Online Appendix (not for publication)

College Admissions as Non-Price Competition: The Case of South Korea

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1. Additional Background on the Korean Admissions System

1.1 College Rankings

Panel A of Table A.1 reports college ranking based on the score on the nationwide test needed for an applicant to have a realistic chance of being admitted as an undergraduate law major. We use a placement guideline made by Daesung, a well-known cram school in South Korea. This guideline is distributed to all high schools in South Korea and used by high school teachers. We expect that this guess will be correlated with an applicant's quality given major-college. We report ranking based on a law major because almost all colleges (except for Postech) have an undergrad law major, and the law schools do not select applicants jointly with another schools. (e.g., for the economics major, some colleges select students as economics majors and others select them as social science majors). In all years from 1994 to 1999, Seoul National was ranked 1, meaning that the law major requires the highest score on the nationwide test. A college's ranking is very stable over time (pair-wise spearman correlation ranges from 0.92 to 0.99).

Panel B is based on a survey of how great a contribution a college makes to Korean society, conducted by Joongang Daily (herein JEDI, available at http://jedi.re.kr/).

Table A.1 College Ranking

	Panel A. Daesung						Panel B. JEDI		
	1994	1995	1996	1997	1998	1999	2005	2010	2011
Seoul National	1	1	1	1	1	1	1	1	1
Korea	2	2	2	2	2	2	2	3	2
Yonsei	3	3	3	3	3	3	3	2	4
Postech	-	-	-	-	-	-	4	5	5
Sogang	5	4	4	4	4	5	8	8	9
Ewha women's	6	5	4	5	5	6	9	10	8
Pusan	8	7	6	6	7	7	10	14	14
Kyungbook	8	7	6	6	6	7	11	15	13
Joongang	7	6	5	6	5	6	13	9	10
Hanyang	5	5	4	4	4	5	5	7	7
Kyunghee	7	6	5	4	6	6	14	11	12
Seongkyunkwan	4	4	4	4	4	4	7	6	6
Hankook	7	6	6	6	6	6	12	12	11

Note: Postech does not offer an undergrad law major. JEDI omitted KAIST from its 2005 report but included it in the 2010 and 2011 reports, in which KAIST was ranked 4th and 3rd, respectively.

JEDI has announced college rankings every year since 2005 (equivalent to the *US News* college ranking). JEDI constructs several indexes based on a combination of surveys about how prestigious/visible a school is and other measures such as publication records and funding. Instead of using the overall ranking, we used people's perceptions of a school's contribution to Korean society as a quality measure because the measure is stable over time. There has been some controversy over using publication records and funding for college evaluation because JEDI uses a simple average instead of adjusting for the quality of a journal and so on.

College ranking based on JEDI is highly correlated with the results in Panel A and fairly stable over time. For example, pairwise spearman correlations between JEDI and Daesung's ranking range from 0.80 to 0.98.

1.2 Distribution of Seats in Regular Admissions

Table A.2 reports what fraction of regular admission seats was allocated to each exam day. Most colleges chose only one exam day for their regular admissions. The remaining a few colleges chose two days. For example, in 1997, Korea University allocated 16 percent of its regular seats to Date A and the rest to Date B. This is because Korea University chose Date A for its highly competitive majors, such as law majors.

Table A.2 Percent of Regular Admission Seats Allocated to Each Day

Panel A: 1994 to 1997

		1994			1995			1996			1997		
		A	B	C	A	B	C	A	B	C	A	B	C
1	Seoul National	100	0	0	100	0	0	100	0	0	100	0	0
2	Korea	100	0	0	100	0	0	0	100	0	16	84	0
	Yonsei	100	0	0	100	0	0	0	100	0	0	100	0
	Postech	0	100	0	0	100	0	0	100	0	0	100	0
3	Sogang	100	0	0	100	0	0	0	100	0	0	100	0
	Ewha women's	100	0	0	100	0	0	0	100	0	0	100	0
	Pusan	100	0	0	100	0	0	0	100	0	0	100	0
	Kyungbook	100	0	0	100	0	0	0	100	0	0	0	100
	Joongang	100	0	0	100	0	0	100	0	0	100	0	0
	Hanyang	100	0	0	100	0	0	0	97	3	0	95	5
	Kyunghee	100	0	0	100	0	0	100	0	0	0	100	0
	Seongkyunkwan	0	0	100	100	0	0	100	0	0	0	100	0
	Hankook	0	0	100	0	0	100	0	0	100	0	0	100

Panel B: 1998 to 2001

		1998			1999			2000			2001		
		A	B	C	A	B	C	A	B	C	A	B	C
1	Seoul National	100	0	0	100	0	0	100	0	0	100	0	0
2	Korea	11	89	0	0	100	0	0	100	0	0	100	0
	Yonsei	0	100	0	0	100	0	0	100	0	11	89	0
	Postech	0	100	0	0	100	0	0	100	0	0	100	0
3	Sogang	60	40	0	100	0	0	100	0	0	100	0	0
	Ewha women's	0	100	0	0	100	0	0	100	0	0	100	0
	Pusan	0	100	0	0	100	0	24	76	0	28	72	0
	Kyungbook	0	100	0	0	100	0	0	100	0	0	100	0
	Joongang	100	0	0	100	0	0	92	8	0	100	0	0
	Hanyang	0	97	3	0	95	5	0	97	3	20	80	0
	Kyunghee	0	86	14	0	80	20	0	100	0	0	100	0
	Seongkyunkwan	0	100	0	0	100	0	0	100	0	0	100	0
	Hankook	0	0	1.00	0	0	100	100	0	0	100	0	0

2. Proofs of MLRP Properties

Proof of MLRP Property 1: Consider any value **y*** and show that the FOSD property holds at **y***, i.e. $\mathbf{F_x(y^*)} \leq \mathbf{F_{x'}(y^*)}$. By definition of MLRP, the ratio $\mathbf{f_x(y)} / \mathbf{f_{x'}(y)}$ is increasing in **y**. Define $\alpha = \mathbf{f_x(y^*)} / \mathbf{f_{x'}(y^*)}$.

If $\alpha \leq 1$, then $\mathbf{F_s(v^*)} = \int_0^{v^*} f(v\,|\,s)\,dv \leq \int_0^{v^*} \alpha f(v\,|\,s')\,dv = \alpha \mathbf{F_{x'}(y^*)}$ and so $\mathbf{F_x(y^*)} \leq \mathbf{F_{x'}(y^*)}$.

If $\alpha > 1$, then $1 - \mathbf{F_x(y^*)} = \int_{v^*}^{\infty} f(v\,|\,s)\,dv \geq \int_{v^*}^{\infty} \alpha f(v\,|\,s')\,dv = 1 - \alpha \mathbf{F_{x'}(y^*)}$ and so $\mathbf{F_x(y^*)} \leq \mathbf{F_{x'}(y^*)}$.

Proof of MLRP Property 2: This is a standard implication of First Order Stochastic Dominance.

Proof of MLRP Property 3: The conditional distribution $\mathbf{f(v \mid x, y < r)} = \gamma\, \mathbf{f(y \mid x)}$ for $\mathbf{y < r}$ where γ is a renormalizing constant given by $\gamma = 1 / \mathbf{F_x(r)}$. So the ratio $\mathbf{f(y \mid x, y < r)} / \mathbf{f(y \mid x', y < r)} = \gamma \mathbf{f_x(y)} / \gamma' \mathbf{f_{x'}(y)}$, where γ and γ' are constants. So since $\mathbf{f_x(y)} / \mathbf{f_{x'}(y)}$ is increasing in **v** by MLRP, $\mathbf{F_x(y \mid y < r)} / \mathbf{f_{x'}(y \mid y < r)}$ is also increasing in v. Thus, MLRP for **x** and **y** continues to hold given the new information that $\mathbf{y < r}$. Then by Properties 1 and 2, the conditional expectation $\mathbf{E(y \mid x, y < r)}$ is increasing in **x**.

Proof of MRLP Property 4:

The ratio $\mathbf{P_x(y > r_1)} / \mathbf{P_x(y > r_2)}$ can be written as $\dfrac{\int_{r_1}^{\infty} f(v\,|\,s)\,dv}{\int_{r_2}^{\infty} f(v\,|\,s)\,dv} = \mathbf{P(y > r_1 \mid y > r_2, x_i = x)}$.

As discussed in the proof of MLRP Property 3, the MLRP property for **y** and **x** carries over to conditional distributions of **y** – in this case, the conditional distribution given that $\mathbf{y > r_2}$. Thus, by MLRP Property 1, $\mathbf{P(y > r_1 \mid y > r_2, x_i = x)}$ is increasing in **x**.

3. Results when Colleges Can Admit Students on Multiple Regular Admission Dates

Proposition: In Regime 1, if each college chooses the number of students to admit on each of the two admissions dates, then in the limit as $\mathbf{u_2} \to 1$, the colleges will choose to admit all students on the same date.

Proof: STEP 1: College 1 does not admit any students who apply originally to College 2. College 2 uses a higher admission threshold on Day 1 than Day 2 if it admits students on both days.

Suppose that College 1 and College 2 assign a positive number of admission slots to both Day 1 and Day 2. In equilibrium, the colleges will use admission thresholds, which we index by $\mathbf{y^{jt}}$ for college $\mathbf{j}$ at time $\mathbf{t}$, and the students will choose where to apply on the basis of these (anticipated) thresholds. Then the admission thresholds will emerge endogenously to ensure that each college actually admits the announced number of students on each admissions day.

If $\mathbf{y^{12}} \leq \mathbf{y^{21}}$, so that College 1 applies a lower threshold for admission at Day 2 than that of College 2 at Day 1, then no student would apply to College 2 on Day 1, since it would be preferable instead to wait to apply to College 1. So in equilibrium, it must be that $\mathbf{y^{12}} > \mathbf{y^{21}}$. But given that $\mathbf{y^{12}} > \mathbf{y^{21}}$, students who apply to College 2 at Day 1 and are not admitted would also not be admitted to College 1 at Day 2. That is, College 1 only admits students (on either Day 1 or Day 2) who apply to College 1 immediately on Day 1. That is, any equilibrium where College 1 admits students on both days is equivalent to an equilibrium where College 1 admits students only on Day 1. Thus, we define College 1's ultimate admissions threshold as $\mathbf{y^1} = \min(\mathbf{y^{11}}, \mathbf{y^{12}})$ and we observe that $\mathbf{y^1} > \mathbf{y^{21}}$ is necessary for any students to apply to College 2 on Day 1 in equilibrium.

If $\mathbf{y^{22}} \leq \mathbf{y^{21}}$, then once again no student would apply to College 2 on Day 1, since it would be preferable to apply to College 1 on Day 1 and then to College 2 on Day 2 if not admitted to College 1. So in equilibrium, it must also be that $\mathbf{y^{22}} > \mathbf{y^{21}}$.

STEP 2: Students with highest x-values apply to College 1 on Day 1.

Applying to College 1 on Day 1 yields utility (to applicant **i**) of $\mathbf{u_1}$ if $\mathbf{y_i} \geq \mathbf{y^1}$, $\mathbf{u_2}$ if $\mathbf{y^{22}} \leq \mathbf{y_i} < \mathbf{y^1}$ and 0 if $\mathbf{y_i} < \mathbf{y^{22}}$. By contrast, applying to College 2 on Day 1 yields utility $\mathbf{u_2}$ if $\mathbf{y_i} \geq \mathbf{y^{21}}$ and 0 otherwise. Defining the function $\mathbf{h}(\mathbf{y_i})$ to be the difference in these utilities,

$$\mathbf{h}(\mathbf{y_i}) = -\mathbf{u_2} \text{ if } \mathbf{y^{21}} \leq \mathbf{y^i} < \mathbf{y^{22}}; \quad \mathbf{h}(\mathbf{y_i}) = \mathbf{u_1} \text{ if } \mathbf{y^1} \leq \mathbf{y^i}; \quad \mathbf{h}(\mathbf{y_i}) = 0 \text{ otherwise.}$$

As shown in the proof of MLRP Property 3, MLRP between **x** and **y** also applies to the conditional distribution of **y** given that $\mathbf{x} \geq \mathbf{y^{21}}$. Since $\mathbf{h}(\mathbf{y_i})$ is an increasing function for $\mathbf{y} \geq \mathbf{y^{21}}$, (and since MLRP implies First Order Stochastic Dominance), $\mathbf{E}(\mathbf{h}(\mathbf{y_i}) \mid \mathbf{x}, \mathbf{y} \geq \mathbf{y^{21}})$ is increasing in **x**. Thus, there is some cutoff **x'** such that $\mathbf{E}(\mathbf{h}(\mathbf{y_i}) \mid \mathbf{x}, \mathbf{y} \geq \mathbf{y^{21}})$, is positive iff $\mathbf{x} \geq \mathbf{x'}$. Finally, since $\mathbf{h}(\mathbf{y_i}) = 0$ for $\mathbf{y} < \mathbf{y^{21}}$, $\mathbf{E}(\mathbf{h}(\mathbf{y_i}) \mid \mathbf{x})$ is just a scaled version of $\mathbf{E}(\mathbf{h}(\mathbf{y_i}) \mid \mathbf{x}, \mathbf{y} \geq \mathbf{y^{21}})$ and is also positive iff $\mathbf{x} \geq \mathbf{x'}$. That is, applicants will apply to College 1 on Day 1 if $\mathbf{x} \geq \mathbf{x'}$ and otherwise will apply to College 2 on Day 1.

<u>STEP 3: In the limit as $\mathbf{u_2}$ approaches $\mathbf{u_1}$, College 2 achieves higher expected payoff if it admits all students on one day than if it admits some students on each admissions day.</u>

Define $\mathbf{y^{1S}}$ and $\mathbf{y^{2S}}$ as the admissions thresholds for the two colleges when both colleges admit all students on Day 1. We know from above that $\mathbf{y^{22}} > \mathbf{y^{21}}$ if College 2 admits students on both admissions days. We also know from Proposition 2 that the two admission thresholds converge to the same value $\mathbf{y_{(2K)}}$ in the limit as $\mathbf{u_2}$ approaches $\mathbf{u_1}$.

If College 2 assigns some of its admissions slots to Day 2, then fewer than 2**K** students will be admitted on Day 1. Further, since $\mathbf{y^{21}} < \mathbf{y^1}$ (and all students apply to one college or the other on Day 1), then $\mathbf{y^1}$ must be strictly greater than $\mathbf{y_{(2K)}}$ – otherwise more than 2**K** students will be admitted on Day 1. Then in the limit as $\mathbf{u_2}$ approaches $\mathbf{u_1}$, College 1 attracts more applicants, uses a higher admissions threshold and achieves higher expected utility when College 2 admits students on both admissions days rather than just on Day 1.

From the perspective of College 2, since (1) $\mathbf{y^{22}} > \mathbf{y^{21}}$ and (2) all students who are rejected by College 1 on Day 1 then apply to College 2 on Day 2, $\mathbf{y^{22}}$ must be strictly greater than $\mathbf{y_{(2K)}}$ as otherwise once again more than 2**K** students will be admitted in total to the two colleges. In the limit as $\mathbf{u_2}$ approaches $\mathbf{u_1}$, the aggregated utility of the two colleges is less when College 2 admits students on both days than when all students are admitted on Day 1, since the limiting equilibrium with simultaneous admissions produces an efficient admissions rule (students with $\mathbf{y_i} \geq \mathbf{y_{(2K)}}$ are admitted and others are not) and the limiting equilibrium where College 2 admits students on both days does not. So we know that College 1 gains utility and aggregate utility for the two colleges declines in equilibrium if College 2 admits students on both days rather than just on Day 1. Thus, it must be that College 2 loses in expected utility by admitting students on both days rather than just on Day 1.

4. Proof of Existence of Equilibrium in Regimes 2M and 2S:

Step 1: *College 2 has a unique best response [$\mathbf{e_{C2}}(\mathbf{e_{C1}}, \mathbf{v_{C1}})$, $\mathbf{v_{C2}}(\mathbf{e_{C1}}, \mathbf{v_{C1}})$] to each pair of thresholds for College 1 in Regime 2M.*

Proof: Given ($\mathbf{e_{C1}}$, $\mathbf{v_{C1}}$, $\mathbf{e_{C2}}$) such that College 2 admits **K** or fewer applicants early, there is a unique choice of regular admission threshold, $\mathbf{v_{C2}}$, for College 2 to fill its class. A marginal early admit to College 2 is either (1) a student who would otherwise be admitted as a regular applicant to College 1 or (2) a student who would not be admitted to College 1 or College 2 as a regular applicant. So the marginal value of an early admit to College 2 is a weighted average of (A) E($\mathbf{v_{i2}}$ | $\mathbf{y_i} = \mathbf{e_{C2}}$) and (B) E($\mathbf{v_{i2}}$ | $\mathbf{y_i} = \mathbf{e_{C2}}$, $\mathbf{v}_{i1} < \mathbf{v_{C1}}$, $\mathbf{v_{i2}} < \mathbf{v_{C2}}$), where the weights are proportional to (A) P($\mathbf{v}_{i1} > \mathbf{v_{C1}}$ | $\mathbf{y_i} = \mathbf{e_{C2}}$) and (B) P($\mathbf{v_{i1}} < \mathbf{v_{C1}}$, $\mathbf{v_{i2}} < \mathbf{v_{C2}}$ | $\mathbf{y_i} = \mathbf{e_{C2}}$).

Fix ($\mathbf{e_1}$, $\mathbf{v_{C1}}$) and suppose that College 2 increases $\mathbf{e_{C2}}$ to $\mathbf{e_{C2}}' > \mathbf{e_{C2}}$ but does not change $\mathbf{v_{C2}}$. Then comparing values (A) and (B) and their weights in the marginal value computation when we change the marginal early test score for $\mathbf{e_{C2}}$ to $\mathbf{e_{C2}'} > \mathbf{e_{C2}}$, the conditional expectations (A) and (B) increase and weight for (A) increases while weight for (B) falls since test scores and abilities satisfy MLRP. So the expected ability of the marginal early admit will increase with $\mathbf{e_{C2}}$ under these circumstances.

In fact, for College 2 to maintain enrollment at **K**, it must reduce $\mathbf{v_{C2}}$ when it increases $\mathbf{e_{C2}}$ – that is, it must increase the number of regular admits to compensate for a reduction in early admits. This change in $\mathbf{v_{C2}}$ eliminates some applicants from the earlier computation of expected value – specifically those with $\mathbf{v_{i2}}$ values just below $\mathbf{v_{C2}}$, which could change the outcome of the comparative static computation. But if the average marginal value of an early admit to College 2 is greater or equal to $\mathbf{v_{C2}}$, candidates with $\mathbf{v_{i2}} < \mathbf{v_{C2}}$ are worse than the average marginal value and so eliminating them can only help College 2. That is, if the expected value of a marginal early admit to College 2 is (weakly) greater than $\mathbf{v_{C2}}$, then this expected value is increasing in $\mathbf{e_{C2}}$. Further, since $\mathbf{v_{C2}}$ must fall when $\mathbf{e_{C2}}$ rises for fixed ($\mathbf{e_{C1}}$, $\mathbf{v_{C1}}$), there can be at most one combination ($\mathbf{e_2}$, $\mathbf{v_{C2}}$) for each ($\mathbf{e_1}$, $\mathbf{v_{C1}}$) such that College 2 is indifferent between admitting an early applicant with $\mathbf{y_2} = \mathbf{e_2}$ and a regular applicant with $\mathbf{v_{i2}} = \mathbf{v_{C2}}$.

Now consider the boundary options for College 2 in response to ($\mathbf{e_{C1}}$, $\mathbf{v_{C1}}$). At the lowest plausible early application cutoff, College 2 fills its class with early applicants and sets its regular admission cutoff to $\mathbf{v_{C2}} = \overline{V}$. At this point, College 2 clearly prefers its marginal regular applicant to its marginal early applicant.[1] As we increase $\mathbf{e_{C2}}$ to its highest plausible value $\mathbf{e_{C2}} = \mathbf{e_{C1}}$, there is either a single interior optimum (where marginal early and regular applicants have the same expected ability v_{i2}), or College 2 prefers its marginal regular applicant at $\mathbf{v_{i2}} = \mathbf{y_{C2}}$ to its marginal early applicant at $\mathbf{y_i} = \mathbf{e_{C2}}$ for each possible $\mathbf{e_{C2}}$, in which case, College 2's optimal policy is to admit only regular applicants. In either case, there is a unique best response for College 2 to College 1's fixed thresholds ($\mathbf{e_{C1}}$, $\mathbf{v_{C1}}$).

Step 2: *In Regime 2M, there exists an equilibrium [($\mathbf{e_1}$, $\mathbf{r_1}$), ($\mathbf{e_2}$, $\mathbf{r_2}$)], where each college's choice of admission thresholds is a best response to the other college's admission thresholds.*

Proof: When students can submit multiple regular applications, the marginal value of an early admit to College 1 when $\mathbf{e_1} = \mathbf{y_1}$ is simply $\mathbf{E}(\mathbf{v_{i1}} \mid \mathbf{s} = \mathbf{e_1})$. For any interior choice of early admission cutoff with $\mathbf{e_{1L}} < \mathbf{e_1} < \overline{S}$, College 1 must be indifferent between a marginal early applicant and a marginal regular applicant, so the regular admission threshold is implicitly defined by the early admission threshold i.e. $\mathbf{r_1}(\mathbf{e_1}) = \mathbf{E}(\mathbf{v_{i1}} \mid \mathbf{y_i} = \mathbf{e_1})$.

[1] Intuitively, since we assume full support over ($\mathbf{y_i}$, $\mathbf{v_{ij}}$), College 2 can always find a higher ability regular applicant with a low score $\mathbf{y_i}$ who is preferred to a marginal early applicant.

From Lemma 2 we know that College 2 has a unique best response [$\mathbf{e_2}(\mathbf{e_1}, \mathbf{r_1})$, ($\mathbf{v_{C2}}(\mathbf{e_1}, \mathbf{r_1})$)] to each pair of thresholds ($\mathbf{e_1}$, $\mathbf{r_1}(\mathbf{e_1})$) for College 1. By construction each college is indifferent between its marginal early and regular applicants, so if College 1 enrolls exactly $\mathbf{K}$ students given these thresholds [($\mathbf{e_1}$, $\mathbf{r_1}(\mathbf{e_1})$), [$\mathbf{e_2}(\mathbf{e_1}, \mathbf{r_1}(\mathbf{e_1}))$, ($\mathbf{r_2}(\mathbf{e_1}, \mathbf{r_1}(\mathbf{e_1}))$)]], then we have identified an equilibrium.

College 1's enrollment is continuous in $\mathbf{e_1}$ given thresholds [($\mathbf{e_1}$, $\mathbf{v_{C1}}$), [$\mathbf{e_2}(\mathbf{e_1}, \mathbf{v_{C1}})$, ($\mathbf{r_2}(\mathbf{e_1}, \mathbf{v_{C1}})$)]] and further College 1's enrollment is greater than $\mathbf{K}$ at $\mathbf{e_1} = \mathbf{e_{1L}}$, since it would still admit additional regular applicants in addition to $\mathbf{K}$ early applicants. Thus, there is either an interior equilibrium at some $\mathbf{e_1}$ with $\mathbf{e_{1L}} < \mathbf{e_1} < \mathbf{S}$, or College 1 always admits too many students for each ($\mathbf{e_1}$, $\mathbf{r_1}$), including the boundary case with $\mathbf{e_1} = \mathbf{S}$, $\mathbf{r_1} = \mathbf{E}(\mathbf{v_{i1}} \mid \mathbf{e_1} = \mathbf{S})$. If there is no interior equilibrium, then consider options with ($\mathbf{e_1} = \mathbf{S}$, $\mathbf{r_1} > \mathbf{E}(\mathbf{v_{i1}} \mid \mathbf{e_1} = \mathbf{S})$), where College 1 strictly prefers its marginal regular applicant to its marginal early applicant, but does not enroll any early applicants. In this case, simply increase $\mathbf{r_1}$ until the point where College 1 exactly fills its class with regular applicants (and no early admits), and by construction, this cutoff for regular admission to College 1 produces an equilibrium.

Step 3: *There is a unique equilibrium in the early application game in Regime 2S.*

Proof: In Regime **2S**, early applicants with $\mathbf{y_i} > \mathbf{e_{Cj}}$ enroll at school $\mathbf{j}$ and have expected contribution $\mathbf{E}(\mathbf{v_{ij}} \mid \mathbf{y_i})$, while regular applicants to school $\mathbf{j}$ with $\mathbf{v_{ij}} > \mathbf{r_{Cj}}$ enroll at school $\mathbf{j}$ (as they can't apply to any the other school) with contribution $\mathbf{v_{ij}}$. In equilibrium, each school must be indifferent between admitting marginal early and marginal regular applicants as they set the early and regular admission thresholds simultaneous. That is, $\mathbf{r_{Cj}} = \mathbf{E}(\mathbf{v_{ij}} \mid \mathbf{y_i} = \mathbf{e_{Cj}})$ so that each college's regular threshold is a (strictly decreasing) function of its early threshold. Assume for the rest of this proof the colleges use admission thresholds of form ($\mathbf{e_{Cj}}$, $\mathbf{r_{Cj}} = \mathbf{E}(\mathbf{v_{ij}} \mid \mathbf{y_i} = \mathbf{e_{Cj}})$). Then we can summarize each college's strategy by its early admission threshold.

For each possible value of $\mathbf{e_{C1}}$, College 2's enrollment is strictly decreasing in $\mathbf{e_{C2}}$. Thus, there is a unique value $\mathbf{e_{C2}}(\mathbf{e_{C1}})$ that yields enrollment $\mathbf{K}$ for College 2. Further, $\mathbf{e_{C2}}(\mathbf{e_{C1}})$ is strictly increasing in $\mathbf{e_{C1}}$ as an increase in $\mathbf{e_{C1}}$ expands the pool of applicants, both early and regular, who are available to College 2. By construction, this value $\mathbf{e}_{C2}(\mathbf{e}_{C1})$ is College 2's unique best response to $\mathbf{e}_{C1}$. Assuming this best response by College 2 to $\mathbf{e}_{C1}$, College 1's implied regular threshold $\mathbf{r}_{C1}$ and both of College 2's admission thresholds are strictly increasing in $\mathbf{e}_{C1}$, so total enrollment at the two schools is strictly decreasing in $\mathbf{e}_{C1}$. For $\mathbf{e}_{C1}$ very small, more than **2K** students will be admitted, so either (1) there is a unique choice of $\mathbf{e}_{C1}$ between 0 and 1 that yields total enrollment 2K or (2) total enrollment is greater than **2K** for all possible values of $\mathbf{e}_{C1}$, including $\mathbf{e}_{C1}$ = 1. In Case (1), each college enrolls **K** students and by construction, we have identified a unique equilibrium. In Case (2), there is a boundary solution where College 1 does not admit any early applicants and chooses $\mathbf{r}_{C1} > \mathbf{E}(\mathbf{v}_{ij} \mid \mathbf{y}_i = \mathbf{e}_{Cj})$ to admit exactly **K** students. In either case, we have identified a unique equilibrium in Regime **2S**.