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Technical Appendix:
Transition to FDI Openness—Reconciling Theory and Evidence*

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* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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Chapter 1.

Multicountry Model

In this chapter, I provide more details on computing equilibrium paths for the dynamic general equilibrium model analyzed in the main text. Specifically, I report the first-order conditions before and after detrending variables that grow over time. I then discuss the algorithm used to compute the equilibrium paths on a parallel processor. Codes and documentation are available at my website (www.minneapolisfed.org/research/economists/emcgrattan.html).

1.1. Maximization problems

The model used here is a version of McGrattan and Prescott (2010) with constant taxes and a simplified portfolio choice. I let i index countries, $i = 1, \ldots, I$, and j index multinational companies. Without loss of generality, I will work with a representative multinational, where the index j denotes the country of incorporation.

1.1.1. Multinational problem

Multinational j solves

$$\max \sum_t p_t \left(1 - \tau_d\right) D_t^j,$$

where

$$D_t^j = \sum_i \Big\{ \left(1 - \tau_{pi}\right) \left(Y_{it}^j - W_{it} L_{it}^j - \delta_T K_{T,it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j \right) - \left(K_{T,i,t+1}^j - K_{T,it}^j \right) \Big\}$$

and $\sum_i \chi_i^j = 1$,

$$K_{T,i,t+1}^j = \left(1 - \delta_T\right) K_{T,it}^j + X_{T,it}^j$$

$$K_{I,i,t+1}^j = \left(1 - \delta_I\right) K_{T,it}^j + X_{I,it}^j$$

$$M_{t+1}^j = \left(1 - \delta_M\right) M_t^j + X_{M,t}^j.$$

Outputs are given by

$$Y_{it}^{j} = A_{it}^{j} \left(N_{it} M_{t}^{j}\right)^{\phi} \left(Z_{it}^{j}\right)^{1-\phi}$$

$$Z_{it}^{j} = \left(K_{T,it}^{j}\right)^{\alpha_T} \left(K_{I,it}^{j}\right)^{\alpha_I} \left(L_{it}^{j}\right)^{1-\alpha_T-\alpha_I},$$

where N_i is the number of locations in country i, M^j is the stock of technology capital, Z_i^j is a composite input to multinationals j in country i, A_i^j is the level of technology parameter faced by multinationals j in country i, $K_{T,i}^j$ is the stock of tangible capital used by multinationals j in country i, $K_{I,i}^j$ is the stock of intangible capital used by multinationals j in country i, and L_i^j is the labor supplied to multinationals j in country i. Below, I assume that $A_{it}^j = A_i(1+\gamma_A)^t$ if $i = j$ and $A_i \sigma_{it}(1+\gamma_A)^t$ otherwise.

The first-order conditions for multinational j with respect to the labor inputs and capital stocks are given by

$$W_{it} = (1-\phi)(1-\alpha_T-\alpha_I)\ Y_{it}^j / L_{it}^j, \quad i = 1, \ldots, I$$

$$\begin{aligned} \frac{p_t}{p_{t+1}} &= 1 + (1-\tau_{pi})\left((1-\phi)\,\alpha_T Y_{i,t+1}^j / K_{T,i,t+1}^j - \delta_T\right) \\ &\equiv 1 + (1-\tau_{pi})\left(r_{T,i,t+1}^j - \delta_T\right), \quad i, j = 1, \ldots, I \end{aligned}$$

$$\begin{aligned} \frac{p_t}{p_{t+1}} &= (1-\phi)\,\alpha_I Y_{i,t+1}^j / K_{I,i,t+1}^j + 1 - \delta_I \\ &\equiv r_{I,i,t+1}^j + 1 - \delta_I, \quad i, j = 1, \ldots, I \end{aligned}$$

$$\begin{aligned} \frac{p_t}{p_{t+1}} &= \sum_i (1-\tau_{pi})\left(\phi Y_{i,t+1}^j / M_{t+1}^j + \chi_i^j\,(1-\delta_M)\right) / (1-\tau_{pj}) \\ &\equiv \sum_i (1-\tau_{pi})\left(r_{M,i,t+1}^j + \chi_i^j\,(1-\delta_M)\right) / (1-\tau_{pj}). \end{aligned}$$

1.1.2. Household problem

Households choose sequences of consumption C_{it}, labor L_{it}, and assets B_{it+1} to solve the following problem:

$$\max \sum_t \beta^t U\left(C_{it}/N_{it}, L_{it}/N_{it}\right) N_{it}$$

subject to

$$\sum_t p_t \left[C_{it} + B_{i,t+1} - B_{it}\right] \leq \sum_t p_t \left[\left(1 - \tau_{li}\right) W_{it} L_{it} + \left(1 - \tau_d\right) D_t^i + r_{bt} B_{it} + \kappa_{it}\right],$$

where τ_{li} and τ_d are tax rates on labor and company distributions, and r_{bt} is the after-tax return on lending/borrowing. I assume that country i has a population of size N_{it}. Note that the measure of a country's production locations is proportional to its population. Hence, I use the same notation for both variables and set the constant of proportionality equal to one (without loss of generality).

If $U(c, l) = \log c + \psi \log(1 - l)$, then the first-order conditions with respect to C_i, L_i, and B_i for the household in country i are

$$\lambda p_t = \beta^t U_{c,it} = \beta^t N_{it}/C_{it}$$

$$\lambda \left(1 - \tau_{li}\right) W_{it} p_t = \beta^t U_{l,it} = \psi \beta^t / \left(1 - L_{it}/N_{it}\right)$$

$$\frac{p_t}{p_{t+1}} = 1 + r_{b,t+1}$$

where λ_i is the Lagrange multiplier associated with household i's budget constraint.

1.1.3. Market clearing

Markets must clear for goods, assets, and labor. The worldwide resource constraint for goods is

$$\sum_i \left\{ C_{it} + \sum_j \left[X_{T,it}^j + X_{I,it}^j\right] + X_{M,t}^i \right\} = \sum_{i,j} Y_{it}^j.$$

The asset market clearing condition is

$$\sum_i B_{it} = 0$$

for all t, and the labor market clearing conditions are

$$L_{it} = \sum_j L_{it}^j, \quad i = 1, \dots, I.$$

1.2. Detrended variables

I'll use lowercase letters for growth-detrended variables. Specifically, let

$$c_{it} = \frac{C_{it}}{N_{it}\left(1+\gamma_y\right)^t}$$

$$y_{it}^j = \frac{Y_{it}^j}{N_{it}\left(1+\gamma_y\right)^t}$$

$$l_{it} = \frac{L_{it}}{N_{it}}, \ l_{it}^j = \frac{L_{it}^j}{N_{it}}$$

$$w_{it} = \frac{W_{it}}{\left(1+\gamma_y\right)^t}$$

$$b_{it} = \frac{B_{it}}{N_{it}\left(1+\gamma_y\right)^t}$$

$$k_{\cdot,it}^j = \frac{K_{\cdot,it}^j}{N_{it}\left(1+\gamma_y\right)^t}$$

$$x_{\cdot,it}^j = \frac{X_{\cdot,it}^j}{N_{it}\left(1+\gamma_y\right)^t}$$

$$x_{M,t}^j = \frac{X_{M,t}^j}{\left(1+\gamma_Y\right)^t}$$

$$m_t^j = \frac{M_t^j}{\left(1+\gamma_Y\right)^t}$$

$$d_t^j = \frac{D_t^j}{\left(1+\gamma_Y\right)^t}$$

$$a_{it}^j = \frac{A_{it}^j}{\left(1+\gamma_A\right)^t},$$

where γ_Y is the growth rate of output, γ_y is the growth rate of per capita output, and γ_A is the growth rate of total factor productivity (TFP). Using the production technology, I can determine the growth rate of total output on the balanced growth path:

$$\begin{aligned}(1+\gamma_Y) &= (1+\gamma_A)(1+\gamma_N)^{\phi}(1+\gamma_Y)^{\phi}(1+\gamma_Y)^{\alpha_T(1-\phi)} \\ &\quad \cdot (1+\gamma_Y)^{\alpha_I(1-\phi)}(1+\gamma_N)^{(1-\alpha_T-\alpha_I)(1-\phi)} \\ &= (1+\gamma_A)^{\frac{1}{(1-\alpha_T-\alpha_I)(1-\phi)}}(1+\gamma_N)^{\frac{1-(\alpha_T+\alpha_I)(1-\phi)}{(1-\alpha_T-\alpha_I)(1-\phi)}},\end{aligned}$$

where γ_N is the growth rate of the population (and locations) on a balanced growth path.

1.3. Detrended first-order conditions

Substituting detrended variables into first-order conditions implies that

$$\begin{aligned}d_t^j &= \sum_i \left\{(1-\tau_{pi}) N_{it}\left(y_{it}^j - w_{it} l_{it}^j - \delta_T k_{T,it}^j - x_{I,it}^j\right)\right\} \\ &\quad - x_{M,t}^j \sum_i (1-\tau_{pi})\chi_i^j - \sum_i N_{it}\left\{(1+\gamma_Y) k_{T,i,t+1}^j - k_{T,it}^j\right\} \qquad (1.3.1)\end{aligned}$$

$$k_{T,i,t+1}^j = \left[(1-\delta_T) k_{T,it}^j + x_{T,it}^j\right] N_{it} / \left[(1+\gamma_Y) N_{i,t+1}\right] \qquad (1.3.2)$$

$$k_{I,i,t+1}^j = \left[(1-\delta_I) k_{I,it}^j + x_{I,it}^j\right] N_{it} / \left[(1+\gamma_Y) N_{i,t+1}\right] \qquad (1.3.3)$$

$$m_{t+1}^j = \left[(1-\delta_M) m_t^j + x_{M,t}^j\right] N_{it} / \left[(1+\gamma_Y) N_{i,t+1}\right] \qquad (1.3.4)$$

$$y_{it}^j = a_{it}^j \left(m_t^j\right)^{\phi} \left(\left(k_{T,it}^j\right)^{\alpha_T} \left(k_{I,it}^j\right)^{\alpha_I} \left(l_{it}^j\right)^{1-\alpha_T-\alpha_I}\right)^{1-\phi} \qquad (1.3.5)$$

$$w_{it} = (1-\phi)(1-\alpha_T-\alpha_I) y_{it}^j / l_{it}^j \qquad (1.3.6)$$

$$r_{T,it}^j = (1-\phi)\alpha_T y_{it}^j / k_{T,it}^j \qquad (1.3.7)$$

$$r_{I,it}^j = (1-\phi)\alpha_I y_{it}^j / k_{I,it}^j \qquad (1.3.8)$$

$$r_{M,it}^j = \phi N_{it} y_{it}^j / m_t^j \qquad (1.3.9)$$

$$c_{it} = (1-\tau_{li}) w_{it} l_{it} + (1-\tau_d) d_t^i / N_{it}$$

$$+ (1 + r_{bt})\, b_{it} - (1 + \gamma_Y)\, b_{i,t+1} N_{it+1}/N_{it} + \kappa_{it} \tag{1.3.10}$$

$$p_t/p_{t+1} = 1 + (1 - \tau_{pi}) \left(r^j_{T,i,t+1} - \delta_T \right) \tag{1.3.11}$$

$$p_t/p_{t+1} = 1 + r^j_{I,i,t+1} - \delta_I \tag{1.3.12}$$

$$p_t/p_{t+1} = \sum_i (1 - \tau_{pi}) \left(r^j_{M,i,t+1} + \chi^j_i \,(1 - \delta_M) \right) / \,(1 - \tau_{pj}) \tag{1.3.13}$$

$$p_t/p_{t+1} = (1 + \gamma_y)\, c_{i,t+1}/\,(\beta c_{it}) \tag{1.3.14}$$

$$(1 - \tau_{li})\, w_{it} = \psi\,(1 + \tau_{ci})\, c_{it}/\,(1 - l_{it}) \tag{1.3.15}$$

$$p_t/p_{t+1} = 1 + r_{b,t+1} \tag{1.3.16}$$

$$\sum_{i,j} N_{it} y^j_{it} = \sum_i N_{it} \Big(c_{it} + \sum_j x^j_{T,it} + \sum_j x^j_{I,it} \Big) + \sum_j x^j_{M,t} \tag{1.3.17}$$

$$0 = \sum_i b_{it} \tag{1.3.18}$$

$$l_{it} = \sum_j l^j_{it}. \tag{1.3.19}$$

$$\kappa_{it} = \tau_{pi} \sum_j \left(y^j_{it} - w_{it} l_{it} - \delta_T k^j_{T,it} - x^j_{I,it} \right) - \tau_{pi} x^i_{M,t}/N_{it}$$
$$+ \tau_{li} w_{it} l_{it} + \tau_d d^i_t / N_{it}. \tag{1.3.20}$$

1.4. Adding adjustment costs

In the 104-country version of the model, small adjustment costs to investment were added to aid in the computations. Numerical problems arise because of investment hitting corners. In this section, I consider adding adjustment costs, which eventually were driven close to zero given a good guess of the final solution.

Let $\varphi_T(X/K)$, $\varphi_I(X/K)$, and $\varphi_M(X/K)$ be the costs of adjusting investment in tangible capital, plant-specific intangible capital, and technology capital, respectively. The capital accumulation equations are now given by

$$K^j_{T,i,t+1} = (1-\delta_T) K^j_{T,it} + X^j_{T,it} - \varphi_T\left(X^j_{T,it}/K^j_{T,it}\right) K^j_{T,it}$$
$$K^j_{I,i,t+1} = (1-\delta_I) K^j_{T,it} + X^j_{I,it} - \varphi_I\left(X^j_{I,it}/K^j_{I,it}\right) K^j_{I,it}$$
$$M^j_{t+1} = (1-\delta_M) M^j_t + X^j_{M,t} - \varphi_M\left(X^j_{M,t}/M^j_t\right) M^j_t.$$

The Lagrangian for the multinational j can be written as follows:

$$\begin{aligned}\mathcal{L} = (1-\tau_d)\sum_t p_t\Big\{&\sum_i [Y^j_{it} - W_{it}L^j_{it} - X^j_{T,it} - X^j_{I,it}] \\ &- \tau_{pi}\sum_i [Y^j_{it} - W_{it}L^j_{it} - \delta_T K^j_{T,it} - X^j_{I,it}] - (1-\tau_{pi}) X^j_{M,t}\Big\} \\ &+ \sum_{t,i}\lambda_{Tit}((1-\delta_T) K^j_{T,it} + X^j_{T,it} - \varphi_T\left(X^j_{T,it}/K^j_{T,it}\right) K^j_{T,it} - K^j_{T,i,t+1}) \\ &+ \sum_{t,i}\lambda_{Iit}((1-\delta_I) K^j_{I,it} + X^j_{I,it} - \varphi_I\left(X^j_{I,it}/K^j_{I,it}\right) K^j_{I,it} - K^j_{I,i,t+1}) \\ &+ \sum_{t}\lambda_{Mt}((1-\delta_M) M^j_t + X^j_{M,t} - \varphi_I\left(X^j_{M,t}/M^j_t\right) M^j_t - M^j_{t+1}).\end{aligned}$$

The new first-order conditions for multinational j with respect to the capital stocks are given by

$$\begin{aligned}\left(\frac{1}{1-\varphi'_{Tt}}\right)\frac{p_t}{p_{t+1}} = (1-\tau_{pi})\, r^j_{T,i,t+1} + \tau_{pi}\delta_T \\ + \left(\frac{1}{1-\varphi'_{T,t+1}}\right)\left(1-\delta_T + \varphi'_{T,t+1}\frac{X^j_{T,i,t+1}}{K^j_{T,i,t+1}} - \varphi_{T,t+1}\right)\end{aligned}$$

$$\left(\frac{1}{1-\varphi'_{It}}\right)\frac{p_t}{p_{t+1}} = r^j_{I,i,t+1} + \left(\frac{1}{1-\varphi'_{I,t+1}}\right)\left(1-\delta_I + \varphi'_{I,t+1}\frac{X^j_{I,i,t+1}}{K^j_{I,i,t+1}} - \varphi_{I,t+1}\right)$$

$$\begin{aligned}\left(\frac{1}{1-\varphi'_{Mt}}\right)\frac{p_t}{p_{t+1}} = \frac{1}{1-\tau_{pj}}\sum_i (1-\tau_{pi})\, r^j_{M,i,t+1} \\ + \left(\frac{1}{1-\varphi'_{M,t+1}}\right)\left(1-\delta_M + \varphi'_{M,t+1}\frac{X^j_{M,t+1}}{M^j_{t+1}} - \varphi_{M,t+1}\right),\end{aligned}$$

where terms like φ_{Tt} and φ'_{Tt} are shorthand for the functions evaluated at the time t ratio of investment to capital.

1.5. BEA accounts

I now apply the same procedure as the Bureau of Economic Analysis (BEA) to set up the national and international accounts for the model economy. I'll use the notation J^i to mean the set of multinationals incorporated in i. This implies the following for gross domestic product (GDP) and gross national product (GNP) and their components:

- $\text{GDP}_{it} = \sum_j (Y_{it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j)$

  Income

  Depreciation: $\delta_T \sum_j K_{T,it}^j$

  Compensation: $W_{it} \sum_j L_{it}^j = W_{it} L_{it}$

  Profits:

  Tax liability: $\tau_{pi} \sum_j (Y_{it}^j - W_{it} L_{it}^j - \delta_T K_{T,it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j)$

  Dividends: $\sum_j \{(1-\tau_{pi})(Y_{it}^j - W_{it} L_{it}^j - \delta_T K_{T,it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j)$
  $-(K_{T,i,t+1}^j - K_{T,it}^j)\}$

  Retained earnings: $\sum_j (K_{T,i,t+1}^j - K_{T,it}^j)$

  Product

  Consumption: C_{it}

  Measured investment: $\sum_j X_{T,it}^j$

  Net exports: $\sum_j (Y_{it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j - X_{T,it}^j) - C_{it}$

- $\text{GNP}_{it} = \text{GDP}_{it}$ + Net factor receipts less payments

  Net factor receipts (from $l \neq i$)

  Direct investment: $\sum_{l \neq i} (1-\tau_{pl}) \sum_{j \in J^i} (Y_{lt}^j - W_{lt} L_{lt}^j - \delta_T K_{T,lt}^j - X_{I,lt}^j - \chi_l^j X_{M,t}^j)$

  Portfolio interest: $r_{bt} B_{it}$ if $B_{it} \geq 0$

  Net factor payments (to $l \neq i$)

  Direct investment: $(1-\tau_{pi}) \sum_{j \in J^l} (Y_{it}^j - W_{it} L_{it}^j - \delta_T K_{T,it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j)$

  Portfolio interest: $r_{bt} B_{it}$ if $B_{it} \leq 0$

- Balance of Payments: Current account = Financial account

  Current account = Net exports + net factor receipts less payments

  Net exports: $\sum_j (Y_{it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j - X_{T,it}^j) - C_{it}$

  Net factor receipts (from $l \neq i$)

  Direct investment: $\sum_{l \neq i} (1-\tau_{pl}) \sum_{j \in J^i} (Y_{lt}^j - W_{lt} L_{lt}^j - \delta_T K_{T,lt}^j - X_{I,lt}^j - \chi_l^j X_{M,t}^j)$

  Portfolio interest: $r_{bt} B_{it}$ if $B_{it} \geq 0$

  Net factor payments (to $l \neq i$)

  Direct investment: $(1 - \tau_{pi}) \sum_{j \in J^l} (Y_{it}^j - W_{it} L_{it}^j - \delta_T K_{T,it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j)$

  Portfolio interest: $r_{bt} B_{it}$ if $B_{it} \leq 0$

  Financial account

  Direct investment: $\sum_{l \neq i} \sum_{j \in J^i} (K_{T,l,t+1}^j - K_{T,lt}^j) - \sum_{j \in J^l} (K_{T,i,t+1}^j - K_{T,it}^j)$

  Change in portfolio: $B_{i,t+1} - B_{it}$

Next, consider the accounts for the two-country case that will be central to the propositions that come later. For ease of exposition, let i be Ireland and r be the rest of world. I index companies in Ireland by d, which I'll refer to as "Domestic." I'll index Rest-of-World (ROW) companies by f, which I'll refer to as "Foreign."

The current account (CA) is the sum of net exports (NX) plus net factor receipts (NFR) less net factor payments (NFP):

$$\begin{aligned}
\text{CA}_{it} &= NX_{it} + \text{NFR}_{it} - \text{NFP}_{it} \\
&= \left[Y_{it}^d + Y_{it}^f - X_{I,it}^d - X_{I,it}^f - X_{T,it}^d - X_{T,it}^f - X_{M,t}^d - C_{it}\right] \\
&+ \left[(1 - \tau_{pr}) \left(Y_{rt}^d - W_{rt} L_{rt}^d - \delta_T K_{T,rt}^d - X_{I,rt}^d\right)\right] \\
&- \left[(1 - \tau_{pi}) \left(Y_{it}^f - W_{it} L_{it}^f - \delta_T K_{T,it}^f - X_{I,it}^f\right) - r_b B_{it}\right] \\
&= (1 - \tau_{li}) W_{it} L_{it} + (1 - \tau_d) D_t^d + r_{bt} B_{it} + \kappa_{it} - C_{it} \\
&\qquad + K_{T,r,t+1}^d - K_{T,rt}^d - K_{T,i,t+1}^f + K_{T,it}^f,
\end{aligned} \tag{1.5.1}$$

where

$$\kappa_{it} = \tau_{li} W_{it} L_{it} + \tau_d D_t^d + \tau_{pi} \left(Y_{it}^d - W_{it} L_{it}^d - \delta_T K_{T,it}^d - X_{I,it}^d\right)$$

$$+\tau_{pi}\left(Y_{it}^f - W_{it}L_{it}^f - \delta_T K_{T,it}^f - X_{I,it}^f\right),$$

Y_{it} is total production in country i, $K_{T,it} = \sum_j K_{T,it}^j$ the total tangible capital stock in country i, and $X_{I,it} = \sum_j X_{I,it}^j$ is total plant-specific investment in country i. In writing net factor payments, I assume that $B_{it} < 0$ and therefore net factor interest is paid by Ireland to rest of world. I also assume that multinationals expense their investment of technology capital at home.

Next, consider the financial account (FA) which is the change in assets and given by

$$\text{FA}_{it} = K_{T,r,t+1}^d - K_{T,rt}^d - K_{T,i,t+1}^f + K_{T,it}^f + B_{i,t+1} - B_{it}. \tag{1.5.2}$$

By the balance of payments, FA less CA is equal to zero and therefore

$$\begin{aligned}
0 &= FA_{it} - CA_{it} \\
&= K_{T,r,t+1}^d - K_{T,rt}^d - K_{T,i,t+1}^f + K_{T,it}^f + B_{i,t+1} - B_{it} \\
&\qquad - (1-\tau_{li})\, W_{it}L_{it} - (1-\tau_d)\, D_t^d - r_{bt}B_{it} - \kappa_{it} + C_{it} \\
&\qquad\qquad - K_{T,r,t+1}^d + K_{T,rt}^d + K_{T,i,t+1}^f - K_{T,it}^f \\
&= C_{it} + B_{i,t+1} - B_{it} \\
&\qquad - (1-\tau_{li})\, W_{it}L_{it} - (1-\tau_d)\, D_t^d - r_{bt}B_{it} - \kappa_{it}.
\end{aligned}$$

As the last equation makes clear, the balancing of payments internationally is consistent with the balancing of budgets domestically. (Compare the last equation to the budget in Section 1.2).

1.6. Propositions for a two-country version

In this section, I provide more details in proofs of the main propositions (repeated here) for the two-country model.

In this version of the model, there are two countries: i (e.g., Ireland) and r (e.g., ROW). I assume from the start that Ireland is small in size relative to the ROW. Companies in Ireland are indexed by d (Domestic), and companies from the ROW are indexed by f (Foreign).

I compute the transition of these countries as they go from fully closed ($\sigma_t = 0$) to sufficiently open ($\sigma_t \geq \sigma^*$) after some date $t = t^*$ so that the small country shuts down investment in technology capital. In this case, there is only a small amount of multinational activity of Irish firms in the rest of the world. Even though the firms have built up technology capital at home, which could be used abroad, they have no intangible or tangible capital abroad when the countries open up. Given they are planning to decumulate the technology capital, it is not worth it for these firms to build up a lot of tangible and intangible capital abroad for temporary use. Therefore, when countries open up, the amount of foreign direct investment (FDI) done by Irish firms is small.

Therefore, to make the mathematics simpler, I assume from the start that Irish companies do not operate in the Rest of World (that is, $Y_{rt}^d = 0$ for all t). This assumption makes the mathematical derivations easier and changes the quantitative results little.

In addition to restrictions on foreign direct investment, there could also be restrictions on portfolio flows. I start with the case of free flows in portolio investments, which is simpler to analyze. I then show how things change if I restrict portfolio investments along with foreign direct investments.

1.6.1. Without constraints on portfolio flows

Domestic multinationals in Ireland (with index d) solve

$$\max \sum_t p_t \left(1 - \tau_d\right) D_t^d,$$

where

$$D_t^d = (1-\tau_{pi})\left(Y_{it}^d - W_{it}L_{it}^d - \delta_{\scriptscriptstyle T} K_{\scriptscriptstyle T,it}^d - X_{\scriptscriptstyle I,it}^d - X_{\scriptscriptstyle M,t}^d\right) - \left(K_{\scriptscriptstyle T,i,t+1}^d - K_{\scriptscriptstyle T,it}^d\right)$$

with the capital accumulation equations as given above. ROW multinationals f solve

$$\max \sum_t p_t \left(1-\tau_d\right) D_t^f,$$

where

$$\begin{aligned} D_t^f &= (1-\tau_{pr})\left(Y_{rt}^f - W_{rt}L_{rt}^f - \delta_{\scriptscriptstyle T} K_{\scriptscriptstyle T,rt}^f - X_{\scriptscriptstyle I,rt}^f - X_{\scriptscriptstyle M,t}^f\right) - K_{\scriptscriptstyle T,r,t+1}^f + K_{\scriptscriptstyle T,rt}^f \\ &\quad + (1-\tau_{pi})\left(Y_{it}^f - W_{it}L_{it}^f - \delta_{\scriptscriptstyle T} K_{\scriptscriptstyle T,it}^f - X_{\scriptscriptstyle I,it}^f\right) - K_{\scriptscriptstyle T,i,t+1}^f + K_{\scriptscriptstyle T,it}^f. \end{aligned}$$

The household problem for the Irish is

$$\begin{aligned} &\max \sum_t \beta^t [\log\left(C_{it}/N_{it}\right) + \psi \log\left(1 - L_{it}/N_{it}\right)] N_{it} \\ &\text{subj. to } \sum_t p_t \left[C_{it} + B_{i,t+1} - B_{it}\right] \\ &\qquad \leq \sum_t p_t \left[(1-\tau_{li}) W_{it} L_{it} + (1-\tau_d) D_t^d + r_{bt} B_{it} + \kappa_{it}\right]. \end{aligned}$$

The ROW households solve a similar problem except that they also receive an additional income $\{\epsilon_t\}$, which is added to household income in order to implement a numerical "trick" described later. The problem is

$$\begin{aligned} &\max \sum_t \beta^t [\log\left(C_{rt}/N_{rt}\right) + \psi \log\left(1 - L_{rt}/N_{rt}\right)] N_{rt} \\ &\text{subj. to } \sum_t p_t \left[C_{rt} + B_{r,t+1} - B_{rt}\right] \\ &\qquad \leq \sum_t p_t \left[(1-\tau_{lr}) W_{rt} L_{rt} + (1-\tau_d) D_t^f + r_{bt} B_{rt} + \kappa_{rt} + \epsilon_t\right]. \end{aligned} \tag{1.6.1}$$

The additional income given to ROW households is equal to

$$\epsilon_t = \tilde{C}_{it} + \tilde{C}_{rt} + \tilde{X}_{\scriptscriptstyle T,it}^d + \tilde{X}_{\scriptscriptstyle T,it}^f + \tilde{X}_{\scriptscriptstyle T,rt}^f + \tilde{X}_{\scriptscriptstyle I,it}^d + \tilde{X}_{\scriptscriptstyle I,it}^f + \tilde{X}_{\scriptscriptstyle I,rt}^f + \tilde{M}_t^d + \tilde{M}_t^f - \tilde{Y}_{it} + \tilde{Y}_{rt},$$

where the tilde $(\tilde{\ })$ denotes the equilibrium values for a model with $r_{bt} = (1+\gamma_y)/\beta - 1$ for $t = 1, \dots, t^*$. Alternatively, the income adjustment can be written in terms of incomes rather than products as follows:

$$\epsilon_t = \tilde{C}_{it} + \tilde{C}_{rt} - (1-\tau_{li})\,\tilde{W}_{it}\tilde{L}_{it} - (1-\tau_{lr})\,\tilde{W}_{rt}\tilde{L}_{rt} - (1-\tau_d)\left(\tilde{D}_t^d + \tilde{D}_t^f\right) - \tilde{\kappa}_{it} - \tilde{\kappa}_{rt}.$$

Adding the income to the Rest-of-World budget in (1.6.1) ensures that the equilibrium of the ϵ-economy is the same as one with $\epsilon_t = 0$ for all t and r_{bt} equal to $(1+\gamma_y)/\beta - 1$ for $t \leq t^*$.

For Propositions 1–3, I assume that the ϵ_t are nonzero for $t = 1, \dots, t^*$. For $t > t^*$, $\epsilon_t = 0$. I refer to this case as the ϵ-economy. Figure 1 shows the difference between the interest rates with and without the ROW income adjustment, which is shown in Figure 2.

Proposition 1. The small country's output, labor, and capital stocks in the ϵ-economy are at or below their historical trends between $t = 1$ and $t = t^*$, while consumption is above. The reverse is true for the large country.

Proof. Suppose that at $t = 1$, consumption c_{it} in Ireland is above its historical trend, $c_{i1} > c_{i0}$. Then, between $t = 2$ and $t = t^*$, $c_{it} = c_{i1}$ since

$$\begin{aligned} c_{it} &= \frac{\beta\,(1+r_{bt})}{1+\gamma_y} c_{i,t-1}, \quad t = 2, \dots, t^* \\ &= c_{i,t-1} \quad t = 2, \dots, t^*, \end{aligned}$$

where the first equation follows from (1.3.14) and (1.3.16), and the second equation follows from the fact that $r_{bt} = (1+\gamma_y)/\beta - 1$ by choice of $\{\epsilon_t\}$ adjustment.

From the intratemporal first-order condition of Irish households (1.3.15), with labor productivity in (1.3.6) substituted for the real wage, I get

$$\frac{y_{it}^d}{l_{it}^d} = \frac{(1+\tau_{ci})}{(1-\tau_{li})\,(1-\phi)\,(1-\alpha_T-\alpha_I)} \frac{c_{it}}{1-l_{it}}.$$

In the period prior to liberalization of the capital account, only domestic firms are operating, so

$$\frac{y_{it}}{l_{it}} = \frac{y_{it}^d}{l_{it}^d} \propto \frac{c_{it}}{1-l_{it}}, \quad t = 1, \ldots, t^*, \tag{1.6.2}$$

where y_{it}/l_{it} is total labor productivity. I can use this relation to determine what happens to employment between $t = 0$ and $t = 1$.

To do that, I take the ratio of labor productivity in the first period relative to its trend (that is, the $t = 0$ level):

$$\frac{y_{i1}/y_{i0}}{l_{i1}/l_{i0}} = \frac{c_{i1}/c_{i0}}{(1-l_{i1})\,/\,(1-l_{i0})}. \tag{1.6.3}$$

Since output depends on beginning of period capital stocks, which are given and equal to historical levels, (1.6.3) implies

$$\left(\frac{l_{i1}}{l_{i0}}\right)^{(1-\phi)(1-\alpha_T-\alpha_I)-1} = \frac{c_{i1}/c_{i0}}{(1/l_{i0}-1)\,/\,(1/l_{i0}-l_{i0}/l_{i1})}. \tag{1.6.4}$$

Let $x = l_{i1}/l_{i0}$, $a = 1-(1-\phi)(1-\alpha_T-\alpha_I)$, and $b = 1/l_{i0}$, $c = c_{i1}/c_{i0}$. Now, I can restate the problem of determining what happens to employment between $t = 0$ and $t = 1$ as finding out whether the value for x in the following equation,

$$cx^a = \frac{b-x}{b-1}, \quad a \in (0,1)\,, b \geq 1, c > 1, \tag{1.6.5}$$

is less or greater than 1, since x is the ratio of labor inputs in the two periods. The left hand side of (1.6.5) is everywhere increasing in x, and the right-hand side is everywhere decreasing, which means there is a unique solution. At $x = 1$, the left-hand side of (1.6.5) exceeds 1 and the right-hand side is equal to 1, which means that the intersection must lie below 1. Thus, $l_{i1} < l_{i0}$. Furthermore, $y_{i1} < y_{i0}$ since capital stocks are fixed. With curvature in the production function, it must be the case that (true) productivity rises initially, $y_{i1}/l_{i1} > y_{i0}/l_{i0}$.

If $t^* > 2$, then in period $t = 2$, output and labor must fall further relative to the historic trend, because domestic capital stocks fall between the first and second periods.

To demonstrate this, I first substitute returns to capital in (1.3.7)–(1.3.9) into the dynamic Euler equations in order to relate capital-output ratios to the return r_{bt}; that is,

$$
\begin{aligned}
r_{bt} &= (1-\tau_{pi})\left((1-\phi)\,\alpha_T y_{it}^d / k_{T,it}^d - \delta_T\right) \\
r_{bt} &= (1-\phi)\,\alpha_I y_{it}^d / k_{I,it}^d - \delta_I \\
r_{bt} &= \phi N_i y_{it}^d / m_t^d - \delta_M
\end{aligned}
$$

for the period $t = 2, \ldots, t^*$. Note that the last equation has only one term because the Irish firms are assumed to operate only domestically. If the return r_{bt} is equal to $(1+\gamma_y)/\beta - 1$, then the capital-output ratios must be equal to their historical levels prior to FDI liberalization. Using this fact along with the production technologies in (1.3.5), it follows that labor productivity in the second period must also be at its historical level,

$$
\frac{y_{i2}}{l_{i2}} = \frac{y_{i0}}{l_{i0}}
$$

since

$$
\begin{aligned}
y_{i2} &= a_{i2}\left(m_2^d\right)^{\phi}\left(\left(k_{T,i2}^d\right)^{\alpha}_{T}\left(k_{I,i2}^d\right)^{\alpha}_{I}\left(l_{i2}^d\right)^{1-\alpha_T-\alpha_I}\right)^{1-\phi} \\
&= a_{i2}\left(m_0^d / y_{i0}\; y_{i2}\right)^{\phi}\left(\left(k_{T,i0}^d / y_{i0}\; y_{i2}\right)^{\alpha}_{T}\left(k_{I,i0}^d / y_{i0}\; y_{i2}\right)^{\alpha}_{I}\left(l_{i2}^d\right)^{1-\alpha_T-\alpha_I}\right)^{1-\phi} \\
&= \left[a_{i2}\left(m_0^d / y_{i0}\right)^{\phi}\left(\left(k_{T,i0}^d / y_{i0}\right)^{\alpha}_{T}\left(k_{I,i0}^d / y_{i0}\right)^{\alpha}_{I}\right)^{1-\phi}\right]^{1/((1-\phi)(1-\alpha_T-\alpha_I))} l_{i2} \\
&= \left(y_{i0}/l_{i0}\right) l_{i2},
\end{aligned}
$$

where I am using the fact that $l_{it} = l_{it}^d$ and $y_{it} = y_{it}^d$ in $t \leq t^*$. If labor productivity is on trend and consumption is equal to the level in period 1, then it follows from (1.6.3) that $l_{i2} < l_{i1}$, which in turn implies that $l_{i2} < l_{i0}$. In other words,

$$
\begin{aligned}
l_{i2} &= 1 - \frac{(1+\tau_c)}{(1-\tau_l)(1-\phi)(1-\alpha_T-\alpha_I)}\frac{c_{i2} l_{i2}}{y_{i2}} \\
&= 1 - \frac{(1+\tau_c)}{(1-\tau_l)(1-\phi)(1-\alpha_T-\alpha_I)}\frac{c_{i1} l_{i0}}{y_{i0}} \\
&< 1 - \frac{(1+\tau_c)}{(1-\tau_l)(1-\phi)(1-\alpha_T-\alpha_I)}\frac{c_{i1} l_{i1}}{y_{i1}} \\
&= l_{i1}.
\end{aligned}
$$

It follows immediately that $y_{i2} < y_{i1}$ which in turn implies that $y_{i2} < y_{i0}$.

Since the interest rate r_{bt} does not change between $t = 2$ and t^*, it must be the case that $y_{it} = y_{i2}$ and $l_{it} = l_{i2}$, $t \leq t^*$. This follows from the intratemporal condition and the fact that capital-output ratios and consumptions relative to trend are constant.

The same arguments as above can be made for the large country. However, because the global resource constraint must hold, the paths relative to trend for the large country must be reversed. To see this, recall the global resource constraint prior to FDI openness:

$$N_i c_{it} + N_r c_{rt} = N_i \left(y_{it} - x^d_{T,it} - x^d_{I,it}\right) - x^d_{Mt} + N_r \left(y_{rt} - x^f_{T,rt} + x^f_{I,rt}\right) - x^f_{Mt}. \quad (1.6.6)$$

If consumption is above trend between $t = 1$ and $t = t^*$ in both Ireland and the Rest of World, it follows from the arguments above that output and investments are below trend in both countries. In fact, output and investments must be down by the same percentage between $t = 2$ and $t = t^* - 1$ because capital-output ratios are on trend. In this case, I can rewrite (1.6.6) as

$$\begin{aligned} N_i c_{it} + N_r c_{rt} &= \gamma_i \{N_i \left(y_{i0} - x^d_{T,i0} - x^d_{I,i0}\right) - x^d_{M0}\} + \gamma_r \{N_r \left(y_{r0} - x^f_{T,r0} + x^f_{I,r0}\right) - x^f_{M0}\} \\ &< N_i c_{i0} + N_r c_{r0} \end{aligned} \quad (1.6.7)$$

for $t = 2, \ldots, t^* - 1$, which leads to a contradiction of the claim that both consumptions are initially above their historical trends. The same logic can be used to prove that both consumptions are not below their historical trends initially.

Finally, I need to show that consumption is initially above trend in Ireland, which is the recipient of future foreign direct investment, and initially below trend for the Rest of World, which is the source of the foreign direct investment. This follows from the fact that there is no change in effective TFP for the Rest of World when $\sigma_{it} > 0$ because $y^d_{rt} = 0$ for all t by assumption. For Ireland, on the other hand, effective TFP is higher because multinationals in the Rest of World use their technology capital in Ireland when FDI is allowed and, therefore, $y^f_{it} > 0$ for $t > t^*$.

To summarize, I've shown that

- $c_{i1} > c_{i0}$, $c_{it} = c_{i1}$, $t = 2, \ldots, t^*$;
- $c_{r1} < c_{r0}$, $c_{rt} = c_{r1}$, $t = 2, \ldots, t^*$;
- $y_{i1} < y_{i0}$, $y_{i2} < y_{i1}$, $y_{it} = y_{i2}$, $t = 2, \ldots, t^*$;
- $y_{r1} > y_{r0}$, $y_{r2} > y_{r1}$, $y_{rt} = y_{r2}$, $t = 2, \ldots, t^*$;
- $l_{i1} < l_{i0}$, $l_{i2} < l_{i1}$, $l_{it} = l_{i2}$, $t = 2, \ldots, t^*$;
- $l_{r1} > l_{r0}$, $l_{r2} > l_{r1}$, $l_{rt} = l_{r2}$, $t = 2, \ldots, t^*$;
- $y_{i1}/l_{i1} > y_{i0}/l_{i0}$, $y_{it}/l_{it} = y_{i0}/l_{i0}$, $t = 2, \ldots, t^*$;
- $y_{r1}/l_{r1} < y_{r0}/l_{r0}$, $y_{rt}/l_{rt} = y_{r0}/l_{r0}$, $t = 2, \ldots, t^*$;
- $k^d_{T,i1} = k^d_{T,i0}$, $k^d_{T,it}/y_{it} = k^d_{T,it}/y_{i0}$, $t = 2, \ldots, t^*$;
- $k^f_{T,r1} = k^f_{T,r0}$, $k^f_{T,rt}/y_{rt} = k^f_{T,rt}/y_{r0}$, $t = 2, \ldots, t^*$.

Proposition 2. The small country's GDP and GNP in the ϵ-economy initially, after the announcement, rise above their historical trends and then fall below trend between $t = 2$ and $t = t^*$. The reverse is true for the large country.

Proof. Detrended GDP in the small country, Ireland, is given by

$$\text{GDP}_{it} = N_i \left(y^d_{it} + y^f_{it} - x^d_{I,it} - x^f_{I,it} \right) - X^d_{M,t}.$$

In $t = 1$, assuming $t^* > 1$, $Y^f_{it} = X^f_{I,it} = 0$ and therefore GDP is domestic output less investments in plant-specific intangible capital and technology capital by domestic companies. To show that Irish GDP is above its historical trend in $t = 1$, I must show that intangible investments of domestic firms (indexed by d) fall by more than output. This is shown as follows:

$$\begin{aligned}
\frac{x^d_{I,i1} - x^d_{I,i0}}{x^d_{I,i0}} &= \frac{1+\gamma_Y}{\delta_I+\gamma_Y}\left(\frac{k^d_{I,i2} - k^d_{I,i0}}{k^d_{I,i0}}\right) \\
&= \frac{1+\gamma_Y}{\delta_I+\gamma_Y}\left(\frac{y^d_{i2} - y^d_{i0}}{y^d_{i0}}\right) \\
&< \frac{1+\gamma_Y}{\delta_I+\gamma_Y}\left(\frac{y^d_{i1} - y^d_{i0}}{y^d_{i0}}\right), \\
&\leq \left(\frac{y^d_{i1} - y^d_{i0}}{y^d_{i0}}\right),
\end{aligned}$$

where the first equality uses the capital accumulation equation after detrending all variables, the second equality follows from the fact that the capital-output ratio in the second period is equal to the historical capital-output ratio, the inequality follows from Proposition 1, and the final inequality follows from the fact that $\delta_I \leq 1$. Thus, plant-specific intangible investment must fall by more than output. The same argument can be made for technology capital. Therefore, GDP must be above trend in $t = 1$.

GNP is equal to GDP in the first period because there are no net factor incomes if $B_{i0} = 0$ and FDI income is zero. Thus, in the first period, when the policy is announced, net factor incomes for the period are already determined and GNP must equal GDP.

In the second period, since the capital-output ratios are at their historical trends, GDP in Ireland must be below its own trend by the same amount as output. GNP, on the other hand, is not necessarily equal to GDP because bond repayments are made by Ireland to the Rest of World. In other words,

$$GNP_{it} = GDP_{it} + r_{bt} B_{it},$$

for $t = 1, \ldots, t^* - 1$, where $B_{i1} = 0$, $B_{i2} > 0$, and $B_{it} < 0$ for $t = 3, \ldots, t^* - 1$. The pattern of Irish debt can be determined from net exports in the transition period, since

$$NX_{it} = B_{it+1} - (1 + r_{bt}) B_{it}$$

and since net exports are equal to GDP less domestic consumption and investment—and all three of these variables are constant relative to their historical trends between $t = 2$ and $t = t^* - 1$. Thus, net exports must also be constant relative to its historical trend.

In $t = t^*$, GDP falls further below its historical trend than output has fallen because investment of foreign multinationals in both tangible and plant-specific intangible capital rises above zero. GDP is lower because of the rise in plant-specific intangible investment.

The arguments made for the small country can be made for the large country, but the direction of change is reversed for the periods $t = 1$ to $t = t^*$. ∎

Figures 3 and 4 show the transition paths of consumption, labor, output, capital, GDP, and GNP in the case that income adjustments are not made to the Rest of World budget constraints. These are the analogues of Figures 3 and 4 in the main text, which displayed the results assuming a small adjustment in ϵ_t was made to keep the interest rate constant in transition.

1.6.2. With constraints on portfolio flows

If portfolio flows are restricted over the same period as FDI flows, then the interest rate is no longer (approximately) constant prior to financial liberalization. In this case, deriving specific analytic solutions for all of the paths of variables of interest is not easy. Instead, I consider whether variables are above or below trend and show graphically a comparison of the economies with and without portfolio restrictions.

Proposition 3. The small country's output and labor with full capital account restrictions are below their historical trend between $t = 1$ and $t = t^*$. The reverse is true for the large country.

Proof. If, in $t = 1$, consumption in Ireland rises relative to its historical trend, $c_{i1} > c_{i0}$, then the intratemporal first-order condition of households, namely,

$$\frac{y_{it}}{l_{it}} = \frac{y_{it}^d}{l_{it}^d} \propto \frac{c_{it}}{1 - l_{it}}, \quad t = 1, \ldots, t^*,$$

implies that labor and output fall initially. With capital fixed, labor falls more than output.

With no borrowing or lending allowed across countries, total investment $y_{i1} - c_{i1}$ must be below trend. With returns equated across assets, investment in all three types of assets—tangible capital, plant-specific intangible capital, and technology capital—must be below trend.

In period $t = 2$, output and labor must fall further because domestic capital stocks are lower between the first and second periods when investment in $t = 1$ is below trend. Households cannot borrow from abroad; thus, output, investment, and labor continue to fall until $t = t^*$, and net exports remain equal to zero until the restrictions on FDI are relaxed.

Again, the same arguments can be made for the large country, but because the global resource constraint must hold, the paths relative to trend for the large country must be reversed. Because the small country is the recipient of future FDI while the large country is its source, the initial consumption in the small country must be above its historical trend, and the initial consumption in the large country must be below. Otherwise, the global resource constraint would be violated. $\blacksquare$

Figures 5 through 8 compare the transitions with and without portfolio restrictions. Not surprisingly, the transitions are more gradual with portfolio restrictions, since the small country cannot immediately take advantage of the higher effective TFP that is coming in the future.

1.7. Computation of a general I-country version

Now, I describe the algorithm used to compute equilibrium paths in a general I-country version of the model over T periods.

Let me start with some notation. Let $\mathcal{P}$ be a n_p-dimensional vector of prices and transfers, where $n_p = T - 1 + 2TI$ and the vector includes $T - 1$ interest rates, TI wage rates, and TI transfers. Let $\mathcal{Q}_i$ be a n_q-dimensional vector of quantities for country i, where $n_q = 4T + 2TI$ and the vector includes T country-i consumptions, T country-i aggregate labor supplies, T country-i next period debt holdings, T investments in technology capital made by companies from country i, TI investments in tangible capital made by companies from country i at home and abroad in all other countries, and TI investments in plant-specific tangible capital made by companies from country i at home and abroad in all other countries. Let $\mathcal{Q}$ be a $n_q \times I$ matrix with column i given by $\mathcal{Q}_i$.

Next, consider the steps of the algorithm to compute equilibrium $\mathcal{P}^*$ and $\mathcal{Q}^*$ on a parallel computer. That is, find $\mathcal{P}^*$ and $\mathcal{Q}^*$ such that

$$
\begin{aligned}
0 &= F\left(\mathcal{P}, \mathcal{Q}\right) \\
0 &= G_i\left(\mathcal{P}, \mathcal{Q}_i\right), \quad i = 1, \ldots, I,
\end{aligned}
$$

where F is a n_p-dimensional function and G_i is a n_q-dimensional function.

The first-order conditions stacked up in F are $T - 1$ global resource conditions in (1.3.17), TI market clearing conditions for labor in (1.3.19), and TI conditions relating transfers to tax revenues in (1.3.20).

The first-order conditions stacked up in G_i are the budget constraint (1.3.10), the household intratemporal first-order condition that combines (1.3.15) and (1.3.6), the relation between the interest rate and marginal rates of substitution that combines (1.3.14) and (1.3.16), and all of the dynamic Euler equations that combine (1.3.7) through (1.3.13). Conditions (1.3.1)–(1.3.5) are used to construct dividends, capital stocks, and outputs.

1. On the master node (node 0), read in all inputs and initialize parameter vectors, exogenous time paths, and initial guesses for the vector of prices $\mathcal{P}$ and vector of quantities $\mathcal{Q}$. When first starting, use steady-state values, which are read in. (See below for details on computing the steady states with actual cross-country data.)

2. Broadcast (MPI_BCAST) parameters, initial conditions for capital and debt, and exogenous time paths (for tax rates, technology levels, populations, degrees of openness, and borrowing constraints) to all processors.

3. The outer loop of the code iterates on the equilibrium price vector. The current price vector and an initial guess for the quantity vectors $\mathcal{Q}_i$ are inputs (along with all parameters) to each country i subroutine. (There are two kinds of countries: those with technology capital at a corner and those with positive levels of technology capital. For those at a corner, only a subset of first-order conditions needs to be solved.) Each country is assigned to a processor. Depending on the capacity of the machine, there might be only one country per processor or more than one.

4. The inner loops of the code—which are run inside each of the country i subroutines—iterate on quantity vectors (for a given vector of prices). In other words, $\mathcal{Q}_i^*$ is found (as a function of the current price vector) that solves the set of first-order conditions $G_i(\mathcal{P}, \mathcal{Q}_i) = 0$ given $\mathcal{P}$. The results are passed back to the outer loop along with derivatives of F with respect to all variables.

5. Back in the outer loop, prices are updated via a Newton-Raphson algorithm. If there is convergence, the results are written out. Otherwise, new prices and quantities are broadcast to the processors and the algorithm continues at step 4 above.

6. The results written out by the code are analogues of BEA variables derived above.

If T and I are large, it is best to move gradually from the steady state (with no change in N_{it} or σ_{it} over time) to the parameterization of interest. Also, if analytic derivatives are used in finding the fixed points for $\mathcal{P}$ and $\mathcal{Q}$ that solve the first-order conditions, then the codes run considerably faster.

Chapter 2.

Data Sources

For the 104-country benchmark model, I use the following series that are available from the World Development Indicators:

- GDP in current U.S. dollars (NY.GDP.MKTP.CD)
- GDP in constant 2000 U.S. dollars (NY.GDP.MKTP.KD)
- Total population (SP.POP.TOTL)
- Foreign direct investment, net inflows, in current U.S. dollars (BX.KLT.DINV.CD.WD)
- Portfolio investment, excluding liabilities constituting foreign authorities' reserves, in current U.S. dollars (BN.KLT.PTXL.CD)

The code setupdat.m loads in these raw data and constructs the relevant time series for the analysis.

When I restrict myself to countries with balance of payments data available through the International Monetary Fund (IMF), I use variable X4555 for the inward foreign direct investment and the sum of variables X4652 and X4602 for the net portfolio investment.

Data sources for employment shares used in Figures 6–8 in the main text are the OECD and BEA, specifically,

- OECD.Stat Inward activity of multinationals
- OECD Factbook 2010
- FDIUS Establishment Data for 2002, Table A1.9

- USDIA 2004 Final Benchmark Data, Table I.H3

The codes plotccemp.m, plotusempi.m, and plotusempo.m load in these data and plot predicted and actual employment shares shown in Figures 6–8 in the main text. The figures in the main text use manufacturing data, which are readily available for many countries and relevant given that most FDI over the sample was done by firms in manufacturing industries.

Chapter 3.

Parameterizing the model

Values for A_{i0} and σ_{i0} are set so that the initial values for real per capita GDP and the ratio of inward FDI to GDP are the same in the model and the data. Specifically, the Matlab code compsteady.m reads in the data (generated by the code setupdat.m discussed in the previous section) and finds $\{A_{i0}, \sigma_{i0}\}$ for all i such that the initial conditions for the model per capita GDP and FDI to GDP ratios are the same. Values for N_{i0} are taken from actual data.

Time paths are set as follows. Technology levels are held on trend, $A_{it} = A_{i0}$. Populations are set equal to values in the WDI data discussed earlier. Values for the paths of openness, σ_{it}, are set so that the trends in ratios of inward FDI to GDP are consistent in the model and the data.

Fixed parameters are taken from McGrattan and Prescott (2010) who compare the United States and rest of world. For tax series, I use the averages of the sample paths in their study. In the next section, I do sensitivity analysis to determine if these choices have an impact on the results.

Chapter 4.

Sensitivity Analyses

In this chapter, I report the results of various sensitivity analyses. The first set of experiments considers the impact on the transition to FDI openness as I vary model parameters, including the assumption of perfect-foresight expectations. In the second set of experiments, I vary time paths of tax rates that were fixed in the benchmark parameterization. Finally, I consider alternative sources for employment shares and balance of payments data.

4.1. Alternative parameters

In this section, I describe how the transition path for the two-country version of the model changes as I vary key parameters. Here, I'll consider the impact on GDP of the timing of opening, relative country sizes, the share of technology capital, the degree of openness, and expectations about future openness.

Figure 9 shows how the path changes as I vary four of the key parameters. The dark line in all graphs is the benchmark shown in Figure 4 for the small country. The other lines are results as I vary the date after which FDI is allowed t^*, the relative population sizes N_r/N_i, the share of technology capital ϕ, and the degree of openness after σ^*.

The first set of results in Figure 9 shows that the timing of opening matters little for the magnitude of the decline in GDP unless the enactment of the policy is immediate. However, even in that case, there must still be a decline given the large increase in intangible investments that takes place.

The second set of results in Figure 9 shows that the initial declines and eventual increases in GDP are amplified with the relative populations. This result is not surprising given that the impact of FDI depends on how large the technology capital is abroad. The larger N_r/N_i, the larger the gap.

The third set of results in Figure 9 shows the impact of increasing the share of technology capital. The smaller this share ϕ, the smaller the incentive for FDI and the smaller the impact of FDI openness.

The final set of results in Figure 9 show the impact of increasing the parameter governing how much FDI is allowed in. This parameter, which has a one-to-one impact on country TFP, amplifies the movements—both up and down—in GDP.

Figure 9 and earlier results shown in Figure 8 comparing the transitions with and without portfolio restrictions also provide an answer to the following question: How do the results change if the future path for σ_{it} is not known with certainty? For example, suppose the Irish did not know until t^* that FDI would be allowed in during the following period. In this case, the path of GDP would be on the historical trend before t^* and would then follow the same path as the first curve in panel A of Figure 9. In other words, at t^* there would be a decline due to increased intangible investment, followed by an increase when capital markets opened. The transition would look similar to a situation in which the Irish faced tight constraints on their portfolio investments. There would be little movement in GDP relative to its historical trend until enactment of the policy change.

4.2. Time-varying tax rates

The benchmark simulation has time variation in only two exogenous series: population and the degree of openness. Other exogenous variables such as tax rates were assumed to be constant over the sample. In this section, I show that this choice does not affect the main results.

Specifically, I rerun the benchmark simulation with a time-varying path for each one of the three tax rates. Starting with the tax on dividends, I assume that all countries start in 1980 with a rate of 28 percent, which linearly falls to 0 by 2025. Thus, in 2005, they are at 12 percent and expected to continue falling. If I recompute the annual growth rates relative to the United States between 1980 and 2005, I find that the results are almost exactly the same as before. (See Figure 9 and Table 4 in the main text.) The correlation between this annual growth rate and the initial level of per capita GDP relative to the United States is 0.05, which is the same as in the benchmark simulation. And the regression of growth on intial GDP and the ratio of FDI to GDP produces the same regression results with, in particular, a coefficient on FDI to GDP of 0.046.

If instead of the tax rate on dividends, I assume that the tax on labor and consumption (i.e., the labor wedge) falls linearly from 34 percent to 0 over the period 1980–2025, then I again find similar results to the benchmark. The correlation between growth and initial GDP is 0.055, and the coefficient on FDI to GDP in the growth regression is 0.045.

Finally, I rerun the experiment with time-varying tax rates on profits. Reducing the tax rates on profits for all countries from 37 percent to 0 over the period 1980–2025 implies a correlation of growth and initial GDP of 0.056 and a coefficient in the growth regression of 0.03 percent. Thus, in this case, the impact of FDI on GDP appears to the econometrician to be even smaller than in the benchmark simulation. But in both cases, the economic significance is small.

4.3. Employment share data

As an external check, I compared the model's predictions of employment shares with actual data. I used data for manufacturing industries, which are readily available in many countries and most relevant for FDI over the period 1980–2005. However, for inward FDI in the United States, I have comparable data for both manufacturing and all industries.

The countries with available data cover 88 percent of employment in foreign-owned manufacturing establishments in the United States and 75 percent of employment in all foreign-owned establishments in the United States. Figure 10 shows the predicted and actual shares for manufacturing. (This is the same as Figure 7 in the main text and is shown here for convenience.) Figure 11 shows the predicted and actual shares for all industries. The correlations between the predicted and actual shares are high in both cases.

4.4. IMF balance of payments data

The benchmark simulation is based on a 104-country model parameterized with data from the *World Development Indicators*. In this section, I report results for a 50-country version of the model parameterized with data from the IMF *Balance of Payments*.

Figure 12 is the analogue of Figure 9 in the main text, except that I use the 50-country sample and IMF data. The main difference in country coverage is in terms of countries with low initial levels of per capita GDP relative to the United States. However, the main result is the same: the data generated by the model show no obvious relationship between capital restrictions and economic performance.

Chapter 5.

Summary of Welfare and Growth Gains

In Table 1, I report welfare and growth gains from the conterfactual experiments. These are also shown graphically in Figures 10 and 11 in the main text.

In some cases, binding nonnegativity constraints on investment made it difficult (if not impossible) to run the counterfactuals. Thus, not all countries in the sample are represented in Table 1. But, the countries included span a wide range of sizes relative to the United States. And, as the figures in the main text demonstrate, these gains are inversely related to size.

Appendix Figures

Figure 1. Interest Rate in the Two Country Model, With and Without the Adjustment to Large-Country Income

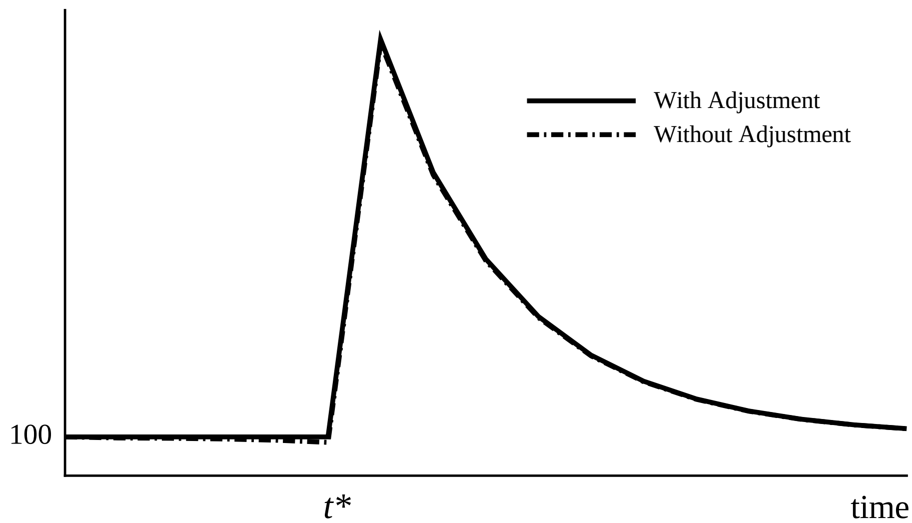

Figure 2. Adjustment to Large Country Income Required to Ensure a Constant Interest Rate Before Economies Open

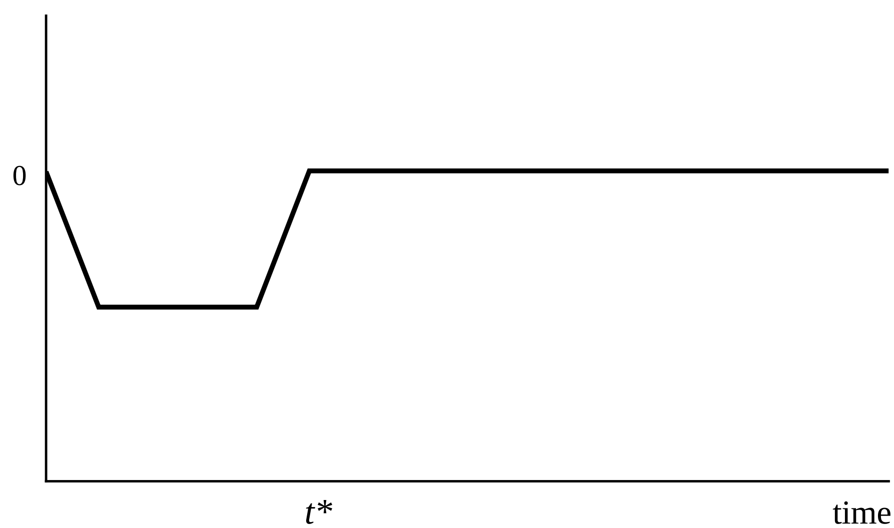

Figure 3. Detrended Consumption, Output, and Labor Over Time
in the Two-Country Model Without Portfolio Restrictions
(Initial Steady State = 100 and ε_t=0 for all *t*)

A. Small Country

B. Large Country

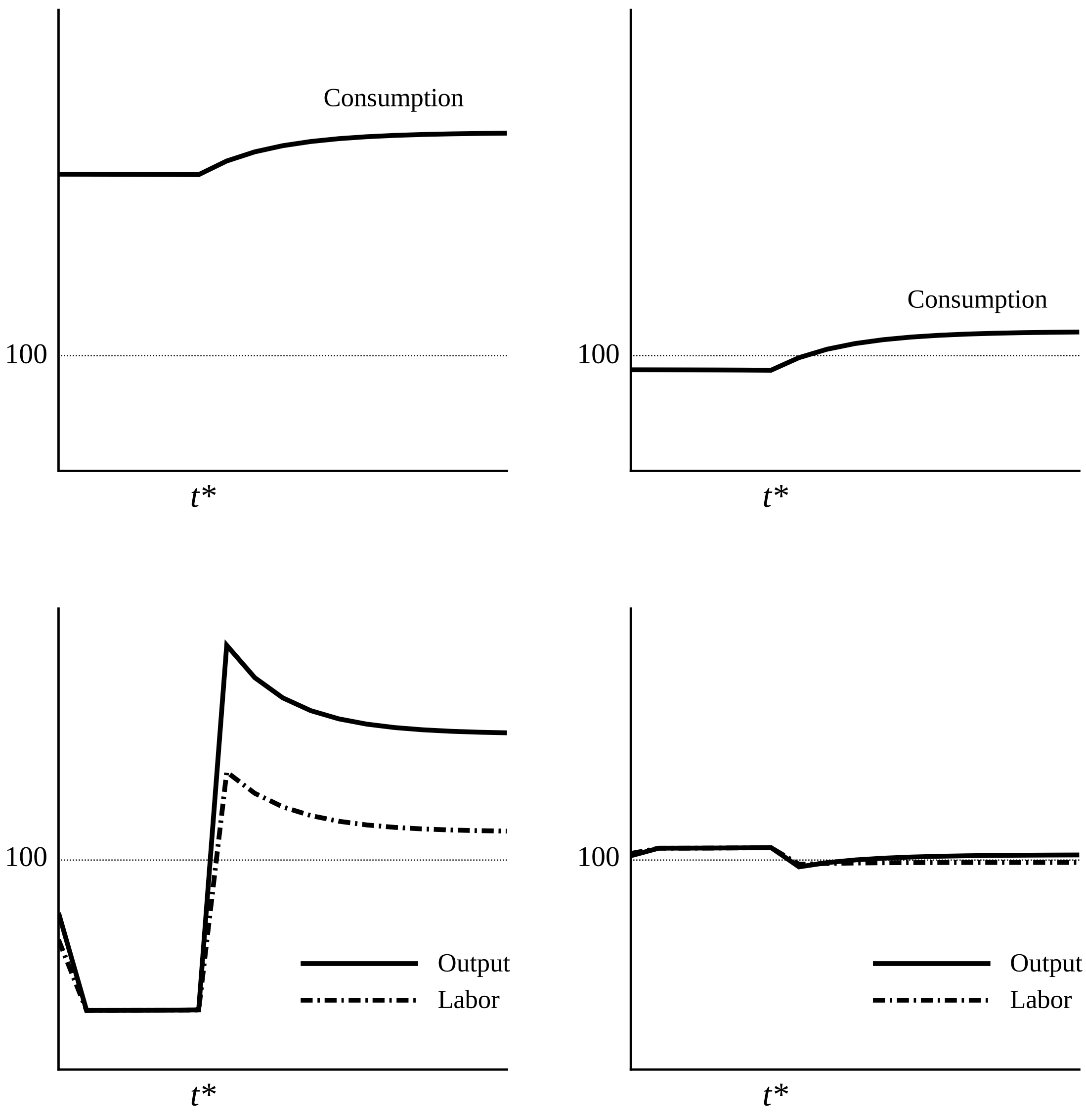

Figure 4. Detrended Capital Stocks, GDP, and GNP Over Time
in the Two-Country Model Without Portfolio Restrictions
(Initial Steady State = 100 and ε_t=0 for all t)

A. Small Country

B. Large Country

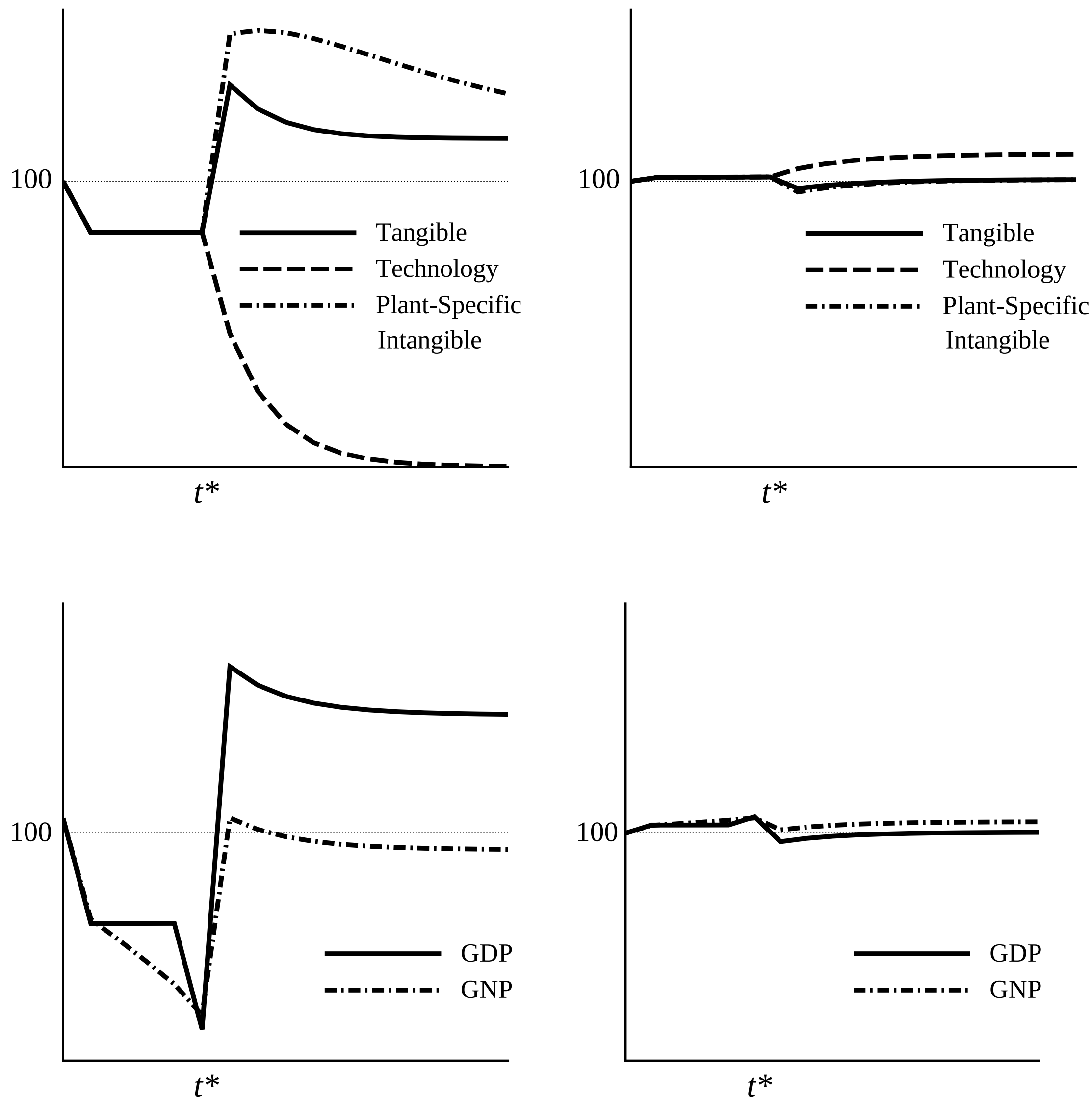

Figure 5. A Comparison of Detrended Consumption and Labor With and Without Portfolio Restrictions in a Two-Country Model (Initial Steady State = 100)

A. Small Country

B. Large Country

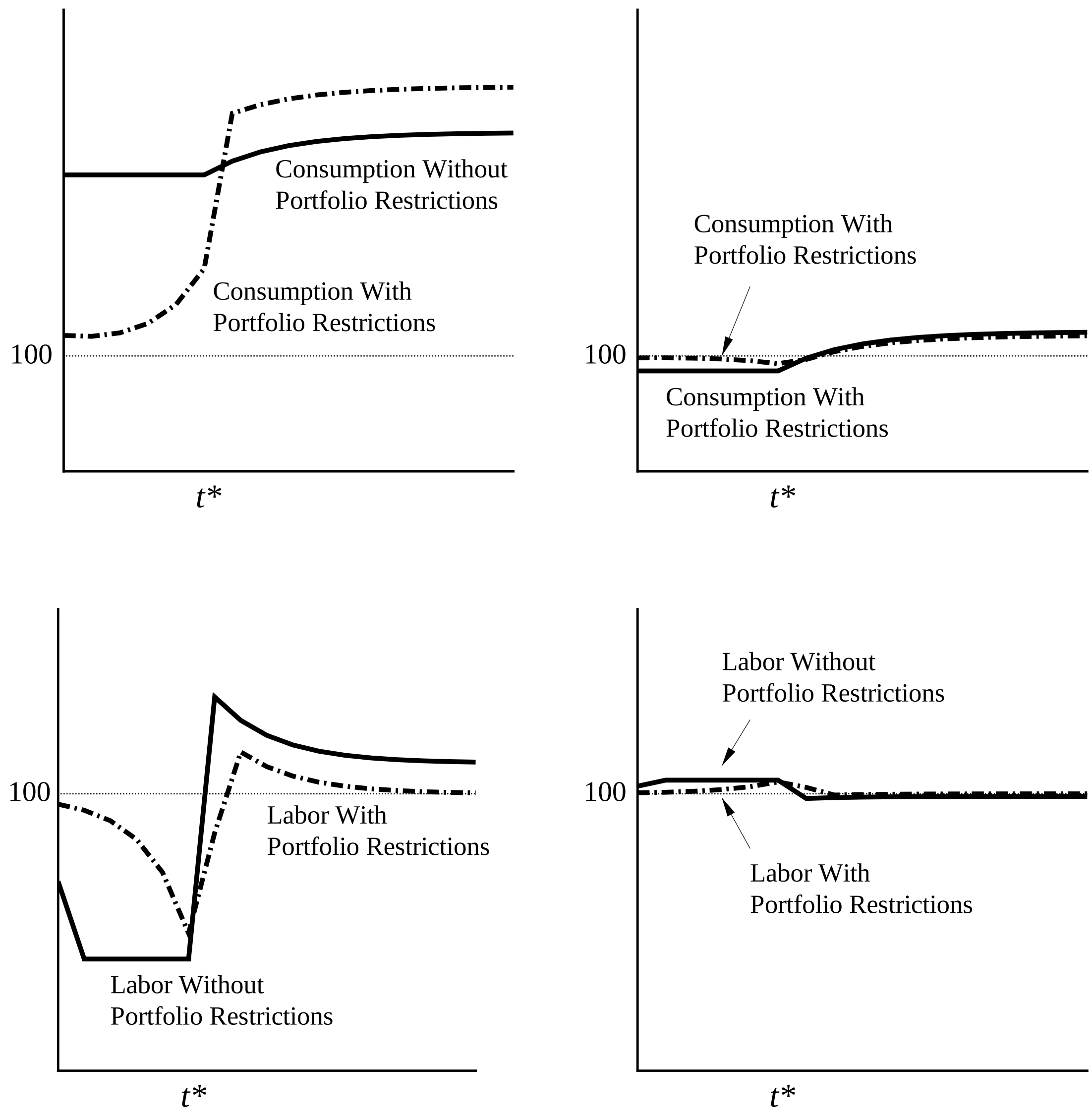

Figure 6. A Comparison of Detrended Output and Tangible Capital With and Without Portfolio Restrictions in a Two-Country Model
(Initial Steady State = 100)

A. Small Country B. Large Country

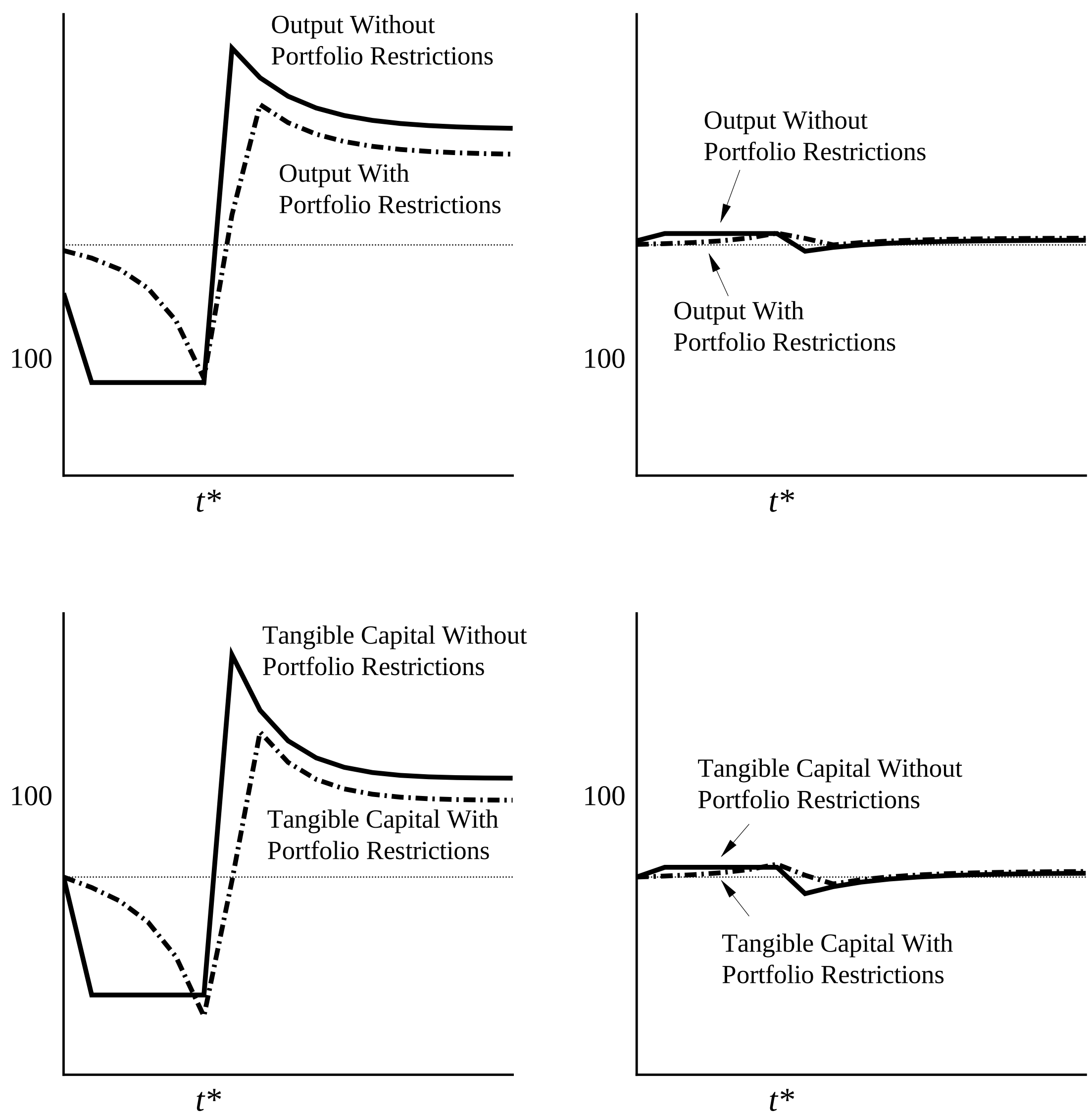

Figure 7. A Comparison of Detrended Intangible Capital Stocks With and Without Portfolio Restrictions in a Two-Country Model (Initial Steady State = 100)

A. Small Country B. Large Country

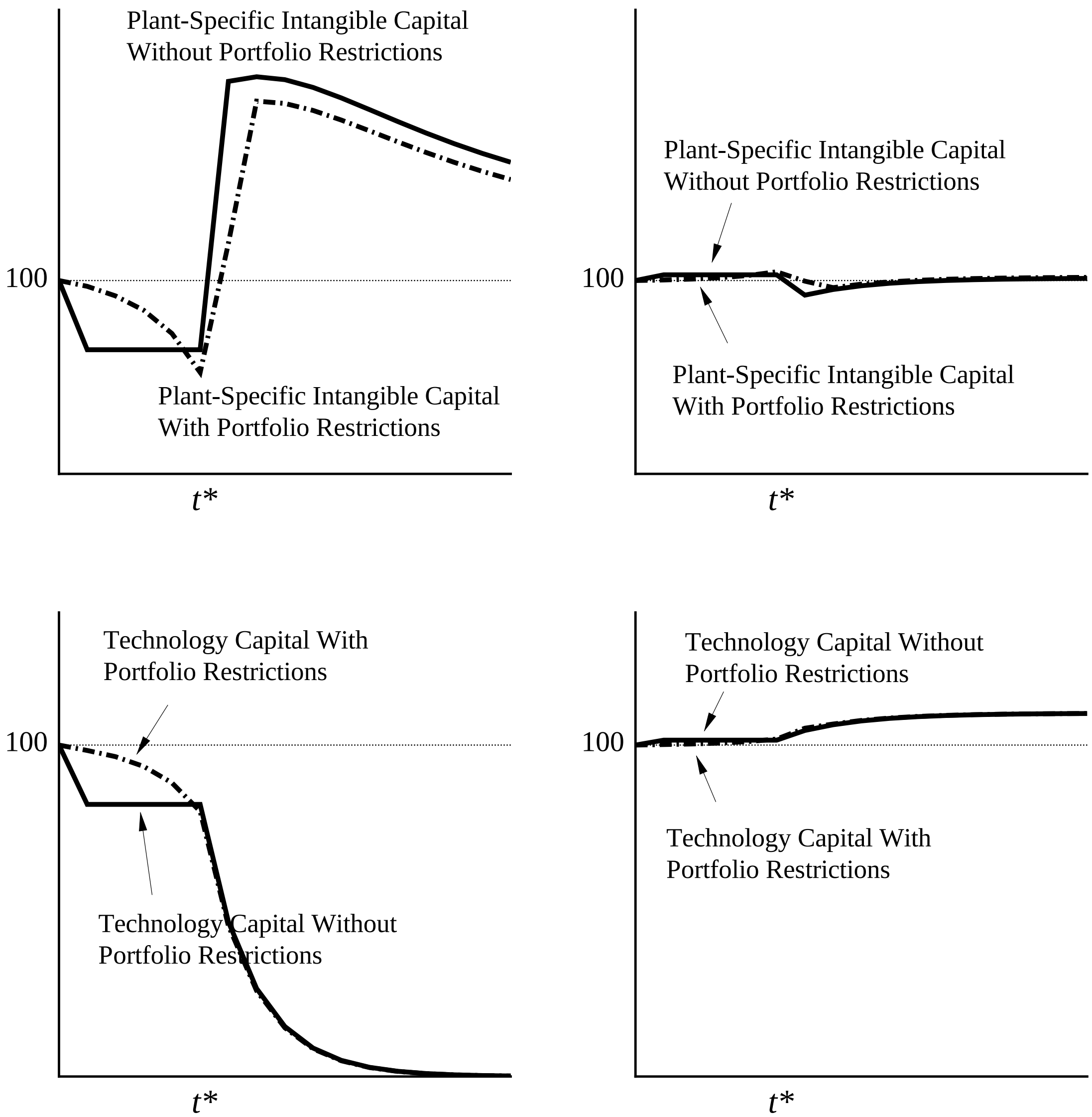

Figure 8. A Comparison of Detrended GDP and GNP With and Without Portfolio Restrictions in a Two-Country Model (Initial Steady State = 100)

A. Small Country

B. Large Country

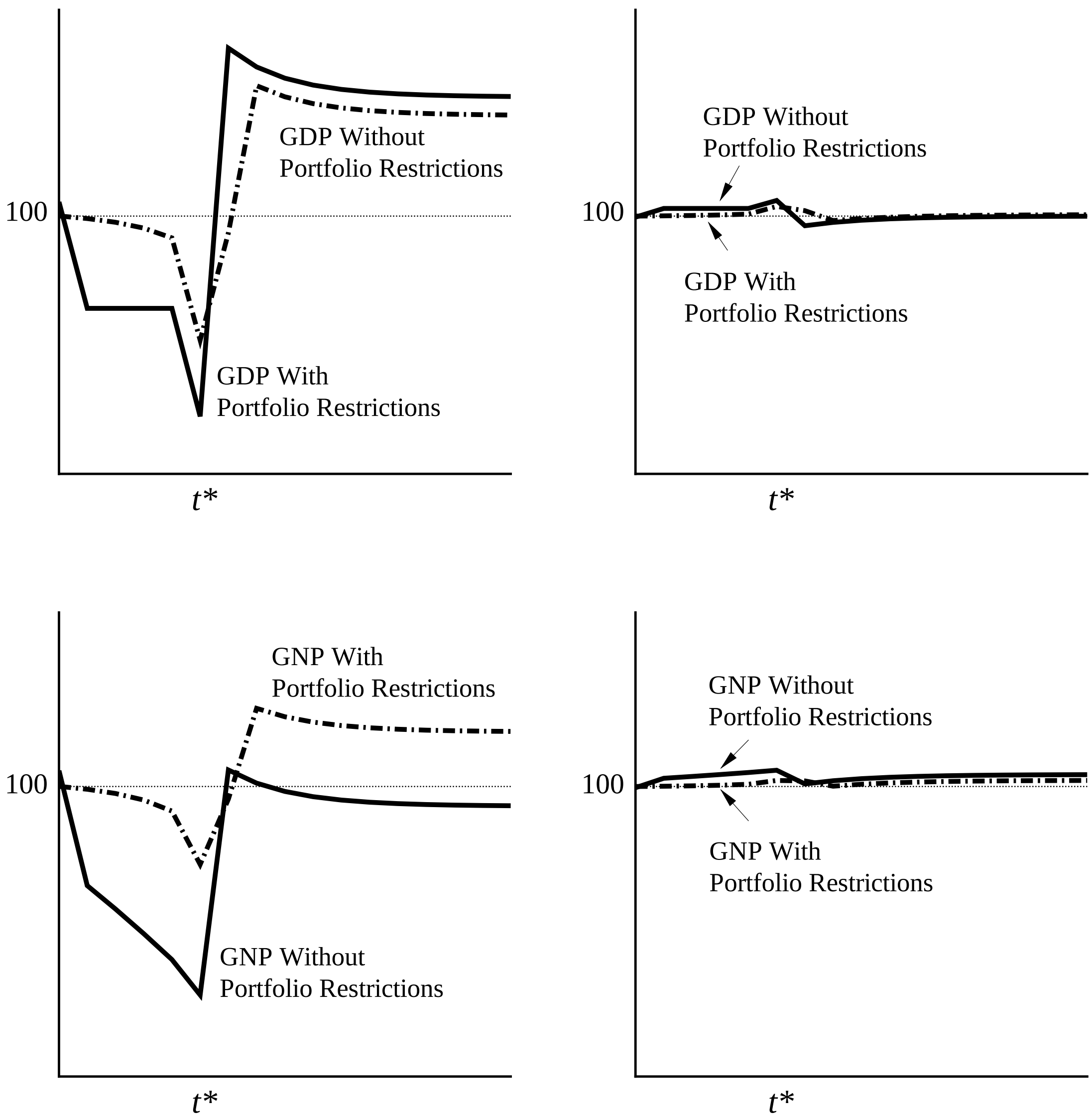

Figure 9. Detrended GDP Over Time in the Two-Country Model Without Portfolio Restrictions and Varying Model Parameters (Initial Steady State = 100)

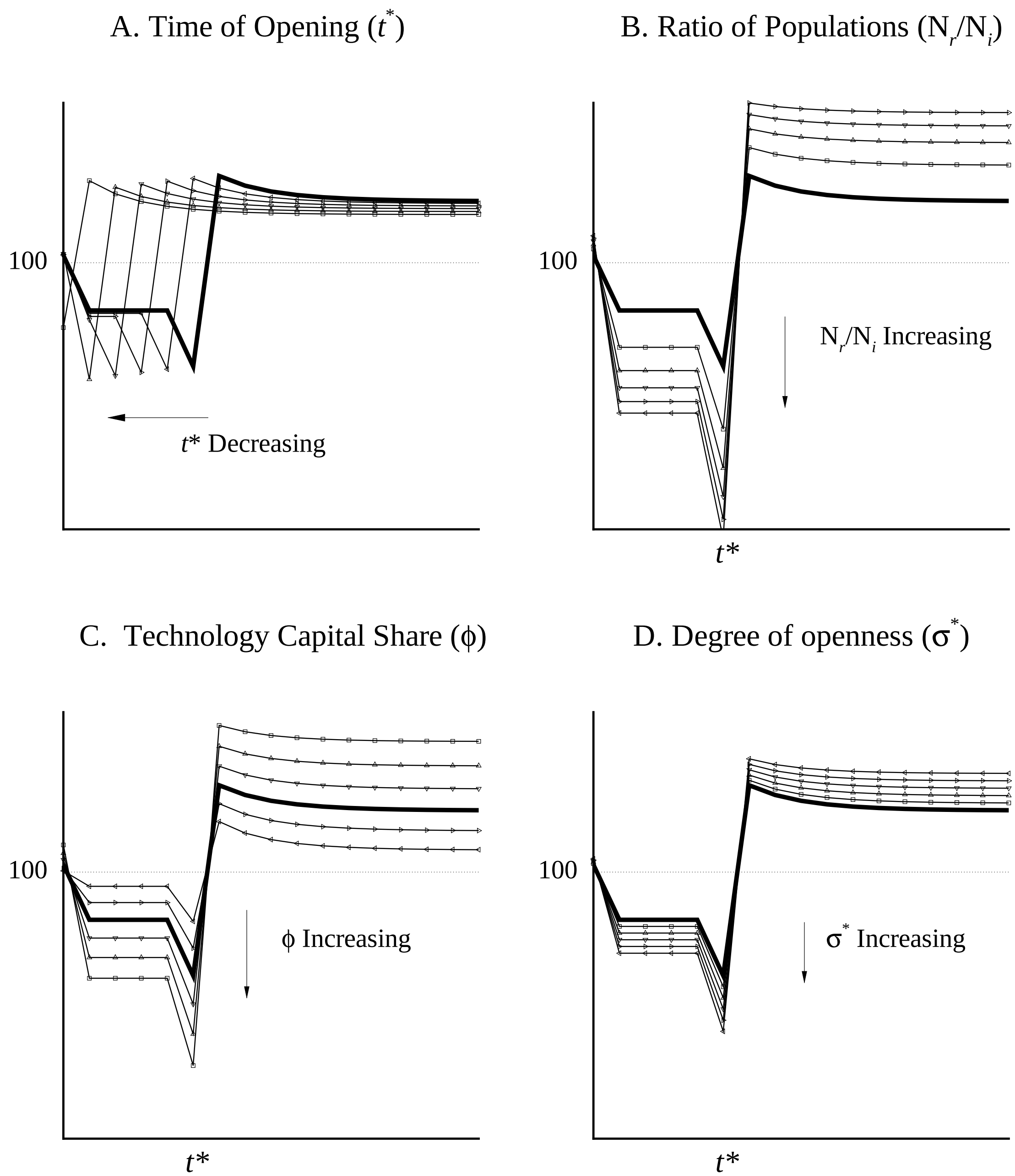

Figure 10. Predicted vs. Actual Share of Total U.S. Employment in Foreign Controlled Establishments by Country

(Source: FDIUS Establishment Data, Manufacturing, Bureau of Economic Analysis, 2002)

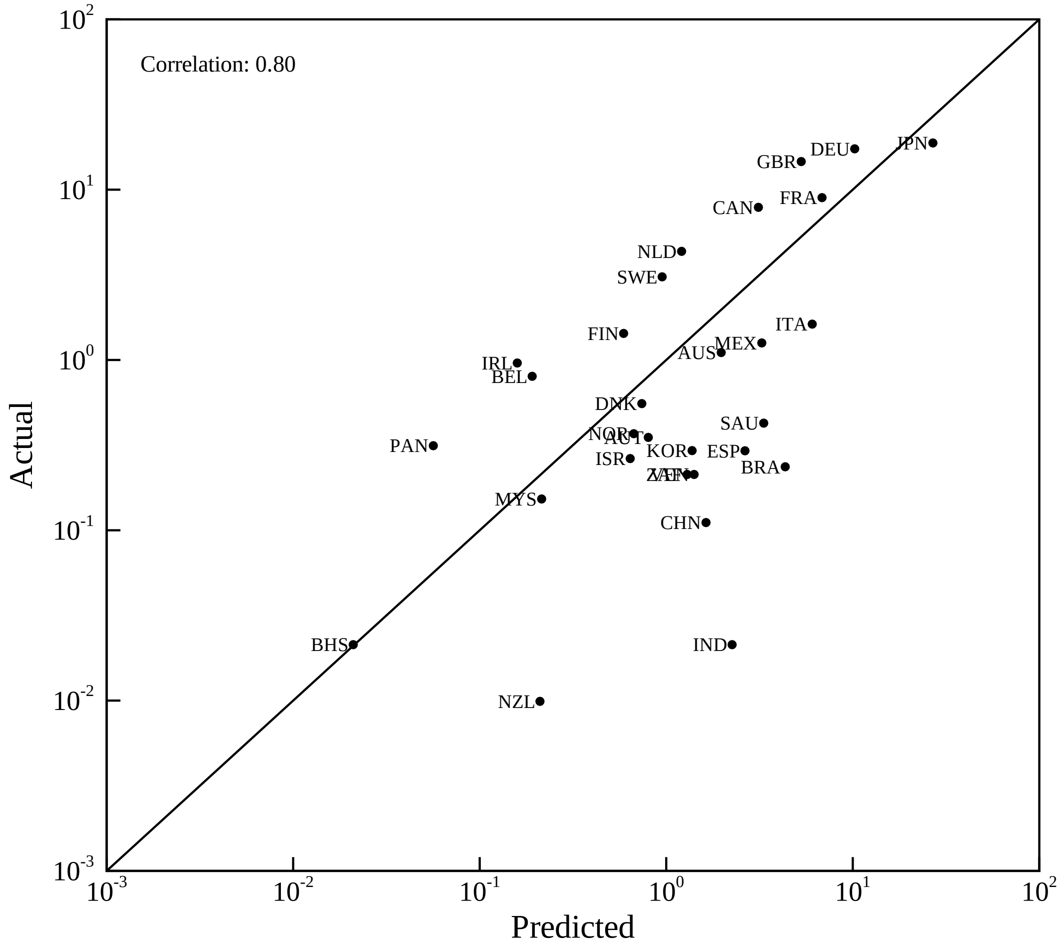

Figure 11. Predicted vs. Actual Share of Total U.S. Employment in Foreign Controlled Establishments by Country

(Source: FDIUS Establishment Data, All Industries, Bureau of Economic Analysis, 2002)

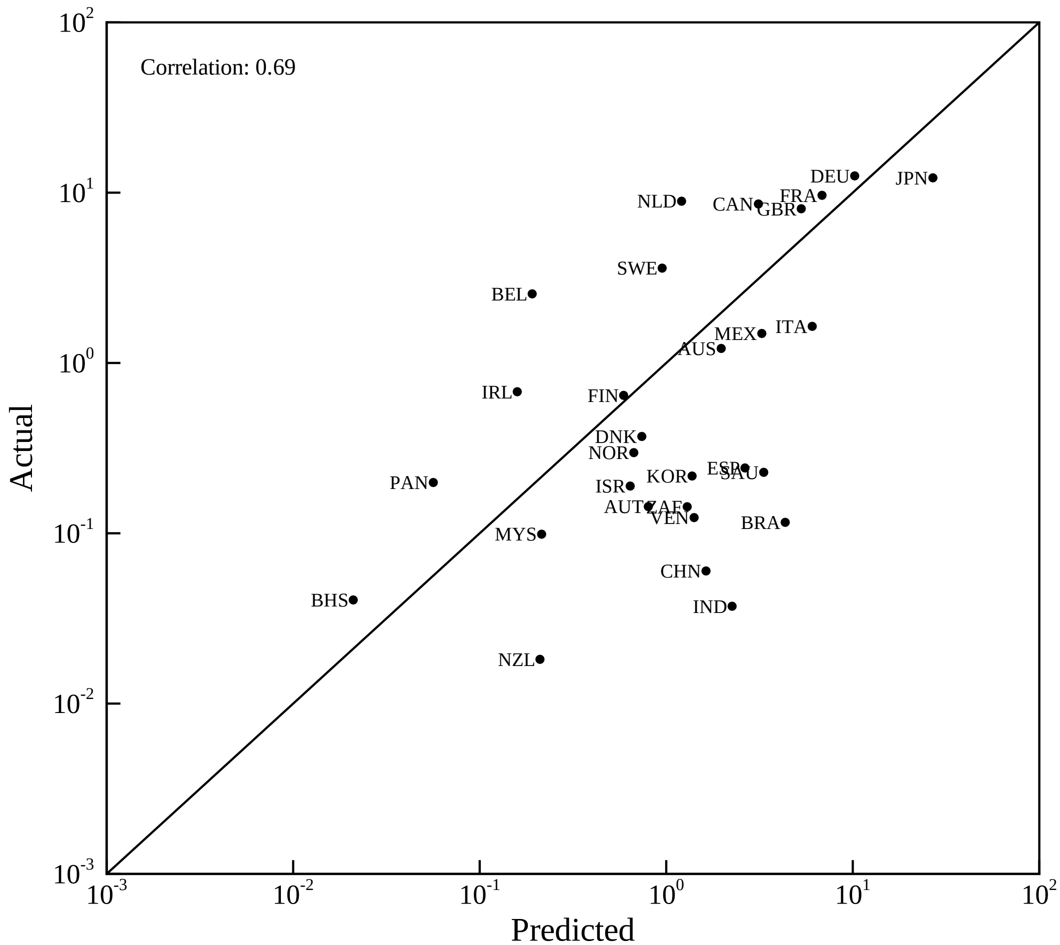

Figure 12. 1980 Real GDP and Predicted Growth Rates, 50 Country Sample

(Countries with average FDI/GDP > 2% labeled)

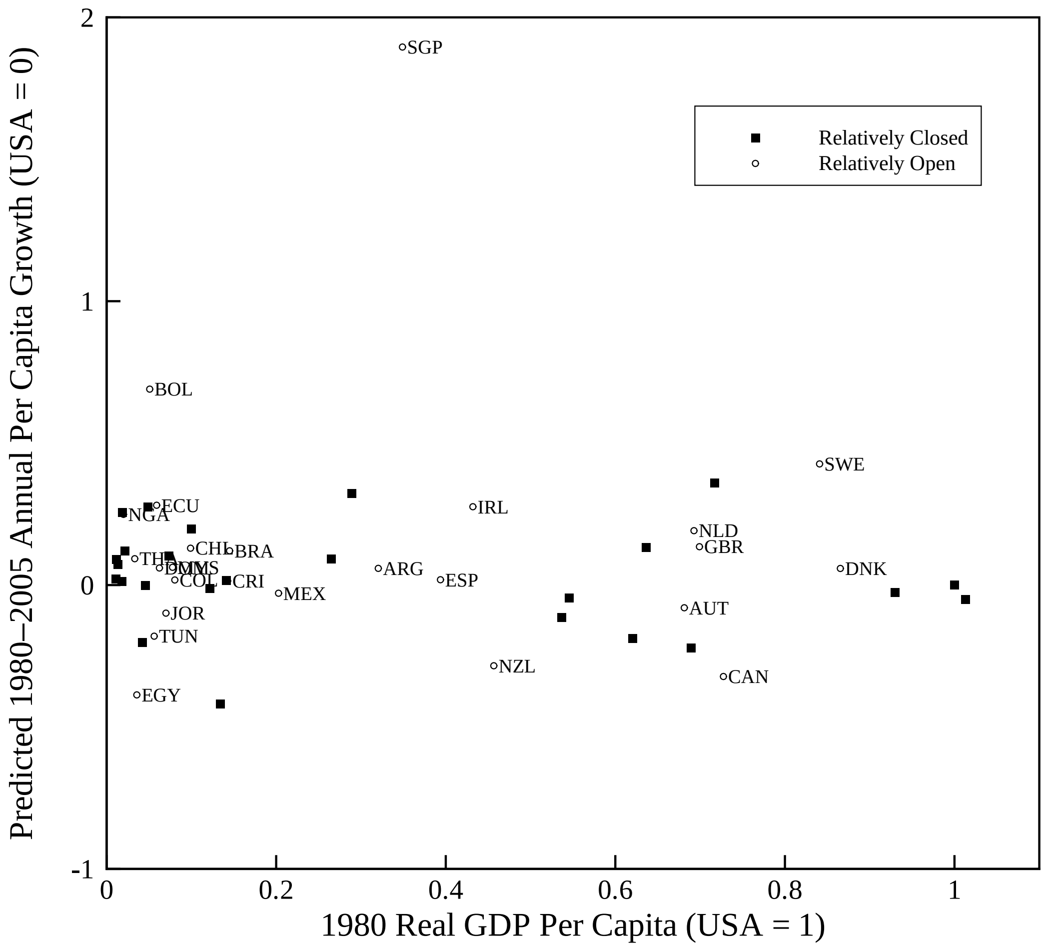

Table 1. Growth and Welfare Gains[a]

Country	Relative Size	Welfare Gains	Growth Gains
United States (USA)	100	4	0.0
Japan (JPN)	52	9	0.0
Germany (DEU)	24	8	1.3
United Kingdom (GBR)	18	15	1.3
France (FRA)	18	13	1.5
Italy (ITA)	15	14	1.6
Brazil (BRA)	11	24	1.4
Canada (CAN)	10	25	1.4
Mexico (MEX)	8.8	26	1.5
Spain (ESP)	7.8	25	1.7
Saudi Arabia (SAU)	6.4	35	1.3
Korea (KOR)	3.5	35	2.0
Venezuela (VEN)	3.4	39	1.6
Australia (AUS)	5.6	32	1.9
Argentina (ARG)	5.6	30	1.7
Netherlands (NLD)	5.5	28	1.5
India (IND)	4.9	29	0.3
Sweden (SWE)	4.0	28	1.7
Turkey (TUR)	3.8	35	1.7
Belgium-Luxembourg (BEL)	3.8	18	1.6
Austria (AUT)	3.0	34	2.4
Denmark (DNK)	2.6	37	2.2
Norway (NOR)	2.3	44	2.3
Finland (FIN)	1.9	41	2.1
Indonesia (IDN)	1.7	50	2.3
Philippines (PHL)	1.7	51	2.2
Colombia (COL)	1.7	49	2.2
Portugal (PRT)	1.6	45	2.4
Peru (PER)	1.4	53	2.2
Algeria (DZA)	1.3	56	2.4
Nigeria (NGA)	1.3	57	2.3
Egypt (EGY)	1.3	59	2.5
Thailand (THA)	1.2	53	2.4
New Zealand (NZL)	1.0	55	2.7
Pakistan (PAK)	1.0	60	2.4
Ireland (IRL)	1.0	56	2.4
Malaysia (MYS)	.94	64	2.4
Chile (CHL)	.87	59	2.5
Morocco (MAR)	.75	60	2.6
Singapore (SGP)	.71	16	0.4
Guatemala (GTM)	.47	77	2.6

[a] Footnotes appear at the end of the table.

Table 1. Growth and Welfare Gains[a] (cont.)

Country	Relative Size	Welfare Gains	Growth Gains
Cote d'Ivoire (CIV)	.47	79	2.6
Ecuador (ECU)	.44	74	2.6
Dominican Republic (DOM)	.40	76	2.9
El Salvador (SLV)	.38	73	3.0
Kenya (KEN)	.34	86	2.8
Costa Rica (CRI)	.33	81	3.1
Tunisia (TUN)	.33	80	3.1
Bolivia (BOL)	.27	83	2.4
Cameroon (CMR)	.29	85	3.7
Sri Lanka (LKA)	.23	83	3.2
Gabon (GAB)	.20	95	3.0
Jordan (JOR)	.20	93	3.5
Honduras (HND)	.20	92	3.3
Haiti (HTI)	.19	93	3.1
Iceland (ISL)	.19	82	3.5
Zambia (ZMB)	.15	97	3.5
Senegal (SEN)	.15	98	3.1
Ghana (GHA)	.14	98	3.3
Cyprus (CYP)	.12	98	3.0
Mozambique (MOZ)	.11	98	3.2
Bahamas (BHS)	.11	102	3.8
Guinea (GIN)	.08	105	1.8
Mali (MLI)	.08	108	3.2
Congo (COG)	.08	108	3.1
Liberia (LBR)	.07	71	2.3
Burkina Faso (BFA)	.06	117	3.4
Benin (BEN)	.06	120	3.3
Malawi (MWI)	.06	121	3.5
Botswana (BWA)	.05	173	4.3
Chad (TCD)	.05	115	3.5
Fiji (FJI)	.05	115	4.0
Togo (TGO)	.05	394	6.9
Central Af. Rep. (CAF)	.05	127	4.1
Mauritania (MRT)	.04	211	6.0
Burundi (BDI)	.03	126	2.9
Swaziland (SWZ)	.03	274	5.7
Gambia (GMB)	.02	287	6.9
Seychelles (SYC)	.01	133	4.5
Solomon Is. (SLB)	.01	458	7.3
Vanuatu (VUT)	.01	173	4.1
St. Vincent (VCT)	.01	211	6.2

[a] Countries are ordered by size relative to the United States. Those not shown encountered numerical problems when computing the counterfactuals.

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