B Additional Analytical Results (Online Appendix)

This appendix describes the aggregate elasticity of substitution between capital and labor in a variety of environments. Web Appendix B.1 describes local elasticities of substitution and Web Appendix B.2 derives a preliminary result under the assumption each plant's production function is homothetic. The assumption of constant returns to scale is relaxed in Web Appendix B.3. Web Appendix B.4 introduces misallocation frictions. Web Appendix B.5 generalizes the demand system to allow for arbitrary elasticities of demand and imperfect pass-through. Web Appendix B.6 relaxes the assumption that production functions are homothetic.

As in Appendix A, we use the following notation for relative factor prices: $\omega \equiv \frac{w}{r}$ and $\mathfrak{q} \equiv \frac{q}{r}$. In addition, we define $p_{ni} \equiv P_{ni}/r$ and $p_n \equiv P_n/r$ to be plant i's and industry n's prices respectively normalized by the rental rate. It will also be useful to define i's cost function (normalized by r) to be

$$z_{ni}(Y_{ni}, \omega, \mathfrak{q}) = \min_{K_{ni}, L_{ni}, M_{ni}} K_{ni} + \omega L_{ni} + \mathfrak{q} M_{ni} \qquad \text{subject to} \qquad F_{ni}(K_{ni}, L_{ni}, M_{ni}) \geq Y_{ni}$$

As in Appendix A, two results will be used repeatedly. First, Shephard's lemma implies that for each i:

$$(1 - s^M_{ni})(1 - \alpha_{ni}) = \frac{z_{ni\omega}(Y_{ni}, \omega, \mathfrak{q})\omega}{z_{ni}(Y_{ni}, \omega, \mathfrak{q})} \tag{23}$$

$$s^M_{ni} = \frac{z_{ni\mathfrak{q}}(Y_{ni}, \omega, \mathfrak{q})\mathfrak{q}}{z_{ni}(Y_{ni}, \omega, \mathfrak{q})} \tag{24}$$

Second, $\alpha_n = \sum_{i \in I_n} \alpha_{ni}\theta_{ni}$, so for any quantity κ_n,

$$\sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\kappa_n \theta_{ni} = 0 \tag{25}$$

B.1 Locally-Defined Elasticities

In our baseline analysis we assumed that plant i produced using a nested CES production function of the form

$$F_{ni}(K_{ni}, L_{ni}, M_{ni}) = \left(\left[(A_{ni}K_{ni})^{\frac{\sigma-1}{\sigma}} + (B_{ni}L_{ni})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}\frac{\zeta-1}{\zeta}} + (C_{ni}M_{ni})^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}$$

In that context, σ was i's elasticity of substitution between capital and labor and ζ was i's elasticity of substitution between materials and i's capital-labor bundle.

When i's production function does not take this parametric form, we define local elasticities of substitution. Suppose that i produces using the production function $Y_{ni} = F_{ni}(K_{ni}, L_{ni}, M_{ni})$ with corresponding cost function z_{ni}. We define σ_{ni} and ζ_{ni} to satisfy

$$\sigma_{ni} - 1 = \left. \frac{d \ln \frac{\alpha_{ni}}{1-\alpha_{ni}}}{d \ln \omega} \right|_{Y_{ni} \text{ is constant}}$$

$$(\alpha_{ni} - \alpha^M)(\zeta_{ni} - 1) = \left. \frac{d \ln \frac{1-s^M_{ni}}{s^M_{ni}}}{d \ln \omega} \right|_{Y_{ni} \text{ is constant}}$$

σ_{ni} and ζ_{ni} measure how i's relative factor usage changes in response to changes in relative factor prices holding i's output fixed (as one moves along an isoquant). That output remains fixed is relevant only if production functions are non-homothetic, in which case a change in a plant's scale would alter its relative factor usage. This section derives expressions for σ_{ni} and ζ_{ni} in terms of i's cost function.

Claim 1 *σ_{ni} and ζ_{ni} satisfy*

$$\begin{aligned}(\alpha_{ni}-\alpha^M)\zeta_{ni} &= -\frac{1}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\\ \sigma_{ni} &= -\frac{1}{\alpha_{ni}}\left\{\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\frac{s_{ni}^M}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\right\}\end{aligned}$$

Proof. Differentiating equation (24) and equation (23) with respect to ω gives

$$\begin{aligned}\frac{d\ln s_{ni}^M}{d\ln\omega} &= \left\{\begin{array}{c}\frac{z_{ni\mathfrak{q}Y}Y_{ni}}{z_{ni\mathfrak{q}}}\frac{d\ln Y_{ni}}{d\ln\omega}+\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}\frac{d\ln\mathfrak{q}}{d\ln\omega}+\frac{d\ln\mathfrak{q}}{d\ln\omega}\\ -\frac{z_{niY}Y_{ni}}{z_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}-\frac{z_{ni\omega}\omega}{z_{ni}}-\frac{z_{ni\mathfrak{q}}\mathfrak{q}}{z_{ni}}\frac{d\ln\mathfrak{q}}{d\ln\omega}\end{array}\right\}\\ \frac{d\ln(1-s_{ni}^M)}{d\ln\omega}+\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} &= \left\{\begin{array}{c}\frac{z_{ni\omega Y}Y_{ni}}{z_{ni\omega}}\frac{d\ln Y_{ni}}{d\ln\omega}+\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}\frac{d\ln\mathfrak{q}}{d\ln\omega}+1\\ -\frac{z_{niY}Y_{ni}}{z_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}-\frac{z_{ni\omega}\omega}{z_{ni}}-\frac{z_{ni\mathfrak{q}}\mathfrak{q}}{z_{ni}}\frac{d\ln\mathfrak{q}}{d\ln\omega}\end{array}\right\}\end{aligned}$$

Imposing $\frac{d\ln Y_{ni}}{d\ln\omega}=0$, $\frac{d\ln\mathfrak{q}}{d\ln\omega}=1-\alpha^M$, and Shephard's lemma, these equations can be written as

$$\begin{aligned}\frac{d\ln s_{ni}^M}{d\ln\omega} &= \frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)+(1-\alpha^M)-(1-s_{ni}^M)(1-\alpha_{ni})-s_{ni}^M(1-\alpha^M)\\ \frac{d\ln(1-s_{ni}^M)}{d\ln\omega}+\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} &= \frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+1-(1-s_{ni}^M)(1-\alpha_{ni})-s_{ni}^M(1-\alpha^M)\end{aligned}$$

Simplifying yields

$$\begin{aligned}\frac{d\ln s_{ni}^M}{d\ln\omega} &= \frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)+(1-s_{ni}^M)(\alpha_{ni}-\alpha^M)\\ \frac{d\ln(1-s_{ni}^M)}{d\ln\omega}+\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} &= \frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\alpha_{ni}-s_{ni}^M(\alpha_{ni}-\alpha^M)\end{aligned}$$

Using $\frac{d\ln 1-s_{ni}^M}{d\ln\omega}=-\frac{s_{ni}^M}{1-s_{ni}^M}\frac{d\ln s_{ni}^M}{d\ln\omega}$ and plugging the first into the second yields

$$\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} = \frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\frac{s_{ni}^M}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]+\alpha_{ni}$$

Finally, the definitions of the elasticities imply $\sigma_{ni}-1=-\frac{1}{\alpha_{ni}}\frac{d\ln(1-\alpha_{ni})}{d\ln\omega}$ and $(\alpha_{ni}-\alpha^M)(\zeta_{ni}-1)=-\frac{1}{1-s_{ni}^M}\frac{d\ln s_{ni}^M}{d\ln\omega}$, so that

$$\begin{aligned}(\alpha_{ni}-\alpha^M)\zeta_{ni} &= -\frac{1}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\\ \sigma_{ni} &= -\frac{1}{\alpha_{ni}}\left\{\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\frac{s_{ni}^M}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\right\}\end{aligned}$$

■

B.2 Industry Substitution and Within-Plant Substitution

We now define i's local returns to scale to be $\gamma_{ni} = \left[\frac{z_{niY} Y_{ni}}{z_{ni}}\right]^{-1}$. The next lemma characterizes the within-plant components of industry substitution.

Lemma 1 *Suppose that plant i produces using the homothetic production function F_{ni}. The industry elasticity of substitution for industry n, σ_n^N, can be written as*

$$\sigma_n^N = (1-\chi_n)\bar{\sigma}_n + \chi_n \bar{s}_n^M \bar{\zeta}_n + \frac{\sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\theta_{ni} \frac{1}{\gamma_{ni}} \frac{d \ln Y_{ni}}{d \ln \omega}}{\alpha_n (1-\alpha_n)}$$

where $\bar{\zeta}_n \equiv \sum_{i \in I_n} \frac{(\alpha_{ni}-\alpha_n)(\alpha_{ni}-\alpha^M) s_{ni}^M}{\sum_{j \in I_n} (\alpha_{nj}-\alpha_n)(\alpha_{nj}-\alpha^M) s_{nj}^M} \zeta_{ni}$ *and* $\bar{s}_n^M \equiv \sum_{i \in I_n} \frac{(\alpha_{ni}-\alpha_n)(\alpha_{ni}-\alpha^M)}{\sum_{j \in I_n} (\alpha_{nj}-\alpha_n)(\alpha_{nj}-\alpha^M)} s_{ni}^M$

Proof. Following the steps of the proof of Proposition 1′, we have

$$\begin{aligned} \sigma_n^N &= (1-\chi_n)\sigma_n + \frac{\sum_{i \in I_n} (\alpha_{ni} - \alpha_n) \frac{d\theta_{ni}}{d \ln \omega}}{\alpha_n (1-\alpha_n)} + \chi_n & (26) \\ \theta_{ni} &= \frac{rK_{ni} + wL_{ni}}{\sum_{j \in I_n} rK_{nj} + wL_{nj}} = \frac{(1-s_{ni}^M) z_{ni}}{\sum_{j \in I_n} (1-s_{nj}^M) z_{nj}} \\ \frac{d \ln (1-s_{ni}^M)}{d \ln \omega} &= s_{ni}^M (\zeta_{ni} - 1)(\alpha_{ni} - \alpha^M) \end{aligned}$$

The change in i's expenditure on all inputs depends on its return to scale and its expenditure shares:

$$\begin{aligned} \frac{d \ln z_{ni}(Y_{ni}, \omega, \mathfrak{q})}{d \ln \omega} &= \frac{Y_{ni} z_{niY}}{z_{ni}} \frac{d \ln Y_{ni}}{d \ln \omega} + \frac{z_{ni\omega} \omega}{z_{ni}} + \frac{z_{ni\mathfrak{q}} \mathfrak{q}}{z_{ni}} \frac{d \ln \mathfrak{q}}{d \ln \omega} \\ &= \frac{1}{\gamma_{ni}} \frac{d \ln Y_{ni}}{d \ln \omega} + (1-s_{ni}^M)(1-\alpha_{ni}) + s_{ni}^M (1-\alpha^M) \\ &= \frac{1}{\gamma_{ni}} \frac{d \ln Y_{ni}}{d \ln \omega} + (1-\alpha_{ni}) + s_{ni}^M (\alpha_{ni} - \alpha^M) & (27) \end{aligned}$$

Putting these pieces together, since $\sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\theta_{ni} \frac{d \ln \sum_{j \in I_n} (1-s_{nj}^M) z_{nj}}{d \ln \omega} = 0$, we have

$$\begin{aligned} \sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\theta_{ni} \frac{d \ln \theta_{ni}}{d \ln \omega} &= \sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\theta_{ni} \left[\frac{d \ln 1 - s_{ni}^M}{d \ln \omega} + \frac{d \ln z_{ni}}{d \ln \omega}\right] \\ &= \sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\theta_{ni} \left[\begin{array}{c} s_{ni}^M (\zeta_{ni} - 1)(\alpha_{ni} - \alpha^M) + \frac{1}{\gamma_{ni}} \frac{d \ln Y_{ni}}{d \ln \omega} \\ +(1-\alpha_{ni}) + s_{ni}^M (\alpha_{ni} - \alpha^M) \end{array}\right] \\ &= \sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\theta_{ni} \left[s_{ni}^M \zeta_{ni} (\alpha_{ni} - \alpha^M) + \frac{1}{\gamma_{ni}} \frac{d \ln Y_{ni}}{d \ln \omega} + (1-\alpha_{ni})\right] \end{aligned}$$

Using the definitions of $\bar{\zeta}_n$ and $\bar{s}_n^M$, this becomes

$$\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega} = \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\bar{s}_n^M\bar{\zeta}_n(\alpha_{ni}-\alpha^M)+\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}+(1-\alpha_{ni})\right]$$

Using the fact that for any constant κ, $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\kappa=0$, we can write this as

$$\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\bar{s}_n^M\bar{\zeta}_n(\alpha_{ni}-\alpha_n)+\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}-(\alpha_{ni}-\alpha_n)\right]$$

Finally, we can plug this back into equation (26) to get

$$\sigma_n^N=(1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)}$$

■

B.3 Returns to Scale

This section relaxes the assumption that each plant's production function exhibits constant returns to scale.

Claim 2 *Suppose that i produces using the production function $Y_{ni}=F_{ni}(K_{ni},L_{ni},M_{ni})=G_{ni}(K_{ni},L_{ni},M_{ni})^\gamma$, where G_{ni} has constant returns to scale and $\gamma\leq\frac{\varepsilon_n}{\varepsilon_n-1}$. Let $x=\frac{\varepsilon_n}{\varepsilon_n+\gamma(1-\varepsilon_n)}$. Then the industry elasticity of substitution is*

$$\sigma_n=(1-\chi_n)\bar{\sigma}_n+\chi_n\left[\bar{s}_n^M\bar{\zeta}_n+(1-\bar{s}_n^M)x\right]$$

and the revenue-cost ratio is $\frac{P_{ni}Y_{ni}}{rK_{ni}+wL_{ni}+qM_{ni}}=\frac{x}{x-1}$.

Proof. i's optimal price is $p_{ni}=\frac{\varepsilon_n}{\varepsilon_n-1}z_{niY}(Y_{ni},\omega,\mathfrak{q})$, so differentiating yields

$$\frac{d\ln p_{ni}}{d\ln\omega}=\frac{z_{niYY}Y_{ni}}{z_{niY}}\frac{d\ln Y_{ni}}{d\ln\omega}+\frac{z_{niY\omega}\omega}{z_{niY}}+\frac{z_{niY\mathfrak{q}}\mathfrak{q}}{z_{niY}}\frac{d\ln\mathfrak{q}}{d\ln\omega}$$

The production function implies that $\frac{z_{niYY}Y_{ni}}{z_{niY}}=\frac{1}{\gamma}-1$, $\frac{z_{niY\omega}\omega}{z_{niY}}=(1-\alpha_{ni})(1-s_{ni}^M)$, and $\frac{z_{niY\mathfrak{q}}\mathfrak{q}}{z_{niY}}=s_{ni}^M$, so this can be written as

$$\frac{d\ln p_{ni}}{d\ln\omega}=\left(\frac{1}{\gamma}-1\right)\frac{d\ln Y_{ni}}{d\ln\omega}+(1-\alpha_{ni})(1-s_{ni}^M)+s_{ni}^M(1-\alpha^M)$$

The change in i's output is then

$$\begin{aligned}\frac{d\ln Y_{ni}}{d\ln\omega} &= -\varepsilon_n\frac{d\ln p_{ni}}{d\ln\omega}+\frac{d\ln Y_np_n^{\varepsilon_n}}{d\ln\omega}\\ &= -\varepsilon_n\left(\frac{1}{\gamma}-1\right)\frac{d\ln Y_{ni}}{d\ln\omega}-\varepsilon_n\left[(1-\alpha_{ni})(1-s_{ni}^M)+s_{ni}^M(1-\alpha^M)\right]+\left[\frac{d\ln Y_np_n^{\varepsilon_n}}{d\ln\omega}\right]\end{aligned}$$

This can be rearranged as

$$\frac{d\ln Y_{ni}}{d\ln\omega}=\gamma x\left[(\alpha_{ni}-\alpha_n)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]+\frac{x\gamma}{\varepsilon_n}\left[\frac{d\ln Y_np_n^{\varepsilon_n}}{d\ln\omega}-\varepsilon_n(1-\alpha_n)\right]$$

Using Lemma 1 and the fact that $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln Y_n p_n^{\varepsilon_n}}{d\ln\omega}-\varepsilon_n(1-\alpha_n)\right]=0$ gives

$$\begin{aligned}\sigma_n^N &= (1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}x\left[(\alpha_{ni}-\alpha_n)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]}{\alpha_n(1-\alpha_n)}\\ &= (1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}x\left[(\alpha_{ni}-\alpha_n)-\bar{s}_n^M(\alpha_{ni}-\alpha^M)\right]}{\alpha_n(1-\alpha_n)}\end{aligned}$$

where the second line uses the definition of $\bar{s}_n^M$. The desired result follows using $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{s}_n^M(\alpha^M-\alpha_n)=0$ and the definition of χ_n.

Finally, since $p_{ni}=\frac{\varepsilon_n}{\varepsilon_n-1}z_{niY}$, the revenue cost ratio is

$$\frac{P_{ni}Y_{ni}}{rK_{ni}+wL_{ni}+qM_{ni}}=\frac{p_{ni}Y_{ni}}{z_{ni}}=\frac{\varepsilon_n}{\varepsilon_n-1}\frac{z_{niY}Y_{ni}}{z_{ni}}=\frac{\varepsilon_n}{\varepsilon_n-1}\frac{1}{\gamma}=\frac{x}{x-1}$$

■

B.4 Adjustment costs and Misallocation Frictions

Suppose that $\{T_{Kni},T_{Lni},T_{Yni}\}_{i\in I_n}$ represent wedges that are paid while $\{\tau_{Kni},\tau_{Lni},\tau_{Yni}\}_{i\in I_n}$ represent wedges that are unpaid. Then, for example, if w is the overall wage level then plant i pays a wage of $T_{Lni}w$, but its shadow cost of labor is $\tau_{Lni}T_{Lni}w$. Then i acts as if it maximizes

$$\tau_{Yni}T_{Yni}P_{ni}Y_{ni}-\tau_{Kni}T_{Kni}rK_{ni}-\tau_{Lni}T_{Lni}wL_{ni}$$

subject to $Y_{ni}\leq F_{ni}(K_{ni},L_{ni})$ and $Y_{ni}\leq Y_nP_n^{\varepsilon_n}D_{ni}P_{ni}^{-\varepsilon_n}$. We define, α_{ni}, θ_{ni}, α_n, and z_{ni} as summarizing payments to factors

$$\begin{aligned}z_{ni} &= T_{Kni}K_{ni}+\omega T_{Lni}L_{ni}\\ \alpha_{ni} &= \frac{T_{Kni}rK_{ni}}{T_{Kni}rK_{ni}+T_{Lni}wL_{ni}}\\ \alpha_n &= \frac{\sum_{i\in I_n}T_{Kni}rK_{ni}}{\sum_{i\in I_n}T_{Kni}rK_{ni}+T_{Lni}wL_{ni}}=\sum_{i\in I_n}\alpha_{ni}\theta_{ni}\\ \theta_{ni} &= \frac{T_{Kni}rK_{ni}+T_{Lni}wL_{ni}}{\sum_{j\in I_n}T_{Knj}rK_{nj}+T_{Lnj}wL_{nj}}=\frac{z_{ni}}{\sum_{j\in I_n}z_{nj}}\end{aligned}$$

We define the same variables with hats to summarize shadow costs:

$$\begin{aligned}\hat{z_{ni}} &= \tau_{Kni}T_{Kni}K_{ni}+\tau_{Lni}T_{Lni}\omega L_{ni}\\ \hat{\alpha_{ni}} &= \frac{\tau_{Kni}T_{Kni}rK_{ni}}{\tau_{Kni}T_{Kni}rK_{ni}+\tau_{Lni}T_{Lni}wL_{ni}}\\ \hat{\alpha_n} &= \frac{\sum_{i\in I_n}T_{Kni}rK_{ni}}{\sum_{i\in I_n}T_{Kni}rK_{ni}+T_{Lni}wL_{ni}}\end{aligned}$$

Claim 3 *Suppose each F_{ni} has a constant elasticity of substitution between capital and labor, σ_{ni}.*

The industry elasticity of substitution, which satisfies $\sigma_n^N - 1 = \frac{d\ln \frac{\sum_{i\in I_n} T_{Kni}K_{ni}}{\sum_{i\in I_n} T_{Lni}\omega L_{ni}}}{d\ln\omega}$, *is*

$$\sigma_n^N = (1-\hat{\chi}_n)\bar{\sigma}_n + \hat{\chi}_n\varepsilon_n$$

where $\hat{\chi}_n \equiv \sum_{i\in I_n} \frac{(\alpha_{ni}-\alpha_n)(\hat{\alpha_{ni}}-\hat{\alpha_n})\theta_{ni}}{\alpha_n(1-\alpha_n)}$

Proof. To compute the industry elasticity of substitution we have

$$\frac{d\ln\alpha_{ni}}{d\ln\omega} = \frac{d\ln \frac{\frac{T_{Kni}K_{ni}}{T_{Lni}\omega L_{ni}}}{\frac{T_{Kni}K_{ni}}{T_{Lni}\omega L_{ni}}+1}}{d\ln\omega} = \frac{T_{Lni}\omega L_{ni}}{T_{Kni}K_{ni}+T_{Lni}\omega L_{ni}}\frac{d\ln\frac{T_{Kni}K_{ni}}{T_{Lni}\omega L_{ni}}}{d\ln\omega} = (1-\alpha_{ni})(\sigma_{ni}-1)$$

and similarly

$$\frac{d\ln\alpha_n}{d\ln\omega} = \frac{d\ln\left[\frac{\sum_{i\in I_n} T_{Kni}K_{ni}}{\sum_{i\in I_n} T_{Lni}\omega L_{ni}} \Big/ \left(\frac{\sum_{i\in I_n} T_{Kni}K_{ni}}{\sum_{i\in I_n} T_{Lni}\omega L_{ni}}+1\right)\right]}{d\ln\omega} = (1-\alpha_n)(\sigma_n^N - 1)$$

Using $d\alpha_n = \sum_{i\in I_n}\theta_{ni}d\alpha_{ni} + \alpha_{ni}d\theta_{ni}$, we can use the same logic as the benchmark case to write

$$\sigma_n^N - 1 = \frac{1}{\alpha_n(1-\alpha_n)}\sum_{i\in I_n}\left[\alpha_{ni}(1-\alpha_{ni})(\sigma_{ni}-1)\theta_{ni} + (\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega}\right] \quad (28)$$

Before computing $\frac{d\ln\theta_{ni}}{d\ln\omega}$, note that

$$\frac{z_{ni}}{\hat{z_{ni}}} = \frac{T_{Kni}K_{ni}+T_{Lni}\omega L_{ni}}{\tau_{Kni}T_{Kni}K_{ni}+\tau_{Lni}T_{Lni}\omega L_{ni}} = \tau_{Kni}^{-1}\hat{\alpha_{ni}} + \tau_{Lni}^{-1}(1-\hat{\alpha_{ni}})$$

Since $\frac{d\ln\frac{\hat{\alpha_{ni}}}{1-\hat{\alpha_{ni}}}}{d\ln\omega} = \frac{d\ln\frac{\tau_{Kni}T_{Kni}K_{ni}}{\tau_{Lni}T_{Lni}\omega L_{ni}}}{d\ln\omega} = \sigma_{ni}-1$, we can differentiate to get

$$\begin{aligned}
\frac{d\ln z_{ni}}{d\ln\omega} - \frac{d\ln\hat{z_{ni}}}{d\ln\omega} &= \alpha_{ni}\frac{d\ln\hat{\alpha_{ni}}}{d\ln\omega} + (1-\alpha_{ni})\frac{d\ln 1-\hat{\alpha_{ni}}}{d\ln\omega} \\
&= \alpha_{ni}(1-\hat{\alpha_{ni}})(\sigma_{ni}-1) - (1-\alpha_{ni})\hat{\alpha_{ni}}(\sigma_{ni}-1) \\
&= (\alpha_{ni}-\hat{\alpha_{ni}})(\sigma_{ni}-1)
\end{aligned}$$

Constant returns to scale and Shephard's lemma imply that

$$\frac{d\ln\hat{z_{ni}}}{d\ln\omega} = \frac{d\ln Y_{ni}}{d\ln\omega} + (1-\hat{\alpha_{ni}})$$

Thus we have

$$\begin{aligned}
\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega} &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln z_{ni}}{d\ln\omega} \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln\hat{z_{ni}}}{d\ln\omega}+(\alpha_{ni}-\hat{\alpha_{ni}})(\sigma_{ni}-1)\right] \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln Y_{ni}}{d\ln\omega}+(1-\hat{\alpha_{ni}})+(\alpha_{ni}-\hat{\alpha_{ni}})(\sigma_{ni}-1)\right] \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln Y_{ni}}{d\ln\omega}+(\hat{\alpha_n}-\hat{\alpha_{ni}})+(\alpha_{ni}-\hat{\alpha_{ni}})(\sigma_{ni}-1)\right]
\end{aligned}$$

Combining this with equation (28) gives

$$\begin{aligned}
\sigma_n^N-1 &= \frac{1}{\alpha_n(1-\alpha_n)}\sum_{i\in I_n}\left\{\begin{array}{c}\alpha_{ni}(1-\alpha_{ni})(\sigma_{ni}-1)\theta_{ni} \\ +(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln Y_{ni}}{d\ln\omega}+(\hat{\alpha_n}-\hat{\alpha_{ni}})+(\alpha_{ni}-\hat{\alpha_{ni}})(\sigma_{ni}-1)\right]\end{array}\right\} \\
&= \frac{\sum_{i\in I_n}\left\{\alpha_{ni}(1-\alpha_{ni})+(\alpha_{ni}-\alpha_n)(\alpha_{ni}-\hat{\alpha_{ni}})\right\}\theta_{ni}(\sigma_{ni}-1)}{\alpha_n(1-\alpha_n)} \\
&\quad+\frac{\sum_{i\in I_n}\left\{(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}}{d\ln\omega}+(\hat{\alpha_n}-\hat{\alpha_{ni}})\right\}}{\alpha_n(1-\alpha_n)}
\end{aligned}$$

This can be simplified to

$$\sigma_n^N-1 = (1-\hat{\chi}_n)(\sigma_{ni}-1)+\frac{\sum_{i\in I_n}\left\{(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}}{d\ln\omega}+(\hat{\alpha_n}-\hat{\alpha_{ni}})\right\}}{\alpha_n(1-\alpha_n)}$$

or more simply

$$\sigma_n^N = (1-\hat{\chi}_n)\sigma_{ni}+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)}$$

To get at $\frac{d\ln\theta_{ni}}{d\ln\omega}$, we can use $Y_{ni}=Y_nP_n^{\varepsilon_n}D_{ni}P_{ni}^{-\varepsilon_n}$ and $p_{ni}=\frac{1}{\tau_{Yni}T_{Yni}}\frac{\varepsilon_n}{\varepsilon_n-1}\hat{z}_{niY}$ to write

$$\frac{d\ln p_{ni}}{d\ln\omega}=\frac{d\ln\hat{z}_{niY}}{d\ln\omega}=1-\hat{\alpha_{ni}}$$

We thus have

$$\begin{aligned}
\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)} &= \frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}(-\varepsilon_n)\frac{d\ln p_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)} \\
&= \varepsilon_n\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}(\alpha_{ni}-\hat{\alpha_n})}{\alpha_n(1-\alpha_n)} \\
&= \hat{\chi}_n\varepsilon_n
\end{aligned}$$

■

With this we have two results. First, if unpaid wedges do not distort any plant's capital-labor ratio ($\tau_{Kni}/\tau_{Lni}=1$ for each i) then $\hat{\chi}_n=\chi_n$, and the formula for the aggregate elasticity is exactly the same.

Second, in Section 4.4 we describe a thought experiment in which all variation in cost shares of capital is due to unpaid wedges. To do this, we first compute the impact of unpaid wedges as follows: Define α^*_{ni} to be the i's hypothetical capital cost share is there were no unpaid wedges. Thus $\{K^*_{ni}, L^*_{ni}\} = \arg\min_{K_{ni},L_{ni}} T_{Yni}P_{ni}Y_{ni} - T_{Kni}rK_{ni} - T_{Lni}wL_{ni}$ subject to $Y_{ni} \leq F_{ni}(K_{ni}, L_{ni})$ and $Y_{ni} \leq Y_n P_n^{\varepsilon_n} D_{ni} P_{ni}^{-\varepsilon_n}$. This would satisfy $\frac{\alpha_{ni}}{1-\alpha_{ni}} = \left(\frac{\tau_{Kni}}{\tau_{Lni}}\right)^{-\sigma_n} \frac{\alpha^*_{ni}}{1-\alpha^*_{ni}}$ and $\frac{\hat{\alpha_{ni}}}{1-\hat{\alpha_{ni}}} = \frac{\alpha_{ni}}{1-\alpha_{ni}} \frac{\tau_{Kni}}{\tau_{Lni}}$, which together imply

$$\frac{\hat{\alpha_{ni}}}{1-\hat{\alpha_{ni}}} = \left(\frac{\alpha_{ni}}{1-\alpha_{ni}}\right)^{1-\frac{1}{\sigma_n}} \left(\frac{\alpha^*_{ni}}{1-\alpha^*_{ni}}\right)^{\frac{1}{\sigma_n}}$$

In the thought experiment, we set α^*_{ni} equal to the mean industry capital share and compute the resulting industry and aggregate elasticities. In practice, this procedure can generate extremely large and unrealistic wedges; we thus Windsorize all wedges using the 2nd and 98th percentiles.

B.5 Demand

In this section we generalize the demand system to a class of homothetic demand systems in which demand for each good is strongly separable. While this class nests Dixit-Stiglitz demand, it allows for arbitrary demand elasticities and pass through rates. An industry aggregate Y_n is defined to satisfy

$$1 = \sum_{i \in I_n} H_{ni}\left(Y_{ni}/Y_n\right) \tag{29}$$

where each H_{ni} is positive, smooth, increasing, and concave. If P_n is the ideal price index associated with Y_n, then cost minimization implies $\frac{P_{ni}}{P_n} = H'_{ni}\left(\frac{Y_{ni}}{Y_n}\right)$. Define the inverse of H'_{ni} to be $h_{ni}(\cdot) = H'^{-1}_{ni}(\cdot)$. i faces a demand curve; to find its elasticity of demand, we can differentiate:

$$d\ln Y_{ni}/Y_n = -\varepsilon_{ni}(P_{ni}/P_n) d\ln P_{ni}/P_n \tag{30}$$

where the elasticity of demand is $\varepsilon_{ni}(x) \equiv -\frac{h'_{ni}(x)x}{h_{ni}(x)}$. The optimal markup chosen by i will satisfy $\mu_{ni}(P_{ni}/P_n) = \frac{\varepsilon_{ni}(P_{ni}/P_n)}{\varepsilon_{ni}(P_{ni}/P_n)-1}$. It will be useful to define b_{ni} to be i's local relative rate of pass through: the responsiveness of P_{ni} to a change in i's marginal cost. Since $P_{ni} = \mu(P_{ni}/P_n) \times \mathrm{mc}_{ni}$, then $\frac{d\ln P_{ni}}{d\ln \mathrm{mc}_{ni}} = \frac{P_{ni}/P_n \mu'_{ni}}{\mu_{ni}} \frac{d\ln P_{ni}}{d\ln \mathrm{mc}_{ni}} + 1$, so that $b_{ni}(x) \equiv \frac{1}{1-\frac{x\mu'_{ni}(x)}{\mu_{ni}(x)}}$.

Lastly, we define $\alpha^P_n \equiv 1 - \frac{d\ln p_n}{d\ln \omega}$ to be the response of the ideal price index to a change in relative factor prices. The following claim describes the industry elasticity of substitution.

Claim 4 *Suppose that each F_{ni} exhibits constant returns to scale and the demand structure in industry n satisfies equation (29). Then the industry elasticity is*

$$\sigma^N_n = (1-\chi_n)\bar{\sigma}_n + \chi_n \bar{s}^M_n \bar{\zeta}_n + \chi_n(1-\bar{s}^M_n)\bar{x}_n$$

where

$$\bar{x}_n \equiv \frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[(\alpha_{ni}-\alpha^P_n) - s^M_{ni}(\alpha_{ni}-\alpha^M)\right]\varepsilon_{ni}b_{ni}}{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[(\alpha_{ni}-\alpha^P_n) - s^M_{ni}(\alpha_{ni}-\alpha^M)\right]}$$

$$\alpha^P_n = \frac{\sum_{i\in I_n} P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}\left[\alpha_{ni} - s^M_{ni}(\alpha_{ni}-\alpha^M)\right]}{\sum_{i\in I_n} P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}}$$

Proof. Optimal price setting implies $p_{ni} = \mu_i(p_{ni}/p_n)z_{niY}$. Taking logs and differentiating gives

$$\frac{d\ln p_{ni}/p_n}{d\ln\omega} = \frac{\mu_i'(p_{ni}/p_n)p_{ni}/p_n}{\mu_i(p_{ni}/p_n)}\frac{d\ln p_{ni}/p_n}{d\ln\omega} + \frac{d\ln z_{niY}}{d\ln\omega} - \frac{d\ln p_n}{d\ln\omega}$$

Constant returns to scale implies $\frac{d\ln z_{niY}}{d\ln\omega} = (1-\alpha_{ni})(1-s_{ni}^M) + s_{ni}^M(1-\alpha^M) = (1-\alpha_{ni}) + s_{ni}^M(\alpha_{ni} - \alpha^M)$, so this can be written as

$$\frac{d\ln p_{ni}/p_n}{d\ln\omega} = b_{ni}\left[s_{ni}^M(\alpha_{ni}-\alpha^M) - (\alpha_{ni}-\alpha_n^P)\right] \tag{31}$$

The change in output is then

$$\frac{d\ln Y_{ni}/Y_n}{d\ln\omega} = -\varepsilon_{ni}\frac{d\ln p_{ni}/p_n}{d\ln\omega} = \varepsilon_{ni}b_{ni}\left[(\alpha_{ni}-\alpha_n^P) - s_{ni}^M(\alpha_{ni}-\alpha^M)\right]$$

To get at the aggregate elasticity, we compute the following

$$\begin{aligned}
\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}}{d\ln\omega} &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}/Y_n}{d\ln\omega} \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\varepsilon_{ni}b_{ni}\left[(\alpha_{ni}-\alpha_n^P) - s_{ni}^M(\alpha_{ni}-\alpha^M)\right] \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{x}_n\left[(\alpha_{ni}-\alpha_n^P) - s_{ni}^M(\alpha_{ni}-\alpha^M)\right] \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{x}_n\left[(\alpha_{ni}-\alpha_n^P) - \bar{s}_n^M(\alpha_{ni}-\alpha^M)\right] \\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{x}_n\left[(\alpha_{ni}-\alpha_n) - \bar{s}_n^M(\alpha_{ni}-\alpha_n)\right] \\
&= \alpha_n(1-\alpha_n)\chi_n(1-\bar{s}_n^M)\bar{x}_n
\end{aligned}$$

where the third equality uses the definition of $\bar{x}_n$ and the fourth uses the definition $\bar{s}_n^M$. This expression and Lemma 1 give the desired result.

It remains only to compute α_n^P. Since $\sum_{i\in I_n}\frac{P_{ni}Y_{ni}}{P_nY_n}\frac{d\ln Y_{ni}/Y_n}{d\ln\omega} = 0$, we can use equation (30) and equation (31) to write

$$0 = \sum_{i\in I_n}\frac{P_{ni}Y_{ni}}{P_nY_n}\varepsilon_{ni}b_{ni}\left[s_{ni}^M(\alpha_{ni}-\alpha^M) - (\alpha_{ni}-\alpha_n^P)\right]$$

which simplifies to

$$\alpha_n^P = \frac{\sum_{i\in I_n}P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}\left[\alpha_{ni} - s_{ni}^M(\alpha_{ni}-\alpha^M)\right]}{\sum_{i\in I_n}P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}}$$

■

B.6 Non-Homothetic Production

This section analyzes how the industry elasticity of substitution is altered if production is non-homothetic. This requires a more careful definition of the elasticities of substitution. A change in factor prices will have a direct effect on a plant's choice of capital-labor ratio, and may have an indirect impact if the change in factor prices alters a plant's scale. We pursue an approach similar to Joan Robinson: we define a plant's elasticity of substitution to be how a change in relative factor prices alters the plant's capital-labor ratio holding output fixed. Similarly, an industry's elasticity of substitution is the response of the industry's capital labor ratio to a change in relative factor prices holding fixed the industry aggregate, Y_n.

We first characterize the plant-level elasticity of substitution, and then derive an expression for the industry level elasticity. In the interest of space, we restrict attention to the case in which plants do not use materials.

Just as $1-\alpha_{ni}\left(=\frac{z_{ni\omega}\omega}{z_{ni}}\right)$ is the labor share of i's cost, we define $\tilde{\alpha}_{ni}$ so that $1-\tilde{\alpha}_{ni}=\frac{z_{niY\omega}\omega}{z_{niY}}$, the labor share of i's marginal cost.

Since $1-\alpha_{ni}=\frac{z_{ni\omega}(Y_{ni},\omega)\omega}{z_{ni}(Y_{ni},\omega)}$, we have

$$d\ln(1-\alpha_{ni}) = \frac{z_{ni\omega Y}Y_{ni}}{z_{ni\omega}}d\ln Y_{ni}+\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}d\ln\omega+d\ln\omega-\frac{z_{niY}Y_{ni}}{z_{ni}}d\ln Y_{ni}-\frac{z_{ni\omega}\omega}{z_{ni}}$$

Since $\frac{z_{ni\omega Y}Y_{ni}}{z_{ni\omega}}=\frac{z_{ni\omega Y}\omega}{z_{niY}}\frac{z_{niY}Y_{ni}}{z_{ni}}$, this can be arranged as

$$d\ln(1-\alpha_{ni}) = \left(\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+1-(1-\alpha_{ni})\right)d\ln\omega+\left(\frac{1-\tilde{\alpha}_{ni}}{1-\alpha_{ni}}-1\right)\frac{1}{\gamma_{ni}}d\ln Y_{ni}$$

Using $d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}=-\frac{1}{\alpha_{ni}}d\ln(1-\alpha_{ni})$, we have

$$d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}=\left(-\frac{1}{\alpha_{ni}}\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}-1\right)d\ln\omega+\frac{\tilde{\alpha}_{ni}-\alpha_{ni}}{\alpha_{ni}(1-\alpha_{ni})}\frac{1}{\gamma_{ni}}d\ln Y_{ni}$$

By definition, $\sigma_{ni}-1$ is the change in $\frac{\alpha_{ni}}{1-\alpha_{ni}}$ holding Y_{ni} fixed. The plant level elasticity of substitution is

$$\sigma_{ni}=-\frac{1}{\alpha_{ni}}\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}$$

and

$$d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}=(\sigma_{ni}-1)d\ln\omega+\frac{\tilde{\alpha}_{ni}-\alpha_{ni}}{\alpha_{ni}(1-\alpha_{ni})}\frac{1}{\gamma_{ni}}d\ln Y_{ni} \tag{32}$$

Claim 5 *The industry elasticity is*

$$\sigma_n^N=(1-\chi_n)\bar{\sigma}_n+\tilde{\chi}_n\bar{x}_n$$

where χ_n and $\bar{\sigma}_n$ are defined as in Lemma 1 and

$$\tilde{\chi}_n \equiv \sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_n)(\tilde{\alpha}_{ni}-\alpha_n^P)\theta_{ni}}{\alpha_n(1-\alpha_n)}$$

$$\bar{x}_n \equiv \frac{\sum_{i\in I_n}(\tilde{\alpha}_{ni}-\alpha_n)(\tilde{\alpha}_{ni}-\alpha_n^P)\theta_{ni}\frac{\varepsilon_n/\gamma_{ni}}{1+\varepsilon_n\frac{z_{niYY}Y_{ni}}{z_{niY}}}}{\sum_{i\in I_n}(\tilde{\alpha}_{ni}-\alpha_n)(\tilde{\alpha}_{ni}-\alpha_n^P)\theta_{ni}}$$

and α_n^P is defined to satisfy $1-\alpha_n^P=\frac{d\ln p_n}{d\ln\omega}$.

Proof. Following the same logic as in the benchmark, we have

$$d\ln\frac{\alpha_n}{1-\alpha_n}=\sum_{i\in I_n}\frac{\alpha_{ni}(1-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}+\sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}d\ln\theta_{ni}$$

Using equation (32), this becomes

$$\begin{aligned}d\ln\frac{\alpha_n}{1-\alpha_n} &= \sum_{i\in I_n}\frac{\alpha_{ni}(1-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}(\sigma_{ni}-1)d\ln\omega+\sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni}+\sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}d\ln\theta_{ni}\\ &= (1-\chi_n)(\bar{\sigma}_n-1)d\ln\omega+\sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni}+\sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}d\ln\theta_{ni}\end{aligned}$$

where the second line used the definitions of $\bar{\sigma}_n$ and χ_n. Since $\theta_{ni}=z_{ni}/\sum_{j\in I_n}z_{nj}$, we have

$$\begin{aligned}\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}d\ln\theta_{ni} &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{z_{niY}Y_{ni}}{z_{ni}}d\ln Y+\frac{z_{ni\omega}\omega}{z_{ni}}d\ln\omega-d\ln\sum_{j\in I_n}z_{nj}\right]\\ &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{1}{\gamma_{ni}}d\ln Y+(1-\alpha_{ni})d\ln\omega\right]\end{aligned}$$

Plugging this in and combining coefficients gives

$$d\ln\frac{\alpha_n}{1-\alpha_n} = (1-\chi_n)(\bar{\sigma}_n-1)d\ln\omega+\sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_n)\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni}+\sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}(1-\alpha_{ni})d\ln\omega$$

One can easily verify that $\sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)(\alpha_{ni}-1)}{\alpha_n(1-\alpha_n)}\theta_{ni}=\chi_n$. This and $d\ln\frac{\alpha_n}{1-\alpha_n}=d\ln K_n/L_n-d\ln\omega$ imply

$$d\ln K_n/L_n = (1-\chi_n)\bar{\sigma}_n d\ln\omega+\sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_n)\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni} \tag{33}$$

Finally we need to address the changes in scale. i's price is $p_{ni}=\frac{\varepsilon_n}{\varepsilon_n-1}z_{niY}$, so the change in i's price is

$$\begin{aligned}d\ln p_{ni} &= \frac{z_{niYY}Y_{ni}}{z_{niY}}d\ln Y_{ni}+\frac{z_{niY\omega}\omega}{z_{niY}}d\ln\omega\\ &= \frac{z_{niYY}Y_{ni}}{z_{niY}}d\ln Y_{ni}+(1-\tilde{\alpha}_{ni})d\ln\omega\end{aligned}$$

If the change in the industry price index satisfies $d\ln p_n=(1-\alpha_n^P)d\ln\omega$, then the change in output is

$$\begin{aligned}d\ln Y_{ni} &= -\varepsilon_n d\ln\frac{p_{ni}}{p_n}+d\ln Y_n\\ &= -\varepsilon_n\left(\frac{z_{niYY}Y_{ni}}{z_{niY}}d\ln Y_{ni}+(1-\tilde{\alpha}_{ni})d\ln\omega-(1-\alpha_n^P)d\ln\omega\right)+d\ln Y_n\\ &= \frac{\varepsilon_n(\tilde{\alpha}_{ni}-\alpha_n^P)d\ln\omega+d\ln Y_n}{1+\varepsilon_n\frac{z_{niYY}Y_{ni}}{z_{niY}}}\end{aligned}$$

Using the definition of $\bar{x}_n$ and $\tilde{\chi}_n$, we therefore have that

$$\begin{aligned} d\ln K_n/L_n &= (1-\chi_n)\bar{\sigma}_n d\ln\omega + \sum_{i\in I_n} \frac{(\tilde{\alpha}_{ni}-\alpha_n)\theta_{ni}}{\alpha_n(1-\alpha_n)} \frac{1}{\gamma_{ni}} \frac{\varepsilon_n(\tilde{\alpha}_{ni}-\alpha_n^P)d\ln\omega + d\ln Y_n}{1+\varepsilon_n \frac{z_{niYY}Y_{ni}}{z_{niY}}} \\ &= (1-\chi_n)\bar{\sigma}_n d\ln\omega + \tilde{\chi}_n\bar{x}_n d\ln\omega + \sum_{i\in I_n} \frac{(\tilde{\alpha}_{ni}-\alpha_n)\theta_{ni}}{\alpha_n(1-\alpha_n)} \frac{1/\gamma_{ni}}{1+\varepsilon_n \frac{z_{niYY}Y_{ni}}{z_{niY}}} d\ln Y_n \end{aligned}$$

Since σ_n^N is defined to be the change in K_n/L_n in response to a change in ω holding fixed Y_n, we have

$$\sigma_n^N = (1-\chi_n)\bar{\sigma}_n + \tilde{\chi}_n\bar{x}_n$$

■

C Data Notes (Online Appendix)

C.1 Census of Manufactures

We use the 1987 and 1997 Census of Manufactures to estimate plant elasticities of substitution and demand. We remove all Administrative Record plants because these plants do not have data on output or capital. We also eliminate a set of outliers and missing values from the dataset. We first remove all plants born in the given Census year, as well as a small set of plants with missing age data. We then remove plants with zero or missing data on the following variables: average revenue product of capital, average revenue product of labor, capital share, capital labor ratio, and plant level wage. We also remove plants above the 99.5th percentile or below the 0.5th percentile of their 4-digit SIC industry on these variables to remove plants with potential data problems.

For capital costs, we multiply capital stock measures by rental rates of capital. In the 1987 Census, the Census asked plants to report the book value of structures capital separately from equipment capital. Thus, we construct the capital stock for structures capital separately from equipment capital for 1987. Because the book value reported in the Census is a historical gross cost measure (although it accounts for capital retirements), we multiply the book value of capital by a current net cost to historical gross cost deflator based upon estimates of the current net value of capital and historic gross value of capital constructed by the Bureau of Economic Analysis at the 2-digit SIC level. Because this deflator is not base 1987, we then use investment deflators to convert each capital stock to 1987 dollars.

In the 1997 Census, the Census only asked plants to report the total value of capital. We construct capital deflators and rental rates for both structures and equipment capital using the same procedure as 1987. We then average both the capital deflator and rental rate of capital for structures and equipment capital, weighting each type of capital by its share of overall capital based upon data for the plant's 4-digit SIC industry from the NBER Productivity Database.

C.2 Local Wages

We construct measures of the local wage in order to estimate the elasticity of substitution across plants, using two different datasets to measure the local area wage. The primary dataset that we use in the Census 5 percent samples of Americans. The Population Censuses have data on both wages and MSA geographic location for a large sample of workers.

To obtain the local wage, we first calculate the individual wage for prime age men (with age between 25 and 55) who are employed in the private sector as workers earning a wage or salary. We calculate the wage as an hourly wage, defined as total yearly wage and salary income divided by total hours worked. We measure total hours worked as weeks worked per year multiplied by hours worked per week. We remove all individuals with zero or missing income or zero total hours worked. For 1990, incomes above the Census top code of $140,000 are set to the state median of wage and salary income above the top code. For 2000, incomes above the Census top code of $175,000 are set to the state mean of wage and salary income above the top code.

Before calculating local area wages, we adjust measures of local wages for differences in worker characteristics through regressions with the individual log wage as a dependent variable. We include education through a set of dummy variables based upon the worker's maximum educational attainment, which include four categories: college, some college, high school degree, and high school dropouts. We define experience as the individual's age minus an initial age of working that depends upon their education status, and include a quartic in experience in the regression. We also have data on the race of workers and so include three race categories of white, black, and other. We include six occupational categories: Managerial and Professional; Technical, Sales, and Administrative; Service, Farming, Forestry, and Fishing; Precision Production, Craft, and Repairers; and Operatives and Laborers. Finally, we include thirteen industrial categories: Agriculture, Forestry, and Fisheries; Mining; Construction; Manufacturing; Transportation, Communications and Other Public Utilities; Wholesale Trade; Retail Trade; Finance, Insurance, and Real Estate; Business and Retail Services; Personal Services; Entertainment and Recreation Services; Professional and Related Services; and Public Administration. We then calculate the local area wage as the MSA average of residual wages from a regression that includes all of these characteristics, with separate regressions by year. Because the Economic Census is conducted in different years from the Population Censuses, we match the 1987 Census of Manufactures to wages from the 1990 Population Census, and the 1997 Census of Manufactures to wages from the 2000 Population Census.

The second dataset that we use for robustness checks is the Longitudinal Business Database, which contains data on payroll and employment for all US establishments. We construct the establishment wage as total payroll divided by total employment. We measure the local wage as the mean log wage at the county level. We match the 1987 Longitudinal Business Database to the 1987 Census of Manufactures and the 1997 Longitudinal Business Database to the 1997 Census of Manufactures.

C.3 Instruments

We use labor demand instruments for the local wage for robustness checks on our estimates of the elasticity of substitution, based upon the differential impact of national level shocks to industry employment across locations. Positive national shocks to an industry should increase labor demand and wages, more in areas with high concentrations of that industry. Formally, the predicted growth rate in employment for a given location is the sum across industries of the product of the local employment share of this industry and the 10 year change in national level employment for that industry. We use the Longitudinal Business Database, which contains all US establishments, to construct these instruments.

The implicit assumption here is that changes in industry shares at the national level are independent of local manufacturing plant productivity. To help ensure that this assumption holds, we exclude manufacturing industries from the labor demand instrument. We calculate the instrument defining locations by MSAs and industries at the SIC 4 digit level. For 1987, we use the instrument from 1976-1986 because the SIC 4 digit industry definitions change significantly from 1977 to 1987.

C.4 Annual Survey of Manufactures

The Annual Survey of Manufactures tracks about 50,000 plants over five year panel rotations that are more heavily weighted towards large plants. We use the ASM to calculate the heterogeneity indices and materials shares. The ASM has data on plant investment over time as well as book values of the stock of capital, which we use to construct perpetual inventory measures of capital.

We also take into account retirements of the capital stock until 1987, as data on retirements of capital stock are available from 1977-1987 excepting 1986. For 1973-1976 and 1986 we can calculate an imputed value for retirements as end of year capital subtracted from beginning year capital and yearly investment; we lower investment if this value is negative. Plants retire their capital stock at a rate of about 4 percent a year, which is concentrated in a few plants retiring a lot of capital stock. Since firms retiring capital deduct the retirement values from their book value, the book value incorporates depreciation from retirements.

We calculate perpetual inventory measures of capital through the following capital accumulation equation, as in Caballero et al. (1995):

$$K_t = (1 - \delta^a)K_{t-1} + I_t - R_t$$

where K_t is period t capital stock, I_t is period t investment, R_t is period t retirements, and δ^a is the in use depreciation rate. We build separate capital stocks for structures and equipment capital. To calculate the in use depreciation rate δ^a, we first calculate δ^r the average yearly rate of capital retirements (total retired capital stock divided by beginning gross capital stock) across plants from 1977 to 1985 by 2 digit SIC industry. We then initially define the in use depreciation rate as:

$$\delta^a = \delta - \delta^r$$

where δ is the overall 2 digit SIC depreciation rate calculated by the BLS minus this yearly retirement rate.

We account for retirements by building a set of capital vintages for each year that the plant exists in the dataset. Retirements are taken out of the gross capital stock of the earliest vintages of capital, as we assume FIFO retirement of capital. We initialize capital stock by the initial sample year book value, so for the first year that the plant exists in the dataset, capital is set to book value of capital. We deflate this book value by a net current cost to gross historical cost deflator. In subsequent years, each vintage is investment deflated through the investment deflator. Real investment is added to capital, and in use depreciation subtracted from capital. After this process, we recalculate the retirement depreciation rate as capital retired net of in use depreciation divided by net overall capital stock, and then recalculate all of the capital vintages to construct an overall capital measure.

After 1987, retirements are no longer recorded so we calculate perpetual inventory measures of capital without retirements, as in the following capital accumulation equation:

$$K_t = (1 - \delta)K_{t-1} + I_t$$

where δ is the overall depreciation rate. After 1992, the Census no longer records book values for structures and equipment separately, although they do record investment separately by capital type. When we only have a total book value of capital for a plant, we use earlier data from the plant on the share of equipment capital to form separate capital stocks for structures and equipment. If no earlier data from the plant is available, we use the share of equipment capital for the 4 digit industry from the NBER productivity database.

The ASM plant samples also have data on the value of non monetary compensation given to employees, such as health care or retirement benefits, which we use to better measure payments to labor.

C.5 Rental Rates

We define the rental rate using the external real rate of return specification of Harper et al. (1989). The rental rate for industry n is defined as:

$$R_{i,t} = T_{i,t}(p_{i,t-1}r_{i,t} + \delta_{i,t}p_{i,t})$$

where $r_{i,t}$ is a constant external real rate of return of 3.5 percent, $p_{i,t}$ is the price index for capital in that industry, $\delta_{i,t}$ is the depreciation rate for that industry, and $T_{i,t}$ is the effective rate of capital taxation. We calculate $T_{i,t}$ following Harper et al. (1989) as:

$$T_{i,t} = \frac{1 - u_t z_{i,t} - k_{i,t}}{1 - u_t}$$

where $z_{i,t}$ is the present value of depreciation deductions for tax purposes on a dollar's investment in capital type i over the lifetime of the investment, $k_{i,t}$ is the effective rate of the investment tax credit, and u_{it} is the effective corporate income tax rate. We obtained $z_{i,t}$, u_{it}, and $k_{i,t}$ from Dale Jorgenson at the asset year level; we then used a set of capital flow tables at the asset-industry level to convert these to the industry level.

To calculate depreciation rates $\delta_{i,t}$, we take depreciation rates from NIPA at the asset level and use the capital flow tables to convert them to the industry level. Our primary source of prices of capital $p_{i,t}$ are from NIPA, which calculates separate price indices for structures and equipment capital. As an alternative, we also develop a set of rental rates based upon the investment price series of Cummins and Violante (2002).

The capital flow tables and investment price series depend upon the industry definition; because the US switches from SIC basis to NAICS basis during this period, we construct separate rental price series for SIC 2 digit industries and NAICS 3 digit industries. Finally, when we examine the aggregate we have to aggregate all of the rental price series; we do so by calculating Tornqvist indices between equipment and structures capital for each industry, and then a Tornqvist index across rental rates for each industry. The Tornqvist indices allow for the share of equipment capital in industry capital and for the share of different industries in manufacturing capital to change over this period.

C.6 Homogeneous Product Industries

We follow a similar process to Foster et al. (2008) in constructing data on homogeneous product industries for robustness checks on the elasticity of demand. We use six homogeneous products: Boxes, Bread, Coffee, Concrete, Processed Ice and Plywood.[41] All of the products are defined as in Foster et al. (2008). We use data from 1987-1997 as capital data was imputed before 1987 for non-ASM plants, although we do not use data for 1992 for Processed Ice (because of data errors), 1987 for Boxes (because of a product definition change), and 1997 for Concrete (because quantity data was not recorded). We remove Census balancing codes imputed by the Census to make product level data add up to overall revenue data in cases where we can identify them. We also remove receipts for contract work, miscellaneous receipts, resales of products, and products with negative values.

We then remove all plants for which the product's share of plant revenue (measured after removing the balancing codes and other items mentioned above) is less than 50 percent. For each product, we have measures of both total quantity produced and revenue, which allows me to calculate product price as revenue over quantity. We delete all plants for which the ratio of product price to median product price is between .999 and 1.001, as these plants likely have quantity data imputed by the Census. We also remove plants with prices greater than ten times the median price or less than one-tenth the median price as potential mismeasured outliers.

[41] Foster et al. (2008) examine 5 additional products: Carbon Black, Flooring, Gasoline, Block Ice, and Sugar; small samples in the years we study preclude this analysis.

C.7 Cross Country Data

We obtain plant-level data for Chile, Colombia, and India from national plant-level manufacturing censuses. The Chilean data spans 1986 to 1996 with about 5,000 plants per year, the Colombian data from 1981 to 1991 with about 7,000 plants per year, and the Indian data from 2000 to 2003 with about 30,000 plants per year.[42] The Chilean and Colombian data cover all manufacturing plants with at least ten employees, while the Indian data are a sample of all plants with at least ten employees (twenty if without power), with plants with at least one hundred workers sampled with certainty. We define industries at a similar level to two digit US SIC; for Chile and Colombia this is at the three digit ISIC level, and for India at the two digit NIC level.

Capital costs are the most involved variable to construct. For each country, a capital stock is constructed for each type of capital. Capital services is the sum of the stock of each type multiplied by its rental rate plus rental payments. To account for utilization (and especially entry and exit), we multiply capital services by the fraction of the year the plant was open. The capital rental rate is composed of the real interest rate R, depreciation rate δ for that type of capital, and effective corporate tax rate τ:

$$r = \frac{R+\delta}{1-\tau}$$

For corporate tax rates, we use the one year effective tax rate collected by Djankov et al. (2010). Djankov et al. (2010) derive effective tax rates for fiscal year 2004 by asking a major accounting firm to calculate the tax rate for the same fictitious corporation in 85 countries.

Across countries, there are some differences in the construction of capital stocks and depreciation rates. For Chile, we use capital stocks constructed by Greenstreet (2007). Greenstreet (2007) constructed capital stocks for each type of capital using a permanent inventory type procedure. We use his depreciation rates of 5 percent for buildings, 10 percent for equipment, and 20 percent for vehicles.

For Colombia and India, we construct measures of capital services. To construct capital for Colombia, we broadly follow the perpetual inventory procedure of Tybout and Roberts (1996). Because the Indian data is not panel, we use book values of capital for each type of capital. For both Colombia and India, we match the depreciation rates we calculate for US industries to the equivalent industries in Colombia and India for structures and equipment, while for transportation, we follow Greenstreet (2007) and set the depreciation rate to 0.20.[43]

We base the real interest rate on private sector lending rates reported in the IMF Financial Statistics. For Colombia, we have capital deflators over time and so construct separate real interest rates for each type of capital by deducting the realized inflation rates for each type of capital from the lending rate. For India, we do not have investment deflators and so use the GDP deflator.

We then use the average real interest rate over our sample period for the rental rates. For labor costs, we use the available wages and benefits data for each country.

To construct rental rates of capital for our policy experiments, we require real interest rates, corporate tax rates, and depreciation rates for each country at the same point in time. For the real interest rate, we adjust the nominal private sector lending rate for each country from the IMF

[42] We have data from Chile going back until 1979, but we only use the later years to avoid the Chilean financial crisis in the early 1980s.

[43] The US depreciation rates are based on NIPA data on depreciation rates of assets; we then use asset-industry capital tables to construct depreciation rates for structures and equipment for each industry.

International Financial Statistics for inflation, and then average from 1992 to 2011.[44] For corporate tax rates, we again use the one year effective tax rate collected by Djankov et al. (2010). Finally, we set the depreciation rate to 9.46 percent based upon US manufacturing data.

D Additional Empirical Results (Online Appendix)

D.1 Micro Capital–Labor Elasticity Estimates

This section includes tables of plant capital-labor substitution elasticity estimates.

Table VI Elasticities of Substitution between Labor and Capital for Two Digit Industries

Industry	1987	1997	N
20: Food Products	0.67 *(0.10)*	0.87 *(0.11)*	≈ 10,000
22: Textiles	0.70 *(0.16)*	0.30 *(0.24)*	≈ 3,500
23: Apparel	0.82 *(0.11)*	0.40 *(0.09)*	≈ 12,000
24: Lumber and Wood	0.23 *(0.12)*	0.48 *(0.11)*	≈ 15,000
25: Furniture	0.42 *(0.14)*	0.18 *(0.17)*	≈ 6,000
26: Paper	0.20 *(0.16)*	0.20 *(0.15)*	≈ 4,000
27: Printing and Publishing	0.57 *(0.05)*	0.50 *(0.08)*	≈ 26,000
28: Chemicals	0.41 *(0.15)*	0.51 *(0.21)*	≈ 6,500
29: Petroleum Refining	0.70 *(0.23)*	0.53 *(0.28)*	≈ 1,500
30: Rubber	0.64 *(0.13)*	0.42 *(0.14)*	≈ 8,500
31: Leather	0.43 *(0.28)*	0.46 *(0.36)*	≈ 1,000
32: Stone, Clay, Glass, Concrete	0.47 *(0.11)*	0.80 *(0.16)*	≈ 9,000
33: Primary Metal	0.42 *(0.17)*	0.26 *(0.19)*	≈ 4,000
34: Fabricated Metal	0.33 *(0.09)*	0.25 *(0.09)*	≈ 20,000
35: Machinery	0.54 *(0.08)*	0.52 *(0.11)*	≈ 25,000
36: Electrical Machinery	0.48 *(0.12)*	0.51 *(0.12)*	≈ 8,000
37: Transportation Equip	0.65 *(0.16)*	0.77 *(0.16)*	≈ 5,000
38: Instruments	0.74 *(0.10)*	0.71 *(0.13)*	≈ 4,500
39: Misc	0.43 *(0.13)*	0.38 *(0.12)*	≈ 6,500

Note: All regressions include 4 digit SIC industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the two digit industry-area level. Wages are as defined in the text.

D.2 Demand Elasticity Estimates

This section includes tables of plant demand elasticity estimates.

[44] We employ a discrete time correction as some countries have high inflation rates, so $R = \frac{i_t - \pi_t}{1+\pi_t}$ for lending rate i_t and inflation rate π_t. We use the change in the GDP deflator for inflation.

Table VII Elasticities of Demand for Two Digit Industries

Industry	1987	1997	N
20: Food Products	3.96 *(0.001)*	3.63 *(0.002)*	≈ 10,000
22: Textiles	4.92 *(0.006)*	5.22 *(0.011)*	≈ 3,500
23: Apparel	3.68 *(0.001)*	3.73 *(0.001)*	≈ 12,000
24: Lumber and Wood	4.20 *(0.002)*	4.86 *(0.001)*	≈ 15,000
25: Furniture	3.90 *(0.001)*	4.07 *(0.002)*	≈ 6,000
26: Paper	4.73 *(0.003)*	4.58 *(0.003)*	≈ 4,000
27: Printing and Publishing	3.20 *(0.000)*	3.53 *(0.000)*	≈ 26,000
28: Chemicals	3.05 *(0.001)*	2.94 *(0.001)*	≈ 6,500
29: Petroleum Refining	4.26 *(0.016)*	4.60 *(0.042)*	≈ 1,500
30: Rubber	3.78 *(0.001)*	3.63 *(0.001)*	≈ 8,500
31: Leather	4.01 *(0.009)*	3.71 *(0.009)*	≈ 1,000
32: Stone, Clay, Glass, Concrete	4.00 *(0.001)*	3.81 *(0.001)*	≈ 9,000
33: Primary Metal	4.76 *(0.005)*	3.97 *(0.004)*	≈ 4,000
34: Fabricated Metal	3.99 *(0.000)*	3.74 *(0.000)*	≈ 20,000
35: Machinery	3.93 *(0.000)*	3.82 *(0.000)*	≈ 25,000
36: Electrical Machinery	3.45 *(0.001)*	3.33 *(0.001*	≈ 8,000
37: Transportation Equip	4.34 *(0.003)*	4.29 *(0.004)*	≈ 5,000
38: Instruments	3.02 *(0.001)*	2.91 *(0.001)*	≈ 4,500
39: Misc	3.58 *(0.001)*	3.43 *(0.001)*	≈ 6,500

Note: All estimates are based upon inverting the average markup across plants in an industry; the markup over marginal cost is equal to $\frac{\varepsilon}{\varepsilon-1}$. We define the markup as sales divided by the sum of costs from capital, labor, and materials.

D.3 Local Content of Materials

In our baseline estimates of the elasticity of substitution between materials and non-materials, ζ, we assume that the local wage does not affect the materials price the plant faces. As a robustness check, we examine how sensitive our estimates are to correlation between materials prices and local wages due to local content of materials. The local wage would affect labor costs for locally sourced materials. We use the 1993 Commodity Flow Survey to construct the local content of shipments for every industry included in the survey, defining local as a shipment within 100 miles of the originating factory. We then use the 1992 Input-Output tables to construct the average local content of materials for every manufacturing industry. Assuming that every input industry has the same materials and labor shares and fraction of local content of materials, the elasticity of the materials price with respect to the wage is:

$$\frac{d\log q_i}{d\log w} = (1-\alpha_n)\frac{1-s_n^M l_c}{1-(1-s_n^M)l_c}$$

where l_c is the measure of local content.

We therefore estimate ζ using the regression

$$\log\frac{rK_i+wL_i}{qM_i} = (1-\zeta)\frac{(1-\alpha_i)}{(1-\alpha_n)\frac{1-s_n^M l_c}{1-(1-s_n^M)l_c}}(\log w_i) + CONTROLS + \epsilon_i$$

As we report in the text, we find only slightly lower estimates in estimated elasticities after accounting for the local content of materials.

D.4 Cross Industry Demand Elasticity

The cross industry elasticity of demand characterizes how industry level demand responds to a change in the overall industry price level. To estimate this elasticity, we use panel data on quantity and price at the industry level from the NBER productivity database from 1962 to 2009.

Since least squares estimates conflate demand and supply, we have to instrument for price using supply side instruments that capture industry productivity. The two instruments that we examine is the average product of labor, defined as the amount of output produced per worker, and the average real cost per unit of output produced, which is the appropriate measure of industry productivity in our model. We thus have the following regression specification:

$\log q_{n,t} = -\eta \log p_{n,t} + \alpha_n + \beta_t + CONTROLS + \varepsilon_n$

where $q_{n,t}$ is quantity produced for industry n in period t, $p_{n,t}$ is the price for industry n in period t, α_n are a set of industry fixed effects, and β_t are a set of time fixed effects.

We then examine the cross industry demand elasticity, defining industry at both the four digit and two digit SIC levels. We have 459 four digit industries and 20 two digit industries.[45] For each industry definition, we develop specifications with extra sets of controls to account for potential trends over time that could be correlated with changes in prices. In the four digit specifications,

[45] Since the underlying data is at the four digit industry level, we develop two digit SIC prices and quantities using a Fisher ideal index with base year 1987. We also exclude eight 4 digit industries which disappear because they are excluded after the Census shifts to NAICS basis manufacturing, the most prominent of which is Newspaper Publishing.

these extra controls include either 2 digit industry-year fixed effects, or 4 digit industry linear trends. In the two digit specifications, these extra controls include 2 digit industry linear trends.

Table VIII below contains these estimates, as well as the OLS estimate. As would be expected from simultaneity bias, OLS estimates are lower in magnitude than IV estimates. The IV estimates using four digit industries range between 1.2 and 2.2 and are slightly above estimates using two digit industries. This pattern is consistent with two digit industry varieties being less substitutable than four digit industry varieties.

The two digit industry IV estimates range from 0.75 to 1.15, with three of the four estimates close to one. Because we define industries in our aggregation analysis at the two digit level, the two digit industry estimates are more appropriate. We thus set the cross industry demand elasticity to one. Our results are not extremely sensitive to this elasticity; increasing the elasticity from 1 to 1.5 would increase the US aggregate elasticity by about 0.01.

Table VIII Cross Industry Elasticity of Demand for the Manufacturing Sector

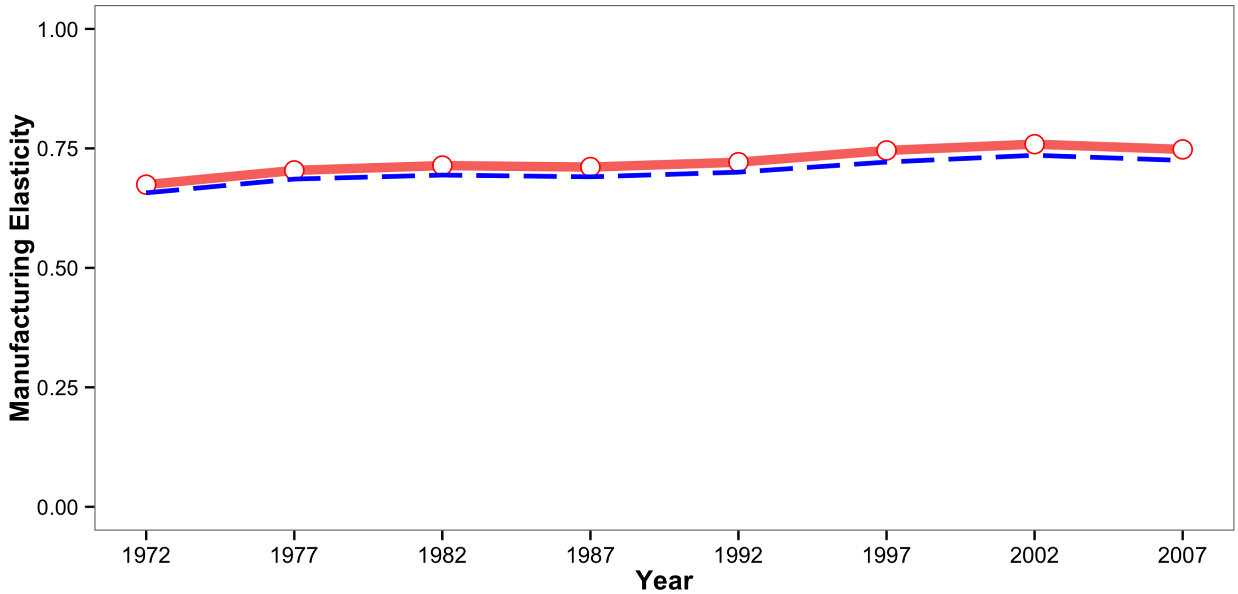
	Industry Definition:				
Instrument	Four Digit			Two Digit	
None	0.99	1.06	0.57	0.91	0.37
	(0.02)	*(0.01)*	*(0.02)*	*(0.03)*	*(0.05)*
APL	1.30	1.28	2.12	1.14	1.05
	(0.01)	*(0.01)*	*(0.03)*	*(0.04)*	*(0.06)*
Avg Cost	1.19	1.22	1.58	1.04	0.77
	(0.01)	*(0.01)*	*(0.02)*	*(0.03)*	*(0.05)*
Industry-Year Controls	None	Two Digit FE	Four Digit Trends	None	Two Digit Trends

Note: Standard errors are in parentheses. The first row contains coefficients from OLS regressions, while the second and third row are IV regressions with either the average product of labor or average real cost per unit produced as instruments. The first three columns are on four digit SIC industries; all regressions contain four digit SIC industry and year fixed effects. The second column also includes two digit industry-year fixed effects and the third column also includes four digit industry linear time trends. The last two columns are on two digit SIC industries; all regressions contain two digit SIC industry and year fixed effects. The last column also includes two digit industry linear time trends.

D.5 Aggregate Elasticity Over Time

Our baseline approach to examining the change in the aggregate elasticity over time fixes plant demand and production elasticities at their 1987 values. In this section, we show how these estimates change if we instead use the production and demand elasticities from 1997. Figure 10 depicts the aggregate elasticity of substitution over time both using the 1997 micro elasticities, in dashed blue, and using the 1987 elasticities in solid red. The 1997 elasticities lower all of the estimates by about 0.01. Almost all of this change is due to the elasticity of materials to non-materials falling from 0.90 to 0.67; just changing the plant demand elasticities and capital-labor substitution elasticities has almost no effect on the aggregate estimates.

Figure 10 Aggregate Elasticity of Substitution Across Time Using 1997 Production and Demand Elasticities



Note: The figure displays the manufacturing level elasticity of substitution by Census year from 1972-2007. The red solid line is based on the 1987 plant demand and substitution elasticities, and the blue dashed line is based on the 1997 elasticities.

D.6 Labor Share Decomposition

D.6.1 Derivation of Decomposition

This section describes the how the theory is used to execute the decomposition of equation (13). Labor's share of value added is $s^{v,L} = \sum_n v_n s_n^{vL}$, where $s_n^{v,L}$ is labor's share of added in industry n and v_n is industry n's share of total value added. Changes in labor's share of value added can come from changes in labor's share within industries or changes between industries:

$$\begin{aligned} ds^{v,L} &= \sum_n s_n^{v,L} dv_n + \sum_n v_n ds_n^L \\ &= \sum_n \left(s_n^{v,L} - s^{v,L}\right) dv_n + \sum_n v_n ds_n^{v,L} \end{aligned}$$

We can decompose each into the components that responded to price changes, a within-industry residual, and a between-industry residual.

$$ds^{v,L} = \frac{\partial s^{v,L}}{\partial w/r} d\ln w/r + \sum_n \left(s_n^{v,L} - s^{v,L}\right)\left(dv_n - \frac{\partial v_n}{\partial \ln w/r}\right) d\ln w/r + \sum_n v_n \left(ds_n^{v,L} - \frac{\partial s_n^{v,L}}{\partial \ln w/r} d\ln w/r\right)$$

where $\frac{\partial s^{v,L}}{\partial \ln w/r} = \sum_n \left(s_n^{v,L} - s^{v,L}\right) \frac{\partial v_n}{\partial \ln w/r} + \sum_n v_n \frac{\partial s_n^{v,L}}{\partial \ln w/r}$.

We now derive expressions for $\frac{\partial s_n^{v,L}}{\partial w/r}$ and $\frac{\partial v_n}{\partial w/r}$. To get at these we will use

$$\begin{aligned}\frac{\partial \ln(1-\alpha_n)}{\partial \ln w/r} &= \alpha(1-\sigma_n) \\ \frac{\partial \ln\left(1-s_n^M\right)}{\partial \ln \omega} &= s_n^M\left(1-\zeta_n^N\right)\left(\alpha^M-\alpha_n\right)\end{aligned}$$

Define $\mu_n = \frac{\varepsilon_n}{\varepsilon_n - 1}$. Since $s_n^{v,L} = (1-\alpha_n)\frac{rK_n+wL_n}{VA_n} = (1-\alpha_n)\frac{1-s_n^M}{\mu_n - s_n^M}$. We then have

$$\begin{aligned}\frac{\partial s_n^{v,L}}{\partial w/r} &= s_n^{v,L}\left\{\frac{\partial \ln 1-\alpha_n}{\partial \ln w/r} + \frac{\mu_n-1}{\mu_n-s_n^M}\frac{\partial \ln\left(1-s_n^M\right)}{\partial \ln w/r}\right\} \\ &= s_n^{v,L}\left\{\alpha_n\left(1-\sigma_n^N\right) + \frac{\mu_n-1}{\mu_n-s_n^M}s_n^M\left(1-\zeta_n^N\right)\left(\alpha^M-\alpha_n\right)\right\}\end{aligned}$$

To get at the between terms, note that

$$v_n = \frac{VA_n}{VA} = \left(\mu_n - s_n^M\right)\left(\frac{rK_n+wL_n+qM_n}{rK+wL+qM}\right)\left(\frac{rK+wL+qM}{VA}\right)$$

Since $\sum_n v_n(s_n^{v,L} - s^{v,L}) = 0$, we have

$$\sum_n v_n(s_n^{v,L}-s^{v,L})\frac{\partial \ln v_n}{\partial \ln w/r} = \sum_n v_n\left(s_n^{v,L}-s^{v,L}\right)\left[\frac{\partial \ln\left(\mu_n - s_n^M\right)}{\partial \ln w/r} + \frac{\partial \ln \frac{rK_n+wL_n+qM_n}{rK+wL+qM}}{\partial \ln w/r}\right]$$

Since $\eta = 1$, $\frac{\partial \ln \frac{rK_n+wL_n+qM_n}{rK+wL+qM}}{\partial \ln w/r} = 0$. Thus

$$\begin{aligned}\sum_n v_n(s_n^{v,L}-s^{v,L})\frac{\partial \ln v_n}{\partial \ln w/r} &= \sum_n v_n\left(s_n^{v,L}-s^{v,L}\right)\frac{1-s_n^M}{\mu_n-s_n^M}\frac{\partial \ln\left(1-s_n^M\right)}{\partial \ln w/r} \\ &= \sum_n v_n\left(s_n^{v,L}-s^{v,L}\right)\frac{\left(1-s_n^M\right)s_n^M}{\mu_n-s_n^M}\left(1-\zeta_n^N\right)\left(\alpha^M-\alpha_n\right)\end{aligned}$$

D.6.2 Labor Share from Production Data

Our benchmark analysis decomposed labor's share of income as measured in the national accounts. This data is built from manufacturing firms. Alternatively, we could analyze the changes in labor share as measured from production data built from manufacturing plants. We will briefly describe the advantages of each and why the analysis based on national accounts is our preferred measure.

The national accounts is built from firm data, so it includes all establishments (including non-manufacturing establishments) of manufacturing firms. This data contains measures of overall labor compensation.

The production data from the NBER CES production database is built from the same manufacturing plant database that we used to compute the aggregate elasticity. Because the aggregate production data does not include benefits, in each year we adjust the payments to labor by the ratio of total compensation to wages and salaries for manufacturing from NIPA.

We prefer using the labor share from the national accounts for two reasons. First, it makes our study comparable to the rest of the literature that has studied the labor share. Second, the production data only includes expenses incurred at the plant level, such as energy and materials costs. It does not include expenses such as advertising, research and development not conducted at the plant, and all expenses at the corporate headquarters. The absence of these expense means that value added, and hence our residual "profit", are both overstated and may have different trends over time.

Nevertheless, Table IX displays the change in the labor share and its components under two alternatives. First, we perform the same analysis as in the text, decomposing the change in the labor share of value added as measured in the production data. Second, we decompose the change in labor's share of the total expenditure on capital and labor, $d\frac{s_l}{s_l+s_k}$, into the contribution from factor prices and the contribution from biased technical change. We believe the latter is more comparable across the two sources.

Table IX Contributions to Labor Share Change using Production Data

	Annual Contribution			Cumulative Contribution		
Period	Labor Share	Factor Prices	Bias	Labor Share	Factor Prices	Bias
	Labor's Share of Value Added					
1970-1999	-0.51	0.09	-0.60	-15.19	2.67	-17.86
2000-2009	-0.57	0.08	-0.65	-5.67	0.80	-6.47
	Labor's Share of Capital and Labor Cost					
1970-1999	-0.17	0.13	-0.30	-5.19	3.86	-9.06
2000-2009	-0.75	0.13	-0.88	-7.52	1.29	-8.82

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points.

Labor's share of value added in the production data declined at about the same rate between 1970-1999 and in the 2000s. Labor's share of capital/labor cost falls much faster in the 2000s, and is more consistent both qualitatively and quantitatively with the overall pattern using national accounts. The decline of the labor share accelerated since 2000, which is mostly accounted for by an acceleration of the bias.

D.6.3 Alternative Rental Prices

Our rental prices are based upon official NIPA deflators for equipment and structures capital. However, Gordon (1990) has argued that the NIPA deflators underestimate the actual fall in equipment prices over time. We examine how this critique might change our results on the bias of technical change by using an alternative rental price series for equipment capital that Cummins and Violante (2002) developed by extending the work of Gordon (1990). Their series extends to 1999, so we compare our baseline to these rental prices during the 1970-1999 period. Using the Cummins and Violante (2002) equipment prices implies that the wage to rental price ratio has increased by 3.8 percent per year, instead of 2.0 percent per year with the NIPA deflators. This change increases the contribution of factor prices to the labor share from 0.08 percentage points per year to 0.14

percentage points per year. Given our estimate of the aggregate elasticity of substitution, changes in factor prices have not been the driving force behind the declining labor share.

Table X Contributions to Labor Share Change with Alternative Rental Price Series

		Annual Contribution			Cumulative Contribution		
Deflator	Annual $\frac{w}{r}$ Change	Labor Share	Factor Prices	Bias	Labor Share	Factor Prices	Bias
NIPA	2.02	-0.26	0.08	-0.34	-7.69	2.39	-10.08
GCV	3.80	-0.26	0.14	-0.40	-7.69	4.32	-12.00

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points. Data covers 1970-1999.

D.7 Aggregate Time–Series Approach

We now compare our methodology to the approach that jointly estimates the aggregate capital-labor elasticity of substitution and bias of technical change using aggregate time series data. This approach uses the following econometric model:

$$\frac{s^{v,L}}{1-s^{v,L}} = \beta_0 + (\sigma^{agg} - 1)\log\frac{r}{w} + \log\phi + \epsilon \tag{34}$$

where $d\log\phi$ is the bias of technical change and ϵ is interpreted as measurement error that is orthogonal to $\log\frac{r}{w}$. It is well known that estimates depend critically on what assumptions are placed on the bias of technical change. Under an assumption of Hicks neutral technical change ($d\log\phi = 0$), the aggregate elasticity is precisely estimated at 1.91. The elasticity is considerably above one because the labor share fell and wages rose relative to capital prices during the sample period.

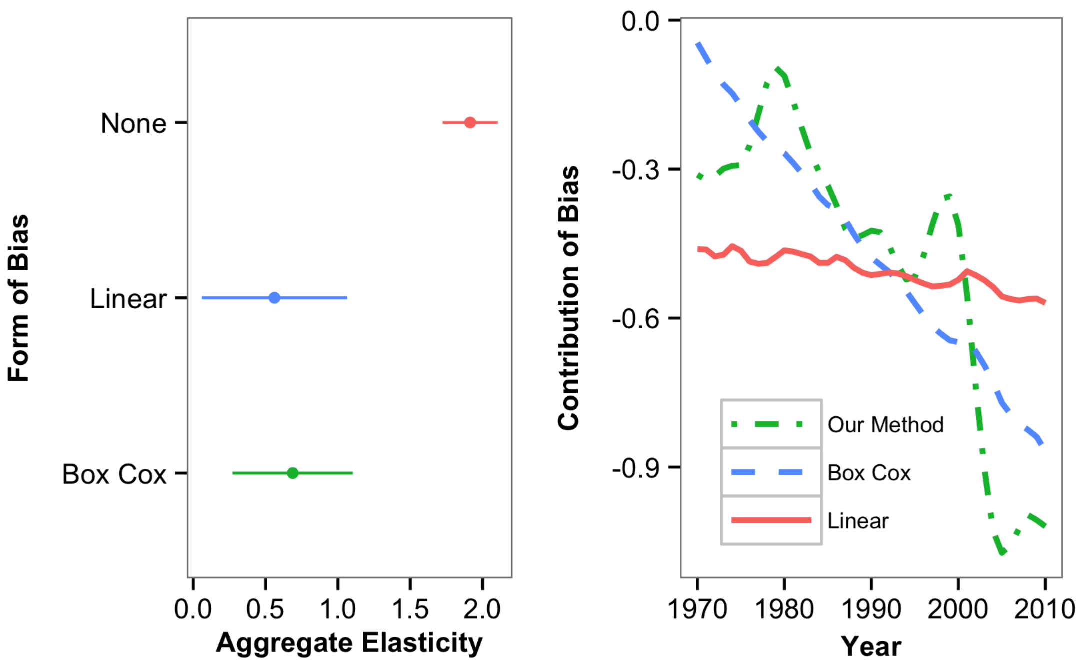
Once we allow for biased technical change, however, estimates of both the bias and aggregate elasticity become imprecise, as shown in Figure 11. The first way we introduce biased technical change is through a constant rate of biased technical change ($d\log\phi$ is constant). This constant rate of bias becomes a time trend in the aggregate regression. The elasticity is then identified by movements in relative factor prices around the trend; short run movements in factor prices are assumed to be uncorrelated with movements in technology. Given a constant bias, the estimate of the aggregate elasticity using least squares regressions is 0.56; the 95 percent confidence interval ranges from 0.05 to 1.07.

Our evidence for a rising rate of biased technical change over time motivates the use of a more flexible specification for the bias. We use a Box–Cox transformation of the time trend, as in Klump et al. (2007), which allows the bias to vary monotonically over time.[46] With the Box–Cox specification, the aggregate elasticity is 0.69, close to our baseline estimates. Again, the range of the confidence interval is large.

[46]The Box–Cox transformation implies that $d\log\phi = \gamma t^{\lambda}$; λ allows the rate of biased technical change to vary over time.

Each methodology provides a measure of the contribution of the bias of technical change to the decline in the labor share, depicted in Figure 11.[47] Assuming a constant rate of biased technical change, the average contribution of bias is about −0.5 percentage points per year and is larger than our average contribution. More importantly, this average misses the timing of the large changes in the contribution of bias over time. The Box–Cox specification implies that the contribution of bias to the labor share has accelerated over time, but does not display the sharp drop at 2000 that the bias estimates from our method have.

Figure 11 Elasticity and Bias Estimates from Aggregate Data



Note: The left plot displays the point estimate and 95 percent confidence interval for the aggregate elasticity of substitution from regressions based on equation (34). Specifications differ in assumptions on the bias of technical change. Technical change is respectively assumed to have no trend, follow a linear time trend, or follow a Box–Cox transformation of the time trend. The right plot displays the contribution to the labor share from the bias of technical change, from either aggregate regressions with a linear or Box-Cox specification of the time trend or from our method that estimates the aggregate elasticity from the micro data.

[47] For the aggregate time series method, the contribution of bias is $s^{v,L}(1 - s^{v,L})\mathrm{d}\log\phi$; thus, the contribution to the labor share can vary over time even if the bias is constant.