\
1 r n & \text { if } E \text { in range } 1, D^{w}=1, E^{r}=E, \\
2 r n & \text { if } E \text { in range } 2, D^{w}=1, E^{r}=E, \\
1 u n & \text { if } E \text { in range } 1, D^{w}=1, E^{r}<E, \\
2 u n & \text { if } E \text { in range } 2, D^{w}=1, E^{r}<E, \\
0 r r & \text { if } E=0, D^{w f}=1, \\
1 r r & \text { if } E \text { in range } 1, D^{w f}=1, E^{r}=E, \\
2 r r & \text { if } E \text { in range } 2, D^{w f}=1, E^{r}=E, \\
1 u u & \text { if } E \text { in range } 1, D^{w f}=1, E^{r}<E, \\
2 u u & \text { if } E \text { in range } 2, D^{w f}=1, E^{r}<E
\end{array} .\right.
$$

Remark 7 (State: Excluded States). In Connecticut welfare and FS assistance programs are managed by the same agency. Accordingly, we do not include states $\{1 u r, 1 r u, 2 u r, 2 r u\}$ because it is not possible to make different earning reports to the same agency. Also, we do not include states $\{0 u n, 0 n u, 0 u u\}$ because it is not possible to under-report zero earnings.

Definition 19 (Job Offers). As in Definition 6.
Definition 20 (Alternative). An alternative is a wage, hours of work, program participation indicators, and earning report tuple ( $W, H, \mathbf{D}, E^{r}$ ).

Definition 21 (Sub-alternative). A sub-alternative is a wage, hours of work, and program participation indicators tuple ( $W, H, \mathbf{D}$ ).

Definition 22 (Alternative Compatible with a State). We say that an alternative ( $W, H, \mathbf{D}, E^{r}$ ) is compatible with state $s$ for woman $i$, if letting $E \equiv W H, s=s\left(E, \mathbf{D}, E^{r}\right)$.
Definition 23 (Alternative Compatible with a State and Available). We say that an alternative $\left(W, H, \mathbf{D}, E^{r}\right)$ is compatible with state $s$ and available for woman $i$ if $\left(W, H, \mathbf{D}, E^{r}\right)$ is compatible with state $s$ and $(W, H) \in \Theta_{i} \cup(0,0)$.

Definition 24 (Utility Function). Define $U_{i}^{t}(H, C, \mathbf{D}, R)$ as the utility woman $i$ derives from the tuple $(H, C, \mathbf{D}, R)$ under regime $t \in\{a, j\}$. Below, when the utility of a tuple $(H, C, \mathbf{D}, R)$ does not vary across policy regimes we omit the superscript $t$.

Definition 25 (Attractiveness of States). We say that a state $s$ is:

1. no better under regime $j$ than under regime $a$ if, for any alternative ( $W, H, \mathbf{D}, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \leq U_{i}^{a}\left(H, C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \text { all } i .
$$

2. no worse under regime $j$ than under regime $a$ if, for any alternative ( $W, H, \mathbf{D}, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \geq U_{i}^{a}\left(H, C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \text { all } i .
$$

3. We say that a state $s$ is equally attractive under regimes $j$ and $a$ if, for any alternative $\left(W, H, \mathbf{D}, E^{r}\right)$ compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right)=U_{i}^{a}\left(H, C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \text { all } i .
$$

Definition 26 (Collections of States). Define

$$
\begin{aligned}
\mathcal{S} & \equiv\{0 n n, 1 n n, 2 n n, 0 n r, 1 n r, 2 n r, 1 n u, 2 n u, 0 r r, 1 r r, 0 r n, 1 r n, 1 u n, 2 u n, 1 u u, 2 u u\}, \\
\mathcal{C}_{0} & \equiv\{0 n n, 1 n n, 2 n n, 0 n r, 1 n r, 2 n r, 1 n u, 2 n u, 1 u u, 2 u u, 1 u n, 2 u n\}, \\
\mathcal{C}_{+} & \equiv\{1 r r, 1 r n\}, \\
\mathcal{C}_{-} & \equiv\{0 r r, 0 r n\}
\end{aligned}
$$

Definition 27 (Welfare Participation State). Let $\mathcal{S}_{w} \equiv\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}$. $\mathcal{S}_{w}$ is the list of latent states that spell out welfare participation only. The states in $\mathcal{S}_{w}$ relate to the states in $\mathcal{S}$ as follows:

$$
s_{w}=h(s)=\left\{\begin{array}{ll}
0 n & \text { if } s \in\{0 n n, 0 n r\} \\
1 n & \text { if } s \in\{1 n n, 1 n r, 1 n u\} \\
2 n & \text { if } s \in\{2 n n, 2 n r, 2 n u\} \\
0 r & \text { if } s \in\{0 r n, 0 r r\} \\
1 r & \text { if } s \in\{1 r n, 1 r r\} \\
1 u & \text { if } s \in\{1 u n, 1 u u\} \\
2 u & \text { if } s \in\{2 u n, 2 u u\}
\end{array},\right.
$$

where the number of each state $s_{w}$ refers to the woman's earnings range, the letter " $n$ " refers to welfare non-participation, the letter " $r$ " refers to welfare participation with truthful reporting of earnings, and the letter " $u$ " refers to welfare participation with under-reporting of earnings.

Definition 28 (Primitives). Let woman $i$ be described by

$$
\theta_{i} \equiv\left(U_{i}^{a}(., ., ., .), U_{i}^{j}(., ., ., .), \kappa_{i}, \omega_{i}, \gamma_{i}, \Theta_{i}, G_{i}^{a}(.), F_{i}(., .), T_{i}(.)\right) .
$$

Consider a sample of $N$ women with children. The sample women have primitives $\left\{\theta_{i}\right\}_{i=1}^{N}$, which are i.i.d. draws from a joint distribution function $\Gamma_{\theta}$ (.).

Definition 29 (Response Probabilities). Let $S_{i}^{t}$ denote woman $i$ 's potential state under regime $t \in\{a, j\}$. Define the proportion of women occupying state $s \in \mathcal{S}$ under regime $t$ as $q_{s}^{t} \equiv P\left(S_{i}^{t}=s\right)$ where $P($.$) is a probability measure induced by the distribution function \Gamma_{\theta}($.$) . Let \pi_{s^{a}, s^{j}}$ denote the proportion of women occupying state $s^{j}$ under regime $j$ among those who occupy state $s^{a}$ under regime $a$, that is, $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$ where $P($.$) is also a probability measure induced$ by the distribution function $\Gamma_{\theta}($.$) .$

Definition 30 (Integrated Response Probabilities). Let $S_{w, i}^{t}$ denote the welfare-only potential state of a woman $i$ whose potential state under regime $t$ is $S_{i}^{t}$; that is, $S_{w, i}^{t}=h\left(S_{i}^{t}\right)$. Define the proportion of women occupying state $s_{w} \in \mathcal{S}_{w}$ under regime $t$ as $p_{s_{w}}^{t} \equiv P\left(S_{w, i}^{t}=s_{w}\right)=$ $\sum_{s \in \mathcal{S}: s_{w}=h(s)} q_{s}^{t}$ where $P($.$) is a probability measure induced by the distribution function \Gamma_{\theta}$ (.). With some abuse of notation (see Definition 29), let $\pi_{s_{w}^{a}, s_{w}^{j}}$ denote the proportion of women who occupy state $s_{w}^{j}$ under regime $j$ among those who occupy state $s_{w}^{a}$ under regime $a$; that is, $\pi_{s_{w}^{a}, s_{w}^{j}} \equiv$ $P\left(S_{w, i}^{j}=s_{w}^{j} \mid S_{w, i}^{a}=s_{w}^{a}\right)$ where $P($.$) is also a probability measure induced by the distribution$ function $\Gamma_{\theta}($.$) .$

Assumption 6 (Preferences). Woman $i$ 's utility functions $U_{i}^{a}(\cdot, \cdot, \cdot, \cdot)$ and $U_{i}^{j}(\cdot, \cdot, \cdot, \cdot)$ satisfy the restrictions:
A. 1 utility is strictly increasing in $C$;
A. $2 U_{i}^{t}(H, C, \mathbf{D}, 1) \leq U_{i}^{t}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D})$ such that $D^{w}+D^{w f}=1$ and all $t \in\{a, j\}$; and $U_{i}^{t}(H, C, \mathbf{D}, 1)=U_{i}^{t}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D})$ such that $D^{f}=1$ and all $t \in\{a, j\}$;
A. $3 \quad U_{i}^{j}(H, C, \mathbf{D}, 1) \leq U_{i}^{a}(H, C, \mathbf{D}, 1)$ for all $(H, C, \mathbf{D})$ such that $D^{w}+D^{w f}=1$;
A. $4 \quad U_{i}^{a}(H, C, \mathbf{D}, 0)=U_{i}^{j}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D}, 0)$ such that $D^{w}+D^{f}+D^{w f}=1$ and $H>0$;
A. $5 \quad U_{i}^{a}(H, C, \mathbf{D}, 0)=U_{i}^{j}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D})$ such that $D^{w}+D^{f}+D^{w f}=0$ and all $t \in\{a, j\}$;
A. $6 \quad U_{i}^{a}\left(H, C_{i}^{a}(E, \mathbf{D}, E), \mathbf{D}, 0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, \mathbf{0}, E), \mathbf{0}, 0\right)$ for all $(H, W)$ such that $E \equiv$ $W H \in\left(F P L_{i}, \bar{E}_{i}\right]$ and $D^{w}+D^{w f}=1$ whenever $\bar{E}_{i}>F P L_{i}$.

Remark 8 (Preferences: Verbalizing Assumption 6). A. 2 reporting zero earnings to the welfare agency yields a hassle disutility, while reporting zero earnings to the FS agency yields no hassle disutility. A. 3 states that regime $j$ 's welfare hassle disutility is no smaller than regime $a$ 's welfare hassle disutility. A. 4 states that the utility value of alternatives entailing FS-only participation is independent of the regime. It also states that utility value of alternatives entailing welfare-only participation, or FS and welfare participation, is independent of the regime whenever reported
earnings are not zero. A. 5 states that the utility value of alternatives entailing no participation in assistance programs is independent of the regime. A. 6 states that under regime $a$ the stigma disutility associated with welfare assistance (alone or in combination with FS) is bounded from below. That is, under $a$ and at earnings levels above $F P L_{i}$, the extra consumption due to the transfer income does not suffice to compensate the woman for the welfare stigma disutility she incurs when on welfare assistance, irrespective of program mix.

Assumption 7 (Under-reporting Earning Penalties). For each woman $i,\left(\kappa_{i}, \omega_{i}, \gamma_{i}\right)>0$.
Assumption 8 (Welfare-Ineligible Earning Levels). No woman may be on welfare assistance and truthfully report earnings above $F P L_{i}$ under regime $j$ or above $\bar{E}_{i}$ under regime a.

Assumption 9 (Utility Maximization). Under regime $t$ woman $i$ makes choices by solving the optimization problem

$$
\max _{(W, H) \in \Theta_{i} \cup(0,0), \mathbf{D} \in\{0,1\}^{3}, D^{f}+D^{w}+D^{w f} \leq 1, E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right)
$$

Assumption 10 (Population Heterogeneity). The distribution $\Gamma_{\theta}$ (.) is unrestricted save for the constraints implied by Assumptions 6-9 and the definition of wage offers (Definition 19).

Assumption 11 (Breaking Indifference). Women break indifference in favor of the same alternative irrespective of the regime.

Assumption 12 (Filing Taxes). A woman files (does not file) for federal income taxes and the EITC irrespective of the regime.

## Intermediate Lemmas

Lemma 6 (Combined Transfer). Under both regimes $j$ and $a$, for every $E^{r}$ such that $G_{i}^{t}\left(E^{r}\right)>$ 0 , the combined welfare plus FS transfer is no smaller than the sole welfare transfer or the sole FS transfer.

Proof. The proof that the combined welfare plus FS transfer is no smaller than the sole welfare transfer is trivial: the FS program has no feed-backs on the welfare program (Definition 16, expressions (136)-(137)) and the FS transfer cannot be negative (Definition 16, expression (142)). The proof that the combined welfare plus FS transfer is no smaller than the sole FS transfer is less obvious because the FS transfer is decreasing in the welfare grant which is counted as income (Definition 16, expressions (142)-(143)). Nevertheless, the FS formula in (142) shows that a $\$ 1$ increase in the welfare grant $(G)$ leads to a less than $\$ 1$ decrease in the FS transfer because $\tau_{1}^{f}<1$ and welfare assistance yields categorical FS eligibility (expression (144)). Thus, a woman whose earnings report makes her eligible for welfare can enjoy a higher transfer income by taking up both welfare and FS as opposed to taking up only FS.

Lemma 7 (Combined Transfer as a Function of Reported Earnings). Under regime a, the combined welfare plus FS transfer is weakly decreasing in reported earnings.

Proof. For any reported earning $E^{r}$, the combined transfer accruing to woman $i$ is a function $B($. defined by $B\left(E^{r}\right) \equiv G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$. Observe: 1) given $E^{r}$, the function $G+F_{i}\left(E^{r}, G\right)$ is weakly increasing in $G$ because a $\$ 1$ increase in the welfare grant $(G)$ leads to a less than $\$ 1$ decrease in the FS transfer due to $\tau_{1}^{f}<1$ (expression 142); 2) $G_{i}^{a}\left(E^{r}\right)$ is weakly decreasing in
$E^{r}$ (expression (136)); 3) given $G, F_{i}\left(E^{r}, G\right)$ is weakly decreasing in $E^{r}$ (expressions (142)-(143)). Together these facts imply that $B($.$) is a weakly decreasing function. { }^{7}$

Lemma 8 (States 2rr and 2rn). Given Assumptions 6, 8, and 9, no woman selects an allocation compatible with states $2 r r$ and $2 r n$.

Proof. Under regime $j$ no alternative is compatible with states $2 r n$ and $2 r r$ by Assumption 8. Consider now a woman with $\bar{E}_{i} \leq F P L_{i}$ under regime $a$. By Assumption 8 she may not be on assistance and truthfully report earnings above $F P L_{i}$ (range 2). Finally, consider a woman with $\bar{E}_{i}>F P L_{i}$ under regime $a$. By Assumption 8 she may not be on assistance and truthfully report earnings above $\bar{E}_{i}$. By A. 6 in Assumption 6 she will not truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ because she can attain a higher utility level by being off welfare assistance (Assumption 9): the extra consumption due to the transfer income does not suffice to compensate the woman for the stigma disutility she incurs when being on welfare assistance.

Lemma 9 (Optimal Reporting). Write woman $i$ 's optimization problem (Assumption 9) as a nested maximization problem:

$$
\begin{equation*}
\max _{(W, H) \in \Theta_{i} \cup(0,0), \mathbf{D} \in\{0,1\}^{3}, D^{f}+D^{w}+D^{w f} \leq 1}\left[\max _{E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right)\right] \tag{146}
\end{equation*}
$$

Focus on the inner maximization problem in (146) for given sub-alternative $(W, H, \boldsymbol{D})$ with $D^{m}=1$ for $m \in\{f, w, w f\}$. Let $E \equiv W H$ and $E_{i}^{r, t, m}=E_{i}^{r, t, m}(W, H)$ denote woman $i$ 's utility maximizing earning report conditional on sub-alternative $(W, H, \boldsymbol{D})$ with $D^{m}=1$. Given Assumptions 6-12, optimal reporting while on assistance is as follows:

## 1. Welfare Only

(a) $E_{i}^{r, j, w}$ entails either truthful reporting, that $i s, E_{i}^{r, j, w}=E$, or under-reporting such that $E>E_{i}^{r, j, w} \in\left[0, F P L_{i}\right]$; in particular, state 1 un is dominated;
(b) $E_{i}^{r, a, w}$ entails either truthful reporting, that is, $E_{i}^{r, a, w}=E$, or under-reporting such that $E>E_{i}^{r, a, w} \in\left[0, \delta_{i}\right] ;$

## 2. FS Only

For any $t \in\{a, j\}, E_{i}^{r, t, f}$ entails either truthful reporting, that is, $E_{i}^{r, t, f}=E$, or underreporting such that $E>E_{i}^{r, t, f} \in\left[0, \min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}\right]$ where $\underline{E}_{i}^{f}$ is the highest level of reported earnings such that the FS transfer is unreduced and $\bar{E}_{i}^{f}$ is the highest level of reported earnings such the FS's eligibility tests are satisfied;

## 3. Welfare and FS

(a) $E_{i}^{r, j, w f}$ entails either truthful reporting, that is, $E_{i}^{r, j, w f}=E$, or under-reporting such that $E>E_{i}^{r, j, w f} \in\left[0, F P L_{i}\right]$; in particular, state $1 u u$ is dominated;

[^4](b) $E_{i}^{r, a, w f}$ entails either truthful reporting, that is, $E_{i}^{r, a, w f}=E$, or under-reporting such that $E>E_{i}^{r, a, w f}=1$ cent, or under-reporting such that $E>E_{i}^{r, a, w f} \in\left[0, \underline{E}_{i}^{w f}\right]$ where $\underline{E}_{i}^{w f}$ is the largest level of reported earnings in $\left[0, \delta_{i}\right]$ such that the corresponding FS transfer is $\bar{F}_{i}$, that is $\chi_{i}\left(\underline{E}_{i}^{w f}, \bar{G}_{i}\right)=0$ (Definition 143), or, if no such earning level exists in $\left[0, \delta_{i}\right], \underline{E}_{i}^{w f}=0$.

Proof. We prove each part of the Lemma in turn.

## 1. Welfare Only

The proofs of statements a.) and b.) mimic the proof of Lemma 2 with the appropriate adjustments in notation, namely, with $\mathbf{D}=(1,0,0)$ in place of $D=1, E_{i}^{r, t, w}$ in place of $E_{i}^{r, t}$ for $t \in\{a, j\}$ and the references to A. 2 in Assumption 6 in place of the references to A. 2 in Assumption 1.

## 2. FS Only

The stand-alone FS program rules are invariant to the regime (Definition 16). The utility associated with any alternative compatible with stand-alone FS assistance is also regime invariant (A. 4 in Assumption 6). Thus, the reported earning level that solves the inner maximization problem in (146) is the same for all $t \in\{a, j\}$ and is that which maximizes consumption. To find such level we make three preliminary observations. First, we observe that the threshold level $\underline{E}_{i}^{f}$ is strictly positive for all $i$. To see this consider a woman $i$ who enjoys no deductions other than the standard deduction, namely, $\beta_{1 i}^{f}=\$ 134$ and $\beta_{2 i}^{f}(0,0)=0$. Then, $\chi_{i}\left(E^{r}, 0\right)=E^{r}\left(1-\tau_{2}^{f}\right)-\$ 134$ (expression (143)), hence any report $E^{r} \leq \$ 134 /\left(1-\tau_{2}^{f}\right)=\$ 167.5$ yields her a FS transfer in the (maximal) amount $\overline{\bar{F}}_{i}$. A woman with deductions other than the standard deduction enjoys an even higher threshold level $\underline{E}_{i}^{f}$. Second, we observe that the threshold level $\bar{E}_{i}^{f}$ is also strictly positive for all i. To see this observe that $\bar{E}_{i}^{f}$ is the smallest level of earnings that engenders ineligibility, formally, $\bar{E}_{i}^{f}=\min \left\{\tau_{3}^{f} F P L_{i}, E^{\prime}\right\}$ where $E^{\prime}$ is such that $\chi_{i}\left(E^{\prime}, 0\right)=F P L_{i}$. Third, we observe that the threshold level $\underline{E}_{i}^{f}$ for a woman with very high deductions may be higher than $\bar{E}_{i}^{f}$. Given $E$, any report $E^{r} \in\left[0, \min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}\right]$ yields the same (maximal) transfer $\overline{\bar{F}}_{i}$ hence woman $i$ enjoys consumption in the amount $T_{i}(E)+\overline{\bar{F}}_{i}-\omega_{i} \mathbf{1}\left[E<E^{r}\right]$. A report $E^{r}>\min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}$ yields transfer $F_{i}\left(E^{r}, 0\right)$ hence woman $i$ enjoys consumption in the amount $T_{i}(E)+F_{i}\left(E^{r}, 0\right)-\omega_{i} \mathbf{1}\left[E<E^{r}\right]$. Because $F_{i}\left(E^{r}, 0\right)<\overline{\bar{F}}_{i}$ for all $E^{r}>\min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}$ and $\omega_{i}>0$ (Assumption 7), and depending on the magnitude of woman $i$ 's under-reporting $\operatorname{cost}\left(\omega_{i}\right)$ and earning level $E$, either reporting $E_{i}^{r, t, f} \in\left[0, \min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}\right]$ or truthful reporting, i.e. $E_{i}^{r, t, f}=E$, maximizes consumption hence solves the inner maximization problem in (146) for all $t \in\{a, j\}$.

## 3. Welfare and FS

The proof of statement a.) mimics the proof of Lemma 2.I.) with the appropriate adjustments in notation, namely, with $\mathbf{D}=(0,0,1)$ in place of $D=1, E_{i}^{r, j, w f}$ in place of $E_{i}^{r, j}$, the references A. 2 and A. 4 in Assumption 6 in place of the references to A. 2 and A. 4 in Assumption 1, and the expressions for consumption equal to $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}$ in place of $T_{i}(E)+\bar{G}_{i}-\kappa_{i}$
(under-reporting) and $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}$ in place of $E+\bar{G}_{i}$ (truthful reporting). In particular, state $1 u u$ is dominated in the extended model because under regime $j$ earnings up to $F P L_{i}$ are fully disregarded in the determination of the combined welfare plus FS transfer (Definition 16, expression 141). Next we prove statement b.).
Consider first a woman $i$ who derives no disutility from hassle under regime $a$ (A. 2 in Assumption 6 holds as an equality). Thus, the utility associated with any alternative compatible with welfare plus FS assistance is regime invariant hence the reported earning level that solves the inner maximization problem in (146) is that which maximizes consumption. Reporting $E^{r} \in\left[0, \underline{E}_{i}^{w f}\right]$ yields woman $i$ the maximal combined transfer $\bar{G}_{i}+\bar{F}_{i}$ with implied consumption $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i} \mathbf{1}\left[E<E^{r}\right]$. A report $E^{r}>\underline{E}_{i}^{w f}$ yields a lower transfer (Lemma 7) with implied consumption $T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i} \mathbf{1}\left[E<E^{r}\right]$. Thus, depending on the magnitude of woman $i$ 's under-reporting cost $\left(\gamma_{i}\right)$ and earning level $E$, either report$\operatorname{ing} E_{i}^{r, a, w f} \in\left[0, \underline{E}_{i}^{w f}\right]$ or truthful reporting, i.e. $E_{i}^{r, a, w f}=E$, solves the inner maximization problem in (146).
Next, consider a woman $i$ who derives some disutility from hassle under regime $a$ (A. $\mathbf{2}$ in Assumption 6 holds as a strict inequality) and such that $\underline{E}_{i}^{w f} \in\left(0, \delta_{i}\right]$. Reporting $E^{r} \in$ $\left(0, \underline{E}_{i}^{w f}\right]$ yields her the maximal combined transfer $\bar{G}_{i}+\bar{F}_{i}$ while higher reports yield a lower transfer (Lemma 7). Depending on the magnitude of woman $i$ 's under-reporting cost and earning level $E$, either reporting $E_{i}^{r, a, w f} \in\left(0, \underline{E}_{i}^{w f}\right]$ or truthful reporting, i.e. $E_{i}^{r, a, w f}=E$, solves the inner maximization problem in equation (146). To show this we next consider $\mathbf{D}=$ $(0,0,1)$ and five mutually exclusive pairs ( $W, H$ ) spanning the range of value for $E \equiv W H$. For convenience, we let $U_{i}^{t}$ serve as shortcut notation for $U_{i}^{t}\left(H, C_{i}^{t}\left(E, \boldsymbol{D}, E^{r}\right), \boldsymbol{D}, R\left(\boldsymbol{D}, E^{r}\right)\right)$. Let $(W, H)$ be:
(a) such that $E=0$.

Woman $i$ 's cannot over-report her earnings (Assumption 146). Thus, $E_{i}^{r, a, w f}=E$.
(b) $(W, H)$ such that $E \in\left(0, \underline{E}_{i}^{w f}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in(0, E) \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have [1] $<[2]$. By $\gamma_{i}>0$ (Assumption 7) we have [2] < [3]. Thus, truthful reporting solves the inner maximization problem (146), that is, $E_{i}^{r, a, w f}=E$.
(c) $(W, H)$ such that $E \in\left(\underline{E}_{i}^{w f}, \delta_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \underline{E}_{i}^{w f}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\underline{E}_{i}^{w f}, E\right) \\
{[4]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have $[1]<[2]$. Because the FS transfer $F_{i}\left(E^{r}, \bar{G}_{i}\right)$ is strictly decreasing in $E^{r}, F_{i}\left(E^{r}, \bar{G}_{i}\right)<\bar{F}_{i}$, hence $[3]<[2]$. Thus,
depending on woman $i$ 's utility function, under-reporting cost $\left(\gamma_{i}\right)$, and earnings $E$, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f} \in\left(0, \underline{E}_{i}^{w f}\right]$ (when [4] $\leq[2]$ ) or by truthful reporting, $E_{i}^{r, a, w f}=E$ (when $[4] \geq[2]$ ).
(d) $(W, H)$ such that $E \in\left(\delta_{i}, F P L_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 6 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \underline{E_{i}^{w f}}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\underline{E}_{i}^{w f}, \delta_{i}\right] \\
{[4]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right) \\
{[5]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have [1] $<[2]$. The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that [4] < $[3]<[2]$ (Lemma 7). Thus, depending on woman $i$ 's utility function, under-reporting cost ( $\gamma_{i}$ ), and earnings $E$, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f} \in$ $\left(0, \underline{E}_{i}^{w f}\right]$ (when $\left.[5] \leq[2]\right)$ or by truthful reporting, $E_{i}^{r, a, w f}=E$ (when $\left.[5] \geq[2]\right)$.
(e) $(W, H)$ such that $E>F P L_{i}$.

Woman $i$ must be under-reporting. Her utility while on welfare and FS depends on reported earnings as follows (A. 6 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, E_{i}^{w f}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(E_{i}^{w f}, \delta_{i}\right] \\
{[4]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right)
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have [1] $<[2]$. The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that $[4]<$ $[3]<[2]$ (Lemma 7). Thus, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f} \in\left(0, \underline{E}_{i}^{w f}\right]$.

Finally, consider a woman $i$ who derives some disutility from hassle under regime $a$ (A.2 in Assumption 6 holds as a strict inequality) and such that $\underline{E}_{i}^{w f}=0$. Depending on women $i$ 's utility function (in particular her hassle disutility), under-reporting cost, and earnings $E$, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f}=1$ cent or by truthful reporting, $E_{i}^{r, a, w f}=E$. Too show this we next consider $\mathbf{D}=(0,0,1)$ and four mutually exclusive pairs $(W, H)$ spanning the range of value for $E \equiv W H$. Again, for convenience, we let $U_{i}^{t}$ serve as shortcut notation for $U_{i}^{t}\left(H, C_{i}^{t}\left(E, \boldsymbol{D}, E^{r}\right), \boldsymbol{D}, R\left(\boldsymbol{D}, E^{r}\right)\right)$. Let $(W, H)$ be:
(a) $(W, H)$ such that $E=0$.

Woman $i$ 's cannot over-report her earnings (Assumption 146). Thus, $E_{i}^{r, a, w f}=E$.
(b) $(W, H)$ such that $E \in\left(0, \delta_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in(0, E) \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

The FS transfer $F_{i}\left(E^{r}, \bar{G}_{i}\right)$ is strictly decreasing in $E^{r}$ which implies that $F_{i}\left(E^{r}, \bar{G}_{i}\right) \leq$ $\bar{F}_{i}$ for all $E^{r} \in(0, E)$ and among these reports that which yields the highest utility is $E^{r}=1$ (the smallest possible denomination). Due to rounding of the FS transfer, $F_{i}\left(1, \bar{G}_{i}\right)=\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$ hence $[1]<[2]$. Thus, whether the inner maximization problem (146) has solution $E_{i}^{r, a, w f}=1$ cent or $E_{i}^{r, a, w f}=E$ depends on whether $\bar{F}_{i}-\gamma_{i} \lessgtr$ $F_{i}\left(E, \bar{G}_{i}\right)$.
(c) $(W, H)$ such that $E \in\left(\delta_{i}, F P L_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right) \\
{[4]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that $[3]<[2]$ (Lemma 7). The FS transfer $F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is also strictly decreasing in $E^{r}$ which implies that among reports in $\left(0, \delta_{i}\right]$ that which yields the highest utility is $E^{r}=1$ cent (the smallest possible denomination). Due to rounding of the FS transfer, $F_{i}\left(1, \bar{G}_{i}\right)=\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$ hence [1] $<[2]$. Thus, whether the inner maximization problem (146) has solution $E_{i}^{r, a, w f}=1$ cent or $E_{i}^{r, a, w f}=E$ depends on whether $\bar{F}_{i}-\gamma_{i} \lessgtr F_{i}\left(E, \bar{G}_{i}\right)$.
(d) $(W, H)$ such that $E>F P L_{i}$.

Woman $i$ must be under-reporting. Her utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, F P L_{i}\right)
\end{array} .\right.
$$

The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that $[3]<[2]$ (Lemma 7). The FS transfer $F_{i}\left(E^{r}, \bar{G}_{i}\right)$ is also strictly decreasing in $E^{r}$ which implies that among reports in $\left(0, \delta_{i}\right]$ that which yields the highest utility is $E^{r}=1$ cent (the smallest possible denomination). Due to rounding of the FS transfer, $F_{i}\left(1, \bar{G}_{i}\right)=\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$ hence $[1]<[2]$. Thus, the inner maximization problem (146) has solution $E_{i}^{r, a, w f}=1$ cent.

Corollary 6 (Optimal Reporting and Policy Invariance). Given Assumptions 6-12, the utility function associated with any alternative compatible with states $\{1 u n, 2 u n, 1 u u, 2 u u\}$ and entailing optimal reporting is regime invariant.

Proof. We examine each state in turn.

1. State 1un
(a) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w}=1$, and $E_{i}^{r, j, w}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, j, w}(W, H)\right)$ is compatible with state 1 un and entails optimal reporting under regime $j$. Let $C_{i}^{j} \equiv$
$C_{i}^{j}\left(E,(1,0,0), E_{i}^{r, j, w}(W, H)\right)$ and $R_{i}^{j} \equiv R\left((1,0,0), E_{i}^{r, j, w}(W, H)\right)$. We next show that $U_{i}^{j}\left(H, C_{i}^{j},(1,0,0), R_{i}^{j}\right)=U_{i}\left(H, C_{i}^{j},(1,0,0), R_{i}^{j}\right)$. By Lemma $9, E_{i}^{r, j, w}(W, H) \in$ $\left(0, F P L_{i}\right]$ or $E_{i}^{r, j, w}(W, H) \in\left[0, F P L_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ which equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6. In the second case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ which also equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $\left.0, F P L_{i}\right]$ and reporting zero earnings, that is, $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 1\right)=U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$.
(b) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w}=1$, and $E_{i}^{r, a, w}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, a, w}(W, H)\right)$ is compatible with state $1 u n$ and entails optimal reporting under regime $a$. Let $C_{i}^{a} \equiv$ $C_{i}^{a}\left(E,(1,0,0), E_{i}^{r, a, w}(W, H)\right)$ and $R_{i}^{a} \equiv R\left((1,0,0), E_{i}^{r, a, w}(W, H)\right)$. We next show that $U_{i}^{a}\left(H, C_{i}^{a},(1,0,0), R_{i}^{a}\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), R_{i}^{a}\right)$. By Lemma $9, E_{i}^{r, a, w}(W, H) \in\left(0, \delta_{i}\right]$ or $E_{i}^{r, a, w}(W, H) \in\left[0, \delta_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ which equals
$U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6. In the second case, the utility woman $i$ enjoys is also
$U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)=U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $0, \delta_{i}$ ] and reporting zero earnings, that is, $U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$.
(c) In 1.(a) and 1.(b) we have shown that any alternative compatible with state $1 u n$ and entailing optimal reporting yields regime-invariant consumption $T_{i}(E)+\bar{G}_{i}-\kappa_{i}$ and regime-invariant utility level $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$.

## 2. State $2 u n$.

The proof that the utility associated with any alternative compatible with state $2 u n$ and entailing optimal reporting is regime invariant is the same as that for state 1 un once we let the pair $(H, W)$ be such that $E \equiv W H$ is in range 2 (Lemma 9$)$.

## 3. State $1 u u$

(a) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w f}=1$, and $E_{i}^{r, j, w f}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, j, w f}(W, H)\right)$ is compatible with state $1 u u$ and entails optimal reporting under regime $j$. Let $C_{i}^{j} \equiv$ $C_{i}^{j}\left(E,(0,0,1), E_{i}^{r, j, w f}(W, H)\right)$ and $R_{i}^{j} \equiv R\left((0,0,1), E_{i}^{r, j, w f}(W, H)\right)$. We next show that $U_{i}^{j}\left(H, C_{i}^{j},(0,0,1), R_{i}^{j}\right)=U_{i}\left(H, C_{i}^{j},(0,0,1), R_{i}^{j}\right)$. By Lemma $9, E_{i}^{r, j, w f}(W, H) \in$ $\left(0, F P L_{i}\right]$ or $E_{i}^{r, j, w f}(W, H) \in\left[0, F P L_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ which equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 4 in Assumption 6. In the second case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ which also equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $0, F P L_{i}$ ] and reporting zero earnings, that is, $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 1\right)=U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$.
(b) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w f}=1$, and $E_{i}^{r, a, w f}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, a, w f}(W, H)\right)$ is compatible with state $1 u u$ and entails optimal reporting under regime $a$. Let $C_{i}^{a} \equiv$ $C_{i}^{a}\left(E,(0,0,1), E_{i}^{r, a, w f}(W, H)\right)$ and $R_{i}^{a} \equiv R\left((0,0,1), E_{i}^{r, a, w f}(W, H)\right)$. We next show that $U_{i}^{a}\left(H, C_{i}^{a},(0,0,1), R_{i}^{a}\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), R_{i}^{a}\right)$. By Lemma $9, E_{i}^{r, a, w f}(W, H) \in$ $\left(0, \underline{E}_{i}^{w f}\right]$ or $E_{i}^{r, a, w f}(W, H) \in\left[0, \underline{E}_{i}^{w f}\right]$ or $E_{i}^{r, a, w f}(W, H)=1$ cent depending on the woman's preferences and $\underline{E}_{i}^{w f}$. In the first case, the utility woman $i$ enjoys is $U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ which equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 6 in Assumption 6 (policy invariance). In the second case, the utility woman $i$ enjoys is also
$U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)=U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $0, \underline{E}_{i}^{w f}$ ] and reporting zero earnings, that is,
$U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$. In the third case, the utility woman $i$ enjoys is also $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ because of the rounding of the FS transfer.
(c) In 3.(a) and 3.(b) we have shown that any alternative compatible with state $1 u u$ and entailing optimal reporting yields regime-invariant consumption $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}$ and regime-invariant utility level $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$.

## 4. State $2 u u$

The proof that the utility associated with any alternative compatible with state $2 u u$ and entailing optimal reporting is regime-invariant is the same as that for state $1 u u$ once we let the pair $(H, W)$ be such that $E \equiv W H$ is in range 2 (Lemma 9$)$.

Remark 9 (Optimal under-Reporting and Alternatives Considered). In what follows, when considering alternatives compatible with states $\{1 u n, 1 u u, 2 u n, 2 u u\}$, it is without loss of generality that we only focus on alternatives entailing optimal (under-) reporting. No woman would select an alternative compatible with states $\{1 u n, 1 u u, 2 u n, 2 u u\}$ not entailing optimal (under-) reporting (Assumption 9). Additionally, it is without loss of generality that we disregard alternatives compatible with states $\{1 u n, 1 u u\}$ under regime $j$. No woman would select an alternative compatible with states $\{1 u n, 1 u u\}$ under regime $j$ because they are dominated (Lemma 9, parts I and III).

Lemma 10 (Revealed Preferences). Consider any pair of states ( $s^{a}, s^{j}$ ) obeying: I) $s^{a} \neq s^{j}$; II) state $s^{a}$ is no worse under regime $j$ than under regime $a$; III) state $s^{j}$ is no better under regime $j$ than under regime $a$. Then, if Assumptions 1 and 5 hold, no woman will pair states $s^{a}$ and $s^{j}$.

Proof. The proof is by contradiction. Suppose that for some pair of states $\left(s^{a}, s^{j}\right)$ satisfying properties I)-III), woman $i$ chooses a tuple ( $H, W, \mathbf{D}, E^{r}$ ) under regime $a$ obeying $s^{a}=s\left(E, \mathbf{D}, E^{r}\right)$ with $E \equiv W H$ and a tuple ( $\left.H^{\prime}, W^{\prime}, \mathbf{D}^{\prime}, E^{r \prime}\right)$ under regime $j$ obeying $s^{j}=s\left(E^{\prime}, \mathbf{D}^{\prime}, E^{r \prime}\right)$ with $E^{\prime} \equiv W^{\prime} H^{\prime}$. For convenience, let $C_{i}^{t}=C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)$ and $C_{i}^{t \prime}=C_{i}^{t}\left(E^{\prime}, \mathbf{D}^{\prime}, E^{r \prime}\right)$. By property II $)$, optimality of the alternative compatible with state $s^{a}$ under regime $a$, and Property III):

$$
\begin{equation*}
U_{i}^{j}\left(H, C_{i}^{j}, \mathbf{D}, R\right) \geq U_{i}^{a}\left(H, C_{i}^{a}, \mathbf{D}, R\right) \geq U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, \mathbf{D}^{\prime}, R^{\prime}\right) \geq U_{i}^{j}\left(H^{\prime}, C_{i}^{j \prime}, \mathbf{D}^{\prime}, R^{\prime}\right) \tag{147}
\end{equation*}
$$

As in the proof of Lemma 4, if any of the inequalities in expression (147) is strict the contradiction ensues. If no inequality is strict, we need to consider $36=7^{2}-13$ possible situations based on the possible values of $\left(\mathbf{D}, R, \mathbf{D}^{\prime}, R^{\prime}\right)$ where we subtract 13 because $R$ is functionally related to $\boldsymbol{D}$ and $R^{\prime}$ is functionally related to $\boldsymbol{D}^{\prime}$. Each of these situations leads to a contradiction based on a woman breaking indifference between two alternatives in favor of the same alternative irrespective of the policy regime (Assumption 11) or based on violation of Property I. Specifically, in each of the following cases (147) simplifies to:

1. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
2. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$ and that the FS-only policy is invariant to the policy regime, hence $C_{i}^{a}=C_{i}^{j}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
3. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :

$$
U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)
$$

where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=$ $C_{i}^{j}=C_{i}$ and $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
4. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$
where we have used the fact that the FS-only policy is invariant to regime, hence $C_{i}^{a}=C_{i}^{j}=$ $C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
5. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C_{i}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
6. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=$ $C_{i}^{j}=C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C_{i},(0,1,0), 0$ ) and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
7. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=$ $C_{i}^{j}=C_{i}$, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)$ and $\bar{F}_{i}=F_{i}^{a}(0)$ hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
8. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
9. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$, and that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
10. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=$ $C_{i}^{j \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
11. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
12. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of ( $H, C_{i},(1,0,0), 0$ ), this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
13. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption $6)$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
14. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. The the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
15. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
16. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
17. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=$ $C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $\left.H, C_{i},(1,0,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
18. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C_{i},(1,0,0), 1$ ) and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
19. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$, and that $\bar{F}_{i}=$ $F_{i}^{a}(0)$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
20. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0):$

$$
U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and ( $\left.H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of ( $\left.H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
21. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and ( $H^{\prime}, C^{\prime}, \mathbf{0}, 0$ ) under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of ( $H^{\prime}, C^{\prime}, \mathbf{0}, 0$ ) under regime $j$.
22. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=$ $C_{i}^{j \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
23. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=$ $C_{i}^{j \prime}=C_{i}^{\prime}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
24. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
25. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6 ). Woman $i$ is thus indifferent between ( $\left.H, C_{i},(0,0,1), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
26. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C_{i},(0,0,1), 1$ ) and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
27. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
28. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
29. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=$ $C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
30. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6 ). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
31. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
32. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and the fact that the FS-only policy is invariant to the policy regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of ( $H, C, \mathbf{0}, 0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
33. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C, \mathbf{0}, 0$ ) and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
34. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and the fact that $\bar{G}_{i}=G_{i}^{a}(0)$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
35.
$(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C, \mathbf{0}, 0$ ) and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
36. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)$ and $\bar{F}_{i}=F_{i}^{a}(0)$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of ( $H, C, \mathbf{0}, 0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.

## Lemma 11 (Policy Impact on Attractiveness of States). Given Assumptions 6-12:

1. the states in $\mathcal{C}_{+}$are no worse under regime $j$ than under regime $a$,
2. the states in $\mathcal{C}_{-}$are no better under regime $j$ than under regime $a$,
3. the states in $\mathcal{C}_{0}$ are equally attractive under regimes $j$ and $a$.

Proof. We prove each statement in turn.

1. The states in $C_{+}$are no worse under $j$ than under regime $a$.

The only two states in $\mathcal{C}_{+}$are $1 r n$ and $1 r r$. The alternatives compatible with these states entail $E$ in range 1 , and, respectively, $\left(D^{w}, E^{r}\right)=(1, E)$ or $\left(D^{w f}, E^{r}\right)=(1, E)$. Thus, the utility function associated with each of these alternatives is invariant to the treatment (A. 4 in Assumption 6). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is not lower under regime $j$ than under regime $a$, that is, $C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right) \geq C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right)$ for all $\left(E, \mathbf{D}, E^{r}\right)$ such that $s\left(E, \mathbf{D}, E^{r}\right) \in \mathcal{C}_{+}$. Consider first state $1 r r \in \mathcal{C}_{+}$so that $\left(E, \mathbf{D}, E^{r}\right)=(E,(0,0,1), E)$. By Lemma 7 part 1$), \bar{G}_{i}+F_{i}\left(E, \bar{G}_{i}\right) \geq$ $G_{i}^{a}(E)+F_{i}\left(E, G_{i}^{a}(E)\right)$ for all $E$ in range $1,{ }^{8}$ thus
$C_{i}^{j}(E,(0,0,1), E)=T_{i}(E)+\bar{G}_{i}+F_{i}\left(E, \bar{G}_{i}\right) \geq T_{i}(E)+G_{i}^{a}(E)+F_{i}\left(E, \bar{G}_{i}\right)=C_{i}^{a}(E,(0,0,1), E)$,
which verifies the desired inequality. Consider next state $1 r n \in \mathcal{C}_{+}$so that $\left(E, \mathbf{D}, E^{r}\right)=$ $(E,(1,0,0), E)$. Because $\bar{G}_{i} \geq G_{i}^{a}(E)$ for all $E$ in range 1,

$$
C_{i}^{j}(E,(1,0,0), E)=T_{i}(E)+\bar{G}_{i} \geq T_{i}(E)+G_{i}^{a}(E)=C_{i}^{a}(E,(1,0,0), E),
$$

which verifies the desired inequality.

## 2. The states in $C_{-}$are no better under $j$ than under regime $a$.

The only two states in $\mathcal{C}_{-}$are $0 r n$ and $0 r r$. It suffices to show that the utility associated with any alternative compatible with states $0 r n$ and $0 r r$ is at least as high under regime $a$ than under regime $j$. Consider a tuple obeying $s\left(E, \mathbf{D}, E^{r}\right) \in C_{-}$. The alternatives compatible with state $0 r n$ are such that $\left(E, \mathbf{D}, E^{r}\right)=(0,(1,0,0), 0)$ hence $C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)=\bar{G}_{i}$ all $t$ and $R\left(\mathbf{D}, E^{r}\right)=1$. The alternatives compatible with state $0 r r$ are such that $\left(E, \mathbf{D}, E^{r}\right)=$

[^5]$(0,(0,0,1), 0)$ hence $C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)=\bar{G}_{i}+\bar{F}_{i}$ all $t$ and $R\left(\mathbf{D}, E^{r}\right)=1$. Thus, it suffices to show that
$$
U_{i}^{a}\left(0, \bar{G}_{i},(1,0,0), 1\right) \geq U_{i}^{j}\left(0, \bar{G}_{i},(1,0,0), 1\right)
$$
and
$$
U_{i}^{a}\left(0, \bar{G}_{i}+\bar{F}_{i},(0,0,1), 1\right) \geq U_{i}^{j}\left(0, \bar{G}_{i}+\bar{F}_{i},(0,0,1), 1\right)
$$

Both inequalities hold by A. 3 in Assumption 6 (hassle disutility is no lower under $j$ than under $a$ ).
3. The states in $\mathcal{C}_{0}$ are equally attractive under regimes $j$ and $a$.

Write $\mathcal{C}_{0}$ as the union of two disjoint collections:

$$
\begin{align*}
& \{0 n n, 1 n n, 2 n n, 0 n r, 1 n r, 2 n r, 1 n u, 2 n u\}  \tag{148}\\
& \{1 u n, 2 u n, 1 u u, 2 u u\} \tag{149}
\end{align*}
$$

The alternatives compatible with states in collection (148) entail no assistance or FS-only assistance. Thus the utility associated with each of these alternatives is invariant to the policy regime (A. 4 and A. 5 in Assumption 6). Accordingly, it suffices to show that the consumption associated with any of these alternatives is the same under regimes $j$ and $a$. Consider first the alternatives compatible with an off-assistance state $s_{i} \in\{0 n n, 1 n n, 2 n n\}$ in collection (148). If $s_{i} \in\{0 n n\}$, consumption is zero. If $s_{i} \in\{1 n n, 2 n n\}$ consumption equals $E$. Thus, consumption is the same under either regime. Consider next the alternatives compatible with a FS-only state $s_{i} \in\{0 n r, 1 n r, 2 n r, 1 n u, 2 n u\}$ in collection (148). If $s_{i} \in$ $\{0 n r\}$, consumption equals $\overline{\bar{F}}_{i}$. If $s_{i} \in\{1 n r, 2 n r\}$, consumption equals $E+F_{i}(E, 0)$. If $s_{i} \in\{1 n u, 2 n u\}$ consumption equals $E+\overline{\bar{F}}_{i}-\omega_{i}$ by optimal reporting (Lemma 9). Thus, consumption is the same under either regime. Finally consider the alternatives compatible with states in collection (149). Given optimal reporting, the utility function associated with all the alternatives compatible with states $\{1 u n, 2 u n, 1 u u, 2 u u\}$ is invariant to the policy regime (Corollary 6). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is the same under regimes $j$ and $a$. If $s_{i} \in\{1 u n, 2 u n\}$, consumption is $E+\bar{G}_{i}-\kappa_{i}$ under both regimes. If $s_{i} \in\{1 u u, 2 u u\}$, consumption is $E+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}$ under both regimes. Thus, consumption is the same under either regime.

## Main Propositions

Proposition 6 (Restricted Pairings). Given Assumptions 6-12, the pairings of states corresponding to the "-" entries in Table A5 are disallowed.

Proof. States $1 u n$ and $1 u u$ are dominated under regime $j$ (Lemma 9). Therefore no woman pairs state $s^{a}$ with state $s^{j} \in\{1 u n, 1 u u\}$ for any $s^{a} \in \mathcal{S}$. Next, by Lemmas 6 and 11, no pairing of state $s^{a}$ with state $s^{j}$ can occur for all $\left(s^{a}, s^{j}\right)$ in the collection

$$
\begin{equation*}
\left\{\left(s^{a}, s^{j}\right): s^{a} \in \mathcal{C}_{0} \cup \mathcal{C}_{+}, s^{j} \in \mathcal{C}_{0} \cup \mathcal{C}_{-}, s^{a} \neq s^{j}\right\} \tag{150}
\end{equation*}
$$

It suffices to show that the properties I)-III) of Lemma 10 are met. Property I) holds trivially and properties II) and III) hold by Lemma 11. Therefore no woman selects any of the pairings in (150).

Proposition 7 (Unrestricted Pairings). Given Assumptions 6-12, the pairings of states corresponding to the non "-" entries in Table A5 are allowed.

Remark 10 (Omitted Proof of Proposition 7). The proof of Proposition 7 would mimic the proof of Proposition 2 in that it would present examples of women who select the pairings corresponding to the non "-" entries in Table A5. We omit the proof of Proposition 7 for two reasons. First, there are 63 allowed pairings in Table A5, which makes the proof exceedingly long. Second, our interest lies in showing that the integrated response matrix of the extended model contains at least as many restrictions as the response matrix of the baseline model (Proposition 9 and Remark 11 below). The proof of Proposition 7 would only serve to confirm the additional result that the integrated response matrix of the extended model contains at most as many restrictions as the response matrix of the baseline model.

Proposition 8 (Response Matrix). Let $\Pi$ denote the matrix of response probabilities $\left\{\pi_{s^{a}, s^{j}}: s^{a}, s^{j} \in \mathcal{S}\right\}$. Given Table A5, $\Pi$ is a $16 \times 16$ matrix with the following zero ( 0 ) and non-zero ( $X$ ) entries:

|  | JF's Experimental Policy: Earnings / Program Participation State |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 0nn | 1 nn | 2nn | 0nr | 1 nr | 2nr | 1nu | 2nu | 0rn | 1rn | 1 un | 2un | Orr | 1 rr | 1uu | 2uu |
| 0nn | X | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 nn | 0 | X | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 nn | 0 | 0 | X | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 0nr | 0 | 0 | 0 | X | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 nr | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 nr | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 nu | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2nu | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 0rn | X | X | X | X | X | X | X | X | X | X | 0 | X | X | X | 0 | X |
| 1 n | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 un | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 un | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | 0 | X | 0 | 0 |
| 0rr | X | X | X | X | X | X | X | X | X | X | 0 | X | X | X | 0 | X |
| 1 rr | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1uu | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 uu | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | X |

Proof. By Definition 29, $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$. By Proposition 6, the pairings of states corresponding to the "-" entries in Table A5 are disallowed. Thus, $\pi_{s^{a}, s^{j}}=0$ for any pairing $\left(s^{a}, s^{j}\right)$ corresponding to a "-" entry in Table A5 because no woman occupies state $s^{a}$ under regime $a$ and state $s^{j}$ under regime $j$. By Proposition 7, the pairings of states corresponding to the non "-" entries in Table A5 are allowed. Thus, $\pi_{s^{a}, s^{j}} \neq 0$ for any pairing $\left(s^{a}, s^{j}\right)$ corresponding to a non "-" entry in Table A5 because some women may occupy state $s^{a}$ under regime $a$ and state $s^{j}$ under regime $j$.

Proposition 9 (Integrated Response Matrix). The matrix of response probabilities over the states in $\mathcal{S}, \Pi$, reduces to the following matrix $\Pi_{w}$ of response probabilities over the states in $\mathcal{S}_{w}$ :

|  | JF's Experimental Policy: Earnings / Program Participation State |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | $0 n$ | $1 n$ | $2 n$ | $0 r$ | $1 r$ | $1 u$ | $2 u$ |
| $0 n$ | $1-\pi_{0 n, 1 r}$ | 0 | 0 | 0 | 0 | $\pi_{0 n, 1 r}$ | 0 |
| $1 n$ | 0 | $1-\pi_{1 n, 1 r}$ | 0 | 0 | $\pi_{1 n, 1 r}$ | 0 | 0 |
| $2 n$ | 0 | 0 | $1-\pi_{2 n, 1 r}$ | $\pi_{2 n, 1 r}$ | 0 | 0 |  |
| $0 r$ | $\pi_{0 r, 0 n}$ | $\pi_{0 r, 1 n}$ | $\pi_{0 r, 2 n}$ | $1-\pi_{0 r, 0 n}-\pi_{0 r, 1 n}-\pi_{0 r, 2 n}$ | $\pi_{0 r, 1 r}$ | 0 | $\pi_{0 r, 2 u}$ |
| $1 r$ | 0 | 0 | 0 | $\pi_{0 r, 1 r}-\pi_{0 r, 2 u}$ |  |  |  |
| $1 u$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $2 u$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

where

$$
\begin{aligned}
\pi_{0 n, 1 r} & \equiv\left(\pi_{0 n n, 1 r n}+\pi_{0 n n, 1 r r}\right) \frac{q_{0 n n}^{a}}{p_{0 n}^{a}}+\left(\pi_{0 n r, 1 r n}+\pi_{0 n r, 1 r r}\right) \frac{p_{0 n}^{a}-q_{0 n n}^{a}}{p_{0 n}^{a}}, \\
\pi_{1 n, 1 r} & \equiv\left(\pi_{1 n n, 1 r n}+\pi_{1 n n, 1 r r}\right) \frac{q_{1 n n}^{a}}{p_{1 n}^{a}}+\left(\pi_{1 n r, 1 r n}+\pi_{1 n r, 1 r r}\right) \frac{q_{1 n r}^{a}}{p_{1 n}^{a}}+\left(\pi_{1 n u, 1 r n}+\pi_{1 n u, 1 r r}\right) \frac{p_{1 n}^{a}-q_{1 n n}^{a}-q_{1 n r}^{a}}{p_{1 n}^{a}}, \\
\pi_{2 n, 1 r} & \equiv\left(\pi_{2 n n, 1 r n}+\pi_{2 n n, 1 r r}\right) \frac{q_{2 n n}^{a}}{p_{2 n}^{a}}+\left(\pi_{2 n r, 1 r n}+\pi_{2 n r, 1 r r}\right) \frac{q_{2 n r}^{a}}{p_{2 n}^{a}}+\left(\pi_{2 n u, 1 r n}+\pi_{2 n u, 1 r r}\right) \frac{p_{2 n}^{a}-q_{2 n n}^{a}-q_{2 n r}^{a}}{p_{2 n}^{a}}, \\
\pi_{0 r, 0 n} & \equiv\left(\pi_{0 r n, 0 n n}+\pi_{0 r n, 0 n r}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 0 n n}+\pi_{0 r r, 0 n r}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 1 n} & \equiv\left(\pi_{0 r n, 1 n n}+\pi_{0 r n, 1 n r}+\pi_{0 r n, 1 n u}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 1 n n}+\pi_{0 r r, 1 n r}+\pi_{0 r r, 1 n u}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 2 n} & \equiv\left(\pi_{0 r n, 2 n n}+\pi_{0 r n, 2 n r}+\pi_{0 r n, 2 n u}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 2 n n}+\pi_{0 r r, 2 n r}+\pi_{0 r r, 2 n u}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 1 r} & \equiv\left(\pi_{0 r n, 1 r n}+\pi_{0 r n, 1 r r}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 1 r n}+\pi_{0 r r, 1 r r}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 2 u} & \equiv\left(\pi_{0 r n, 2 u n}+\pi_{0 r n, 2 u u}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 2 u n}+\pi_{0 r r, 2 u u}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{2 u, 1 r} & \equiv\left(\pi_{2 u n, 1 r n}+\pi_{2 u n, 1 r r}\right) \frac{q_{2 u n}^{a}}{p_{2 u}^{a}}+\left(\pi_{2 u u, 1 r n}+\pi_{2 u u, 1 r r}\right) \frac{p_{2 u}^{a}-q_{2 u n}^{a}}{p_{2 u}^{a}}, \\
\pi_{1 r, 1 r} & =1, \\
\pi_{1 u, 1 r} & =1 .
\end{aligned}
$$

Proof. The response probabilities over the states in $\mathcal{S}_{w}$ are of the form:

$$
\pi_{s_{w}^{a}, s_{w}^{j}} \equiv \operatorname{Pr}\left(S_{w, i}^{j}=s_{w}^{j} \mid S_{w, i}^{a}=s_{w}^{a}\right)=\sum_{s^{j} \in \mathcal{S}: s_{w}^{j}=h\left(s^{j}\right)}\left[\sum_{s^{a} \in \mathcal{S}: s_{w}^{a}=h\left(s^{a}\right)} \operatorname{Pr}\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right) \frac{q_{s^{a}}^{a}}{p_{s_{w}^{a}}^{a}}\right]
$$

Remark 11 (Relationship between the Restrictions in the Baseline and in the Extended Model). The response matrix implied by the baseline model has the same zero and unitary entries as the response matrix $\Pi_{w}$ implied by the extended model.

## 10 Finer Earning Ranges

In this section we consider a finer coarsening of earnings. Specifically, we partition earnings above the federal poverty level into two sub-ranges. We begin with some definitions that supersede those in Section 4 of this Appendix. We conclude with the analytical bounds for two "opt-in" response probabilities. Proofs are omitted because they closely mimic those accompanying the baseline coarsening approach.

Definition 31 (Earning Ranges). Earnings range 0 refers to zero earnings. Earnings range 1 refers to the interval $\left(0, F P L_{i}\right]$ where $F P L_{i}$ is woman $i$ 's federal poverty line. Earnings range $2^{\prime}$ refers to the interval ( $\left.F P L_{i}, 1.2 \times F P L_{i}\right]$. Earning range $2^{\prime \prime}$ refers to the interval $\left(1.2 \times F P L_{i}, \infty\right)$.

Definition 32 (State). Consider the triple $\left(E, D, E^{r}\right)$. The state corresponding to $\left(E, D, E^{r}\right)$ is defined by the function:

$$
s\left(E, D, E^{r}\right)=\left\{\begin{array}{ll}
0 n & \text { if } E=0, D=0 \\
1 n & \text { if } E \text { in range } 1, D=0 \\
2^{\prime} n & \text { if } E \text { in range } 2^{\prime}, D=0 \\
2^{\prime \prime} n & \text { if } E \text { in range } 2^{\prime \prime}, D=0 \\
0 r & \text { if } E=0, D=1 \\
1 r & \text { if } E \text { in range } 1, D=1, E^{r}=E \\
1 u & \text { if } E \text { in range } 1, D=1, E^{r}<E \\
2^{\prime} u & \text { if } E \text { in range } 2^{\prime}, D=1, E^{r}<E \\
2^{\prime \prime} u & \text { if } E \text { in range } 2^{\prime \prime}, D=1, E^{r}<E \\
2^{\prime} r & \text { if } E \text { in range } 2^{\prime}, D=1, E^{r}=E \\
2^{\prime \prime} r & \text { if } E \text { in range } 2^{\prime \prime}, D=1, E^{r}=E
\end{array} .\right.
$$

Definition 33 (Latent and Observed States). Define $\mathcal{S}^{*} \equiv\left\{0 n, 1 n, 2^{\prime} n, 2^{\prime \prime} n, 0 r, 1 r, 1 u, 2^{\prime} u, 2^{\prime \prime} u\right\}$ and $\widetilde{S}^{*} \equiv\left\{0 n, 1 n, 2^{\prime} n, 2^{\prime \prime} n, 0 p, 1 p, 2^{\prime} p, 2^{\prime \prime} p\right\}$ where the mapping between the latent states in $\mathcal{S}^{*}$ and the observed states in $\tilde{\mathcal{S}}^{*}$ is:

$$
g(s)= \begin{cases}s & \text { if } s \in\left\{0 n, 1 n, 2^{\prime} n, 2^{\prime \prime} n\right\} \\ 0 p & \text { if } s=0 r \\ 1 p & \text { if } s \in\{1 u, 1 r\} \\ 2^{\prime} p & \text { if } s=2^{\prime} u \\ 2^{\prime \prime} p & \text { if } s=2^{\prime \prime} u\end{cases}
$$

Proposition 10 (Response Matrix). The matrix of response probabilities over the states in $\mathcal{S}^{*}$ is:

|  | JF: Earnings / Program Participation State |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A F D C$ | $0 n$ | $1 n$ | $2^{\prime} n$ | $2^{\prime \prime} n$ | $0 r$ | $1 r$ | $1 u$ | $2^{\prime} u$ | $2^{\prime \prime} u$ |
| $0 n$ | $1-\pi_{0 n, 1 r}$ | 0 | 0 | 0 | 0 | $\pi_{0 n, 1 r}$ | 0 | 0 | 0 |
| $1 n$ | 0 | $1-\pi_{1 n, 1 r}$ | 0 | 0 | 0 | $\pi_{1 n, 1 r}$ | 0 | 0 | 0 |
| $2^{\prime} n$ | 0 | 0 | $1-\pi_{2^{\prime} n, 1 r}$ | 0 | 0 | $\pi_{2^{\prime} n, 1 r}$ | 0 | 0 | 0 |
| $2^{\prime \prime} n$ | 0 | 0 | 0 | $1-\pi_{2^{\prime \prime} n, 1 r}$ | 0 | $\pi_{2 \prime \prime \prime}{ }^{\prime \prime}{ }^{1} 1 r$ | 0 | 0 | 0 |
| $0 r$ | $\pi_{0 r, 0 n}$ | $\pi_{0 r, 1 n}$ | $\pi_{0 r, 2 n}$ |  | $\begin{gathered} 1-\pi_{0 r, 0 n} \\ -\pi_{0 r, 1 n}-\pi_{0 r, 1 r} \\ -\pi_{0 r, 2^{\prime} n}-\pi_{0 r, 2^{\prime \prime} n} \\ -\pi_{0 r, 2^{\prime} u}-\pi_{0 r, 2^{\prime \prime} u} \end{gathered}$ | $\pi_{0 r, 1 r}$ | 0 | $\pi_{0 r, 2^{\prime} u}$ | $\pi_{0 r, 2^{\prime \prime} u}$ |
| $1 r$ | 0 | 0 | 0 |  | 0 | 1 | 0 | 0 | 0 |
| $1 u$ | 0 | 0 | 0 |  | 0 | 1 | 0 | 0 | 0 |
| $2^{\prime} u$ | 0 | 0 | 0 |  | 0 | $\pi_{2^{\prime} u, 1 r}$ | 0 | $1-\pi_{2^{\prime} u, 1 r}$ | 0 |
| $2^{\prime \prime} u$ | 0 | 0 | 0 |  | 0 | $\pi_{2}{ }^{\prime \prime} u, 1 r$ | 0 | 0 | $1-\pi_{2^{\prime \prime}} u, 1 r$ |

Proof. Omitted. See proof of Propositions 1 and 2.
Corollary 7. The matrix of response probabilities in Proposition 10 implies the following system of equations describing the impact of the JF reform on observable state probabilities:

$$
\begin{align*}
& p_{0 n}^{j}-p_{0 n}^{a}=-\pi_{0 n, 1 r} p_{0 n}^{a}+\pi_{0 r, 0 n} p_{0 p}^{a} \\
& p_{1 n}^{j}-p_{1 n}^{a}=-\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{0 r, 1 n} p_{0 p}^{a} \\
& p_{2^{\prime} n}^{j}-p_{2^{\prime} n}^{a}=-\pi_{2^{\prime} n, 1 r} p_{2^{\prime} n}^{a}+\pi_{0 r, 2^{\prime} n} p_{0 p}^{a} \\
& p_{2^{\prime \prime} n}^{j}-p_{2^{\prime \prime} n}^{a}=-\pi_{2^{\prime \prime} n, 1 r}^{a} p_{2 \prime \prime n}^{a}+\pi_{0 r, 2^{\prime \prime} n} p_{0 p}^{a}  \tag{151}\\
& p_{0 p}^{j}-p_{0 p}^{a}=-\left(\pi_{0 r, 0 n}+\pi_{0 r, 2 / n}+\pi_{0 r, 2^{\prime \prime} n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2^{\prime} u}+\pi_{0 r, 2^{\prime \prime} u}\right) p_{0 p}^{a} \\
& p_{2^{\prime} p p}^{j}-p_{2^{\prime} p}^{a}=\pi_{0 r, 2^{\prime} u} p_{0 p}^{a}-\pi_{2^{\prime} u, 1 r} p_{2^{\prime} p}^{a} \\
& p_{2^{\prime \prime} p}^{j}-p_{2^{\prime \prime} p}^{a}=\pi_{0 r, 2^{\prime \prime} u} p_{0 p}^{a}-\pi_{2^{\prime \prime} u, 1 r} p_{2^{\prime \prime} p}^{a}
\end{align*}
$$

Proof. By an application of the law of total probability given Definition 33.
Corollary 8. The analytical lower bounds of the response probabilities $\pi_{2^{\prime} n, 1 r}$ and $\pi_{2^{\prime \prime \prime} n, 1 r}$ are

$$
\begin{aligned}
\pi_{2^{\prime}, 1 r} & \geq \max \left\{0, \frac{p_{2^{\prime} n}^{a}-p_{2^{\prime} n}^{j}}{p_{2^{\prime} n}^{a}}\right\}, \\
\pi_{2^{\prime \prime} n, 1 r} & \geq \max \left\{0, \frac{p_{2^{\prime \prime} n}^{a}-p_{2^{\prime \prime} n}^{j}}{p_{2^{\prime \prime} n}^{a}}\right\} .
\end{aligned}
$$

Proof. Omitted. See Section 6 in this Appendix.

## References

1. Andrews, Donald and Panle Jia Barwick. 2012. "Inference for Parameters Defined by Moment Inequalities: A Recommended Moment Selection Procedure." Econometrica 80(6): 2805-2826.
2. Barrett, Garry F. and Stephen G. Donald. 2003. "Consistent Tests for Stochastic Dominance." Econometrica 71(1):71-104.
3. Chernozhukov, Victor, Sokbae Lee, and Adam Rosen. 2013. "Intersection bounds: estimation and inference." Econometrica, 81(2), 667-737.
4. Giné, Evarist and Joel Zinn. 1990. "Bootstrapping General Empirical Measures." Annals of Probability 18(2): 851-869.
5. Kosorok, Michael. 2008. Introduction to Empirical Processes and Semiparametric Inference Springer: New York.
6. Murty, Katta. 1983. Linear Programming New York: John Wiley \& Sons.

Table A1: Cross Tabulation of grant-inferred AU size and kidcount

|  | kidcount |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | Total |
| Inferred AU Size |  |  |  |  |  |
| 1 | 0.17 | 0.08 | 0.04 | 0.01 | 0.05 |
| 2 | 0.53 | 0.84 | 0.19 | 0.06 | 0.42 |
| 3 | 0.17 | 0.06 | 0.72 | 0.17 | 0.29 |
| 4 | 0.11 | 0.01 | 0.05 | 0.53 | 0.17 |
| 5 | 0.00 | 0.00 | 0.00 | 0.14 | 0.04 |
| 6 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| 7 | 0.03 | 0.00 | 0.00 | 0.07 | 0.02 |
| 8 | 0.00 | 11,361 | 0.00 | 0.00 | 0.00 |
| \# of monthly observations | 840 |  | 8,463 | 8,043 | 28,707 |

Notes: Analysis conducted on Jobs First sample over quarters 1-7 post-random assignment. Kidcount variable, which gives the number of children reported in baseline survey, is tabulated conditional on non-missing. The AU size is inferred from (rounded) monthly grant amounts. Starting with AU size 5 , the unique correspondence between AU size and rounded grant amount obtains only for units which do not receive housing subsidies. The size inferred during months on assistance is imputed forward to months off assistance and to months that otherwise lack an inferred size.

Table A2: Mean Outcomes Post-Random Assignment

|  | Overall |  |  | Zero Earnings Q7 pre-RA |  |  | Positive Earnings Q7 pre-RA |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jobs First | AFDC | Adjusted Difference | Jobs First | AFDC | Adjusted Difference | Jobs First | AFDC | Adjusted Difference |
| Average Earnings | $\begin{gathered} 1,191 \\ (29) \end{gathered}$ | $\begin{gathered} \hline 1,086 \\ (30) \end{gathered}$ | $\begin{aligned} & \hline 105 \\ & (36) \end{aligned}$ | $\begin{aligned} & \hline 930 \\ & \text { (32) } \end{aligned}$ | $\begin{aligned} & \hline 751 \\ & (30) \end{aligned}$ | $\begin{aligned} & \hline 179 \\ & (42) \end{aligned}$ | $\begin{gathered} 1766 \\ (65) \end{gathered}$ | $\begin{gathered} 1831 \\ (65) \end{gathered}$ | $\begin{gathered} \hline-65 \\ (84) \end{gathered}$ |
| Fraction of quarters with positive earnings | $\begin{gathered} 0.520 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.440 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.080 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.445 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.349 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.096 \\ (0.012) \end{gathered}$ | $\begin{gathered} 0.686 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.647 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.039 \\ (0.017) \end{gathered}$ |
| Fraction of quarters with earnings below 3FPL (AU size implied by kidcount+1) | $\begin{gathered} 0.906 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.897 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.009 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.938 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.940 \\ (0.008) \end{gathered}$ | $\begin{gathered} -0.002 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.837 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.803 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.034 \\ (0.014) \end{gathered}$ |
| Fraction of quarters on welfare | $\begin{gathered} 0.748 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.674 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.074 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.771 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.718 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.053 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.699 \\ (0.014) \end{gathered}$ | $\begin{gathered} 0.577 \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.122 \\ (0.019) \end{gathered}$ |
| Average earnings in quarters with any month on welfare | $\begin{aligned} & 929 \\ & \text { (24) } \end{aligned}$ | $\begin{aligned} & 526 \\ & (19) \end{aligned}$ | $\begin{aligned} & 403 \\ & (28) \end{aligned}$ | $\begin{aligned} & 762 \\ & (25) \end{aligned}$ | $\begin{aligned} & 404 \\ & (18) \end{aligned}$ | $\begin{aligned} & 359 \\ & \text { (30) } \end{aligned}$ | $\begin{gathered} 1316 \\ (53) \end{gathered}$ | $\begin{aligned} & 869 \\ & (43) \end{aligned}$ | 448 <br> (64) |
| Fraction of quarters with no earnings and at least one month on welfare | $\begin{gathered} 0.363 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.437 \\ (0.007) \end{gathered}$ | $\begin{gathered} -0.074 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.426 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.508 \\ (0.009) \end{gathered}$ | $\begin{gathered} -0.082 \\ (0.012) \end{gathered}$ | $\begin{gathered} 0.227 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.272 \\ (0.012) \end{gathered}$ | $\begin{gathered} -0.045 \\ (0.016) \end{gathered}$ |
| \# of cases | 2,318 | 2,324 |  | 1,630 | 1,574 |  | 688 | 750 |  |

Notes: Sample covers quarters 1-7 post-random assignment. Sample cases with kidcount missing are excluded. Adjusted differences are computed via propensity score reweighting. Numbers in parentheses are standard errors calculated via 1,000 block bootstrap replications (resampling at case level).

Table A3: Probability of Earnings / Participation States in AFDC Sample
(Conditional on State=0p in Quarter Prior to Random Assignment)

| Quarter post-RA: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}($ State $=0 n)$ | 0.022 | 0.062 | 0.086 | 0.093 | 0.114 | 0.136 | 0.136 |
| $\operatorname{Pr}($ State $=1 \mathrm{n})$ | 0.021 | 0.045 | 0.058 | 0.079 | 0.084 | 0.101 | 0.112 |
| $\operatorname{Pr}($ State $=2 n)$ | 0.006 | 0.021 | 0.024 | 0.033 | 0.048 | 0.044 | 0.074 |
| $\operatorname{Pr}($ State=0p $)$ | 0.786 | 0.723 | 0.675 | 0.631 | 0.584 | 0.563 | 0.539 |
| $\operatorname{Pr}$ (State=1p) | 0.160 | 0.160 | 0.145 | 0.160 | 0.157 | 0.150 | 0.143 |
| $\operatorname{Pr}($ State=2p) | 0.002 | 0.001 | 0.004 | 0.004 | 0.004 | 0.002 | 0.005 |

Notes: Sample consists of 902 AFDC cases that were not working in the quarter prior to random assignment and were on welfare. Sample units with kidcount missing are excluded. Numbers give the reweighted fraction of sample in specified quarter after random assignment occupying each earnings / welfare paticipation state. Number of state refers to earnings level, with 0 indicating no earnings, 1 indicating earnings below 3 times the monthly FPL, and 2 indicating earnings above 3FPL. The letter $n$ indicates welfare nonparticipation throughout the quarter while the letter $p$ indicates welfare participation throughout the quarter. Poverty line computed under assumption $A U$ size is one greater than amount implied by baseline kidcount variable. Probabilities are adjusted via propensity score reweighting algorithm.

| State under Jobs First |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State <br> under <br> AFDC | On | 1n | 2n | Or | $1 r$ | 1u | $2 u$ | $2 r$ |
| On | No Response | － | － | － | Extensive LS（＋） Take Up Welfare | － | － | － |
| 1n | － | No Response | － | － | Intensive LS（＋／0／－） Take Up Welfare | － | － | － |
| 2n | － | － | No Response | － | Intensive LS（－） <br> Take Up Welfare | － | － | － |
| Or | No LS Response Exit Welfare | Extensive LS（＋） Exit Welfare | Extensive LS（＋） Exit Welfare | No Response | Extensive LS（＋） | － | Extensive LS（＋） Under－reporting | － |
| $1 r$ | － | － | － | － | Intensive LS（＋／0／－） | － | － | － |
| 1u | － | － | － | － | Intensive LS（＋／0／－） <br> Truthful Reporting | － | － | － |
| 2u | － | － | － | － | Intensive LS（－） Truthful Reporting | － | No Response | － |
| $2 r$ | Extensive LS（－） Exit Welfare （Figure A1） | Intensive LS（－） Exit Welfare | Intensive LS（＋／0／－） Exit Welfare | Extensive LS（－） | Intensive LS（－） | － | Intensive LS（＋／0／－） Under－reporting | － |

Notes：This table catalogues the theoretically allowed response margins given the states that a woman may occupy under AFDC and Jobs First when truthful reporting of earnings above the FPL is possible under AFDC，that is，when assumption A． 8 is not maintained．A state is a pair of coarsened earnings（ 0 stands for zero earnings， 1 for positive earnings at or below the FPL，and 2 for earnings strictly above the FPL），and participation in the welfare assistance program along with an earnings reporting decision（ n stands for＂not on assistance＂，r for＂on assistance and truthfully reporting earnings＂，and u for＂on assistance and under－reporting earnings＂）．The cells termed＂no response＂entail the same behavior under the two policy regimes．The cells containing a＂－＂represent responses that are either incompatible with the policy rules or not allowed based on revealed preference arguments derived from the nonparametric model of Section 4 ．Specifically，（a） state 1 u is unpopulated under JF（＂一＂in cells with a horizontally striped background fill），（b）state $2 r$ is not defined under JF（＂一＂in cells with gridded background fill），and（c）a woman will not leave a state at least as attractive under JF a under AFDC for a state that is no more attractive under JF than under AFDC（＂一＂in cells with a solid greyed－out background fill）．The remaining cells represent responses that are allowed by the model．Their content summarizes the three possible margins of responses：（a）the labor supply＂LS＂response（intensive versus extensive and its sign：＂+ ＂for increase，＂ 0 ＂for no change，and＂- ＂for decrease），（b）the program participation response（take up of versus exit from welfare assistance），and（c）the reporting of earnings to the welfare agency margin（to truthfully report versus to under－report）．See Online Appendix for proof．

|  |  |  |  |  |  |  |  | tate under J | st |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State under AFDC | Onn | 1nn | 2nn | Onr | 1nr | 2 nr | 1nu | 2nu | Orn | 1rn | 1 un | 2un | Orr | 1rr | 1uu | 2uu |
| Onn | No Response | - | - | - | - | - | - | - | - | Extensive LS (+) Take Up Welfare | $\underline{ }$ | - | - | $\left\lvert\, \begin{array}{c\|} \text { Extensive LI }(t) \\ \text { Take Up Weffare and } \\ \text { FS } \end{array}\right.$ | - | - |
| 1nn | - | No Response | - | - | - | - | - | - | - | Intensive LS $(+/ 0 /-$-) Take Up Welfare | - | - | - | Intensive LS $(+/ 0 /-$ - Take Up Welfare and FS | - | - |
| 2nn | - | - | No Response | - | - | - | - | - | - | Intensive LS $(-)$ Take | $\underline{ }$ | - | - | Intensive LS $(-)$ Take Up Welfare and FS | ? | - |
| Onr | - | - | - | No Response | - | - | - | - | - | $\begin{aligned} & \text { Extensive LS (+) } \\ & \text { Exit FS, Take Up } \end{aligned}$ Welfare | $\underline{\square}$ | - | - | Extensive LS (+) Take Up Welfare | - | - |
| 1nr | - | - | - | - | No Response | - | - | - | - |  | $\underline{\text { 를 }}$ | - | - | Intensive LS ( $+/ 0 /$ Take Up Welfare | - | - |
| 2nr | - | - | - | - | - | No Response | - | - | - | $\begin{gathered} \text { Intensive LS (-) } \\ \text { Exit FS, Take Up } \\ \text { Welfare } \end{gathered}$ | $\cdots$ | - | - | Intensive LS (-) Take Up Welfare | ? | - |
| 1nu | - | - | - | - | - | - | No Response | - | - | Intensive LS $(+/ / 0 /-)$ Exit FS, Take Up Welfare |  | - | - | Intensive LS (+/0/-) Exit FS, Take Up Welfare | ? | - |
| 2nu | - | - | - | - | - | - | - | No Response | - | $\begin{gathered} \text { Intensive LS (-) } \\ \text { Exit FS, Take Up } \\ \text { Welfare } \end{gathered}$ | $\cdots$ | - | - | (Intensive LS(-) | ? | - |
| Orn | No LS Response Exit Welfare | Extensive LS $(+)$ Exit Welfare | Extensive LL $(+)$ Exit Welfare | No LS Response Exit Welfare, Take Up FS | Extensive LS (+) Exit Welfare, Take Up FS | Extensive LS (t) Exit Welfare, Take Up FS | Extensive LS (+) Exit Welfare Take Up FS Under-report | $\begin{aligned} & \text { Extensive LS }(+) \\ & \text { Exit Welfare } \\ & \text { Take Up FS } \\ & \text { Under-report } \end{aligned}$ | No LS Response Exit Welfare, Take Up FS | Extensive LS(t) | V | Extensive LS (+) <br> Under-report | $\left\lvert\, \begin{gathered} \text { No Ls Response } \\ \text { Take Up Fs } \end{gathered}\right.$ | $\begin{aligned} & \text { Extensive Is (+) } \\ & \text { Take Up Fs } \end{aligned}$ | 厚 | Extensive LS (+) Take Up FS Under-report |
| 1rn | - | - | - | - | - | - | - | - | - | Intensive LS $(+/ 0 /-)$ | ? | - | - | $\begin{aligned} & \text { Intensive } \text { S S }(+0 /- \text { - } \\ & \text { Take UP FS } \end{aligned}$ | ? | - |
| 1un | - | - | - | - | - | - | - | - | - | Intensive LS $(+/ 0 /-$ ) Truthful Report |  | - | - |  | $\underline{\text { nem }}$ | - |
| 2un | - | - | - | - | - | - | - | - | - | Intensive LS (-) Truthful Report | $\underline{\square}$ | No Response | - | Intensive LS (-) Take Up FS Truthful Report | ? | - |
| Orr | No LS Response Exit Welfare | $\begin{aligned} & \text { Extensive LS }(+) \\ & \text { Exit Welfare and } \\ & \text { FS } \end{aligned}$ | Extensive LS $(+)$ <br> Exit Welfare and <br> FS | No LS Response Exit Weffare | Extensive LS (t) Exit Welfare | Extensive LS ( + ) | $\begin{array}{\|l} \text { Extensive LS (+) } \\ \text { Exit Welfare } \\ \text { Under-report FS } \end{array}$ | Extensive LS (t) Exit Welfare Under-report | $\left\lvert\, \begin{gathered} \text { No LS Response } \\ \text { Exit Fs } \end{gathered}\right.$ | $\underset{\text { Exit FS }}{\substack{\text { Extensive LS }(+)}}$ | $\cdots$ | Extensive LS (+) Exit FS Under-report | No Response | Extensive Ls (t) | " | Extensive LS (+) <br> Under-report |
| 1rr | - | - | - | - | - | - | - | - | - |  | $\underline{ }$ | - | - | Intensive LS ( $+10 /-\mathrm{H}$ | $\underline{ }$ | - |
| 1uu | - | - | - | - | - | - | - | - | - | Intensive LS ( $(+/ 0 /-)$ <br> Exit $F$. <br> Truthful report | - | - | - | Intensive LL $(+/ 0 /-$ ) Truthful Report | $\underline{\underline{\underline{2}} \text { + }}$ | - |
| 2uu | - | - | - | - | - | - | - | - | - | Intensive LS (-) Exit FS Truthful report |  | - | - | Intensive LS (-) Truthful Report | " | No Response |

 incorporated. A state isa triplet of coarsened earnings (0 stands for zero earnings, 1 for positive earnings at or below the APL , and 2 for earnings strictly above the FPLL , participation in the weffare assistance program along with an earnings reporting
decision (n stands for "not on assistance" $r$ for "on assistance and truthfully reporting earnings" and u for "on assistance and under-reporting earnings") and participation in the FS assistance program along with an earnings reporting decision (n $r$. decision ( $n$ stands for "not on assistance, $r$ for "on assistance and truthfully reporting earnings, and f for "on assistance and under-reporting earnings"), and participation in the Fs assistance program along wh an earnings reporting decision ( $n, r$,
and $u$ ). When both on welfare and FS assistance, a woman makes only one earning report to the welfare agency, hence states such as e.g. 1ru are ruled out and not included in the table The assumption of lower bounds on the stigma disutilities rules out states $\{2 r \mathrm{rn}, 2 \mathrm{rr}\}$ hence these states are not included in the table. The cells termed "no response" entail the same behavior under the two policy regimes. The cells containing a " - " represent responses that are either incompatible with the policy rules or not allowed based on revealed preference arguments derived from the extended model. Specifically, (a) states 1 uu and 1 un are unpopulated under JF (" - " in cells with a horizontally striped background fill); and (b) a woman will not leave a state at least as attractive under JF as under AFDC for a state that is no more attractive under JF than under AFDC ""-" in cells with a solid greyed-out background fill). The remaining cells represent responses that are allowed by the model. Their
content summarizes the three possible margins of responses: (a) the labor supply "LS" response (intensive versus extensive and its sign: " + " for increase " "0" for no change, and """ for decrease), (b) the program participation response (take up of content summarizes the three possible margins of responses: (a) the labor supply "LS" response (intensive versus extensive and its sign: " 4 " for increase, " 0 " for no change, and " $"$ " for decrease), (b) the program participation response (take up of versus exit from welfare assistance and/or FS assistance), and (c) the reporting of earnings to the welfare agency margin (to truthfully report versus to under-report). See Online Appendix for proof.

Table A6: Probability of Earnings / Participation States

|  | Overall |  |  | Overall - Adjusted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jobs First | AFDC | Difference | Jobs First | AFDC | Difference |
| Pr(State=0n) | 0.127 | 0.136 | -0.009 | 0.128 | 0.135 | -0.007 |
| $\operatorname{Pr}($ State $=1 \mathrm{n}$ ) | 0.076 | 0.130 | -0.055 | 0.078 | 0.126 | -0.048 |
| $\operatorname{Pr}($ State $=2 \mathrm{n}$ ) | 0.021 | 0.032 | -0.011 | 0.022 | 0.031 | -0.010 |
| $\operatorname{Pr}($ State $=2 \mathrm{n}$ ) | 0.047 | 0.067 | -0.020 | 0.048 | 0.065 | -0.017 |
| $\operatorname{Pr}($ State $=0 \mathrm{p})$ | 0.366 | 0.440 | -0.074 | 0.359 | 0.449 | -0.090 |
| $\operatorname{Pr}($ State $=1 \mathrm{p})$ | 0.342 | 0.185 | 0.157 | 0.343 | 0.184 | 0.159 |
| $\operatorname{Pr}($ State $=2 ' p)$ | 0.010 | 0.003 | 0.006 | 0.010 | 0.003 | 0.007 |
| $\operatorname{Pr}\left(\right.$ State $=2 \mathrm{l}{ }^{\prime \prime}$ ) | 0.012 | 0.006 | 0.006 | 0.013 | 0.006 | 0.007 |
| \# of quarterly observations | 16,226 | 16,268 |  | 16,226 | 16,268 |  |

Notes: Sample covers quarters 1-7 post-random assignment during which individual is either always on or always off welfare. Sample cases with kidcount missing are excluded. Number of state refers to earnings level, with 0 indicating no earnings, 1 indicating earnings below 3 times the monthly FPL, $2^{\prime}$ indicating earnings between $3 F P L$ and $1.2 \times 3 F P L$, and $2^{\prime \prime}$ indicating earnings above $1.2 \times \mathrm{FPL}$. The letter $n$ indicates welfare nonparticipation throughout the quarter while the letter $p$ indicates welfare participation throughout the quarter. Poverty line computed under assumption AU size is one greater than amount implied by baseline kidcount variable. Adjusted probabilities are adjusted via propensity score reweighting. Standard errors computed using 1,000 block bootstrap replications (resampling at case level).

Figure A1: Earnings and Participation Choices with Earning Constraints and no Stigma


Notes: Panels a and b are drawn in the earnings (horizontal axis) and consumption equivalent (vertical axis) plane. The consumption equivalent equals earnings plus transfer income from welfare (if any) net of monetized hassle, stigma, work, and under-reporting costs. The welfare stigma and fixed cost of work are set to zero. The cost of under-reporting is set large enough so that under-reporting is a dominated choice. Labor market constraints are imposed in the form of two earnings offers ( $E_{i}^{1}$ and $E_{i}^{2}$ ), both in range 2 (above the FPL). The wage rate is assumed fixed. Because of the labor market constraints, and the fact that a woman may always choose not to work, the only alternatives available are those identified by a solid circular symbol. Vertical lines represent the same earnings levels depicted in Figure 1 but for a situation in which the earnings level at which welfare assistance is exhausted under $\operatorname{AFDC}(\bar{E})$ is above the FPL, that is, for a woman who has access to the unreduced fixed ( $\$ 120$ ) and proportional disregards. It also displays the two earnings offers. Panel a depicts a scenario where under AFDC the woman opts to be on assistance earning $E_{i}^{1}$ and reports truthfully to the welfare agency (point A). She would make the same choice even in the absence of earnings constraints. Under JF, earning $E_{i}^{1}$ on assistance (and reporting truthfully) is no longer feasible because welfare eligibility ends at FPL. Panel b depicts a scenario where, given the earning constraints, the JF reform induces the woman to exit both welfare and the labor force (point B). However, in the absence of earning constraints, she would choose to lower her earnings below the FPL and remain on assistance as evidenced by the fact that the indifference curve through point A lies below the (dashed) JF segment in range 1 (earning levels below FPL).


[^0]:    ${ }^{1}$ Changes in AU size are typically due to a birth or to the fact that a child becomes categorically ineligible for welfare. Under AFDC, the AU size also changes when the adult is removed from the unit due to sanctions for failure to comply with employment-related mandates. Empirically this source of time variation in AU size seems quantitatively minor. Bloom et al. (2002) report that 5 percent of AFDC group members had their benefits reduced owing to a sanction within four years after random assignment.

[^1]:    ${ }^{2}$ Concavity of $v($.$) enables the conditions imposed. For instance, the first condition requires v\left(E_{i}^{k}+\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)<$ $v\left(\bar{G}_{i}\right)-v(0)$ which cannot hold unless $v($.$) is (strictly) concave.$

[^2]:    ${ }^{3}$ Convexity of $v($.$) enables the conditions imposed. For instance, the first condition requires v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-$ $v\left(E_{i}^{k}\right) \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-v\left(E_{i}^{l}\right)$ which cannot hold unless $v$ is convex.

[^3]:    ${ }^{4}$ During the JF demonstration project, $\tau_{f}^{1}=0.30, \tau_{f}^{2}=0.20$ and $\tau_{f}^{3}=1.3$. The JF experimental policy effectively sets $\tau_{f}^{2}=1$ when FS is taken up jointly with welfare. This explains why we write the FS transfer as in (141), that is, as the standard transfer function evaluated at zero earnings. The eligibility formula shows that a woman with earnings above $F P L_{i}$ may be eligible for FS and the transfer formula shows that the FS transfer for which she is eligible may be positive. However, under the JF experimental policy, a woman with earnings above $F P L_{i}$ may not receive both welfare and FS because such earnings disqualify her from welfare.
    ${ }^{5}$ This function is time varying. We dispense with the time subscript for simplicity.
    ${ }^{6}$ This function is time varying. We dispense with the time subscript for simplicity.

[^4]:    ${ }^{7}$ If $B($.$) were differentiable then \frac{d B\left(E^{r}\right)}{d E r}=\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial G} \frac{d G}{d E^{r}}+\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial E^{r}} \frac{d E^{r}}{d E^{r}}$. To show that $\frac{d B\left(E^{r}\right)}{d E^{r}} \leq 0$ it would suffice to show that both $\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial G} \frac{d G}{d E^{r}} \leq 0$ and $\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial E^{r}} \leq 0$. The argument in the proof does exactly this without using calculus because neither $G($.$) nor F(.,$.$) are differentiable functions.$

[^5]:    ${ }^{8}$ There are earning levels in range 1 such that a woman is ineligible for the combined FS plus welfare assistance under JF's control policy. This comparison is meaningful only for earnings that are below the more stringent eligibility threshold; above such threshold state $1 r r$ is ruled out under JF's control policy.

