

# Supplementary Appendix for “Rational Attention Allocation over the Business Cycle”: Model Simulation

Not for publication

In this supplementary appendix, we use a numerical example to illustrate the model’s predictions for the same measures of attention, portfolio dispersion, and performance as the ones we measure in the data. The goal of this exercise is to confirm that the model makes the same qualitative predictions for these observables as for the slightly different measures of attention allocation, portfolio dispersion, and fund performance for which we formally proved our propositions. Notably, we do not attempt to quantitatively account for all time-series and cross-sectional moments of actively managed fund portfolio holdings and returns. Such a task would be beyond the scope of this paper and indeed beyond the current state of the literature. Our model is too stylized along many dimensions to deliver on such a task. For example, it has only three assets and no heterogeneity in risk aversion, prior beliefs, or initial wealth among funds, and no heterogeneity in information capacity among skilled managers. Adding such features could improve the predictions, but only at the cost of obscuring the main mechanisms operating in the model.

## S.1 Parameter Choices

The following explains how we choose the parameters of our model. The simplicity of the model prevents a full calibration. Instead, we pursue a numerical example that matches some salient properties of stock return data. Our benchmark parameter choices are listed in Table S.2. Section S.3 below shows that the qualitative results are robust to a wide range of parameter choices.

Our procedure is to simulate 3000 draws of the shocks  $(x_1, x_2, x_c, s_1, s_2, a)$  in recessions and 3000 draws of the shocks in expansions. Since our model is static, each simulation is best interpreted as different draws of a random variable, and not as a period (months). The model’s recessions differ from expansions in *two respects*.

First, the variance of the aggregate payoff shock  $\sigma_a$  is higher. It is set to replicate the fact the market return volatility is about 25% higher in recessions than in expansions. In the numerical example, the volatility of the market return is 4.0% in expansions and 5.0% in recessions, straddling the observed market return volatility of 4.5%. Setting the variance of the asset supply vector  $\sigma_x = .05^2 \bar{x}$  allows us to match this level of market return volatility.

Second, recessions are also characterized by lower *realized* stock market returns (despite high expected returns). In order to generate lower realized market returns and higher expected returns in a static model, we have to assume that agents are surprised by unexpectedly low returns in recessions. We accomplish this in the numerical example by replicating the bottom  $m = 2.5\%$  of market return realizations among the 3000 simulations of the model in recessions, in effect simulating the economy in recessions for  $3000 * (1 + .025) > 3000$  draws. This choice for  $m$  is conservative because the 0.03% difference between market returns in expansions (0.87% per month) and recessions (0.84% per month) it generates is lower than the 0.20% per month difference in the data. In the robustness section below, we consider a case that generates a 0.20% return difference. The results are qualitatively and quantitatively similar.

To get the average market return right, we choose the mean of asset payoffs  $\mu$  (equal for all assets) and the coefficient of absolute risk aversion,  $\rho$ , to achieve an average equilibrium market return of about 0.85% per month.

We think of assets 1 and 2 as two large industries and the composite asset as summarizing all other industries. Therefore, we normalize the mean asset supply of assets 1 and 2 to 1, and set the supply of the composite asset,  $\bar{x}_c$ , to 7. The variance of the firm-specific shocks is chosen to match the fact that individual industry returns are about 30% more volatile than the market return over our sample from 1980 to 2005. We use data from the 30 industry portfolios of Fama and French (1997). In the example, the average volatility of assets 1 and 2 is 6.5% in recessions and 5.8% in expansions, 29% and 45% higher than that of the market return. This choice matches the proportion of the average industry’s return variance that is idiosyncratic. We choose the asset loadings on the aggregate payoff shock,  $b_1$  and  $b_2$ , to be different from each other so as to generate some spread in asset betas. The chosen values generate average market betas of 0.9 and a dispersion in betas of 33%. This is reasonably close to the average beta of 0.95 and the dispersion of 23% for the 30-industry portfolios.

We set the average risk-free rate equal to 0.22% per month, the average of the 1-month yield minus inflation in our sample. We set initial wealth,  $W_0$ , to generate average holdings in the risk-free asset around 0%.

For simplicity, we set capacity  $K$  for skilled investment managers equal to 1. This implies that learning can increase the precision of one of the idiosyncratic shocks (or the aggregate shock) by 25% (by 18%). We will vary  $K$  in our robustness exercise below. Likewise, we have no strong prior on the fraction of skilled funds,  $\chi$ . In our benchmark, we set it equal to 20%, and we will vary it for robustness. The model is simulated for 800 investors, of which 175 are skilled (20%). We assume that 20% of all investors are non-investment managers (“other investors”). The unskilled managers (60% of the populations) and other investors differ in name only. We note that the parameter conditions in Propositions 2 through 4 are satisfied by these parameter choices.

As in our empirical work on mutual funds in Section 2, we compute all statistics of interest as equally-weighted averages across all investment managers (i.e., without the 20% other investors). We also report results separately for skilled and unskilled investment managers.

## S.2 Main Simulation Results

Every skilled manager ( $K > 0$ ) solves for the choice of signal precisions  $K_{aj} \geq 0$  and  $K_{1j} \geq 0$  that maximize time-1 expected utility (9). We assume that these choice variables lie on a  $25 \times 25$  grid in  $\mathbb{R}_+^2$ . The signal precision choice  $K_{2j} \geq 0$  is implied by the capacity constraint (6).

We simulate a sequence of  $T = 3000$  draws (months) for the random variables in each of the recession and expansion states, as explained above. We form a  $T \times 1$  time series for the three individual asset returns, for the market return, for each fund’s return, and for each fund’s (and the market’s) portfolio weights in each asset. For each asset  $i$ , we then estimate a CAPM regression of the asset’s excess return on the market excess return. This delivers the asset’s CAPM beta,  $\beta_i$ ; one value in expansions and one in recessions. We define the systematic component of returns as  $\beta_i R_t^m$ , for  $t = 1, \dots, T$  and  $i = 1, 2, 3$ . Stacking the different  $i$ s and  $t$ s results in a  $3T \times 1$  vector of systematic returns. Similarly, we define the idiosyncratic return as

$$R_t^i - \beta_i R_t^m.$$

To compute *Ftiming* in equation (10) for fund  $j$ , we stack its portfolio weights in deviation from the market's weights for the three assets and the  $T$  draws into a  $3T \times 1$  vector. We also create a  $3T \times 1$  vector of aggregate shocks by stacking three identical repetitions of each aggregate shock realization  $a$ . We calculate *Ftiming* as the covariance between these two variables. Likewise, we form *Timing* as in Kacperczyk, Van Nieuwerburgh, and Veldkamp (2011) as the covariance between the time series of portfolio weights, in deviation from the market's weights, and the systematic component of returns. The procedure delivers one *Ftiming* and one *Timing* measure per fund in recessions and one set of measures in expansions. We multiply *Ftiming* by 1000 and *Timing* by 10,000 because the aggregate shocks are an order of magnitude larger than the systematic returns.

Table S.3 summarizes the predictions of the model for the main statistics of interest. The left panel shows the results for recessions, while the right panel shows the results for expansions. In each panel, we present three columns. Column *skilled* reports the equally weighted average of the statistic in question for the group of skilled investors (20% of investors have  $K > 0$  in our benchmark parametrization). Column *unskilled* is the equally weighted average across the unskilled funds (60% of investors are unskilled investment managers). Column *all* is the equally weighted average across all funds (80% of investors). The 20% unskilled other investors are excluded from the table because we do not observe them in the data. However, the model's predictions for this group are identical to those for the unskilled funds. These two groups differ in name only.

Rows 1 and 2 of Table S.3 show that *Ftiming* and *Timing* are higher for skilled investors in recessions (left panel) than in expansions (right panel). Because of market clearing, unskilled investors are the flip side of the skilled ones, their *Ftiming* and *Timing* measures are negative. Since no investors learn about the aggregate shock in expansions, *Ftiming* and *Timing* are essentially zero for both skilled and unskilled. The net effect of the skilled and unskilled is listed in Column *all*. This combination of all investment managers, skilled and unskilled, is what we have data on. Hence, the first testable implication of the model is that *Ftiming* and *Timing* should be higher for all funds in recessions than in expansions.

In a similar fashion, we construct *Fpicking* measure, defined in equation (11), and the stock-picking measure *Picking*, defined in Kacperczyk, Van Nieuwerburgh, and Veldkamp (2011). That is, we stack the stock-specific shocks,  $s_i$ , the idiosyncratic returns,  $R_t^i - \beta_i R_t^m$ , and the fund's portfolio weights into  $3T \times 1$  vectors and compute the respective covariances. Rows 3 and 4 summarize the predictions of the model for *Fpicking* (multiplied by 1000) and *Picking* (multiplied by 10,000). Across all funds (skilled and unskilled), the model predicts lower *Fpicking* and *Picking* in recessions. Skilled funds have a high *Fpicking* and *Picking* ability in expansions, when they allocate their attention to stock-specific information. Unskilled investors exhibit a negative *Picking* in expansions for the same reason that they have a negative *Timing* in recessions: Price fluctuations induce them to buy when returns are low and sell when returns are high. The *Fpicking* and *Picking* measures are close to zero for all investors in recessions. Hence, the second testable implication of the model is that *Fpicking* and *Picking* should be lower for all funds in recessions than in expansions.

Next, we turn to the measures of portfolio and return dispersion. Row 5 of Table S.3 shows the results for the *Concentration* measure, defined in equation (13), in our numerical example. We calculate *Concentration* <sup>$j$</sup>  for fund  $j$  by stacking all squared deviations of fund  $j$ 's portfolio from the market portfolio

lio into a  $3T \times 1$  vector, and by summing over its entries, and dividing by  $T$ . We obtain one number for recessions and one for expansions. We find that *Concentration* is higher for all funds in recessions than in expansions. This increase is driven entirely by the informed; the uninformed are all holding the exact same portfolio because of common prior beliefs.

More concentrated portfolios are also less diversified. For each fund  $j$ , we estimate CAPM regression (14) by regressing the fund's excess return on the market's excess return. This delivers the fund's  $\alpha^j$ ,  $\beta^j$ , and  $\sigma_\varepsilon^j$ . We use the idiosyncratic risk  $\sigma_\varepsilon^j$  as our second measure of portfolio dispersion. If all funds held the market portfolio, their idiosyncratic risk would be zero, and there would be zero cross-sectional dispersion. In simulation, the skilled funds take on more idiosyncratic risk than the unskilled ones, and more in recessions than in expansions. As a result, idiosyncratic risk is higher in recessions than in expansions for all funds.

Rows 7 through 9 report the results for the dispersion across funds' abnormal returns, CAPM alphas, and CAPM betas. All three metrics show increasing dispersion in recessions, driven largely by the heterogeneity in the choices of the skilled investors.

Finally, we study performance measures. Rows 10 and 11 of Table S.3 show that skilled investment managers have large excess returns, as measured by abnormal fund returns or fund alphas ( $R^j - R^m$  and  $\alpha^j$ ), at the expense of the uninformed. The average investment manager has a slightly higher alpha in recessions than in expansions. While quantitatively modest (4.6bp per month or 55bp per year), the positive difference in average alphas between recessions and expansions is a robust finding of the model.

The numerical results also reveal that the regression residual variance  $(\sigma_\varepsilon^j)^2$  is higher in recessions. This effect arises because a fund that gets different signal draws (information) in each period holds a portfolio with a beta that varies over time. The CAPM equation (14) estimates an unconditional beta instead. The difference between the true, conditional beta and the estimated, constant beta shows up in the regression residual. Since recessions are times when funds learn more new information each period about the aggregate shock, these are times when true fund betas fluctuate more and the regression residuals are more volatile.

## S.3 Robustness of Simulation Results

This section discusses the robustness of the model to alternative parameter choices. We conduct several experiments in which we vary one key parameter at a time, while holding all other parameters fixed at their benchmark levels. Table S.1 summarizes these robustness checks. For brevity, we only report the results averaged over all investment managers and omit the results broken out for skilled and unskilled managers, separately. We find that none of the comparative statics are sensitive to variation in the key parameters of the model.

**Varying the fraction of skilled managers** In our benchmark model, we assume that 20% ( $\chi = .20$ ) of investors are skilled mutual funds (60% are unskilled mutual funds and 20% unskilled other investors). We first study two different values for the fraction of skilled investment managers:  $\chi = 10\%$  and  $\chi = 30\%$ . When there are fewer skilled funds, they have a comparatively larger advantage over the unskilled. This results in investment choices that exploit their informational advantage more aggressively. *Timing* for the skilled increases from 156 to 210 in recessions while their *Picking* reading in expansions increases from 160 in the baseline to 181. At the same time, there are fewer skilled investors exploiting

more unskilled investors than in the baseline, so that the unskilled investors have less negative average *Timing* values in recessions and less negative *Picking* values in expansions. As a result, the *Timing* value in recessions and *Picking* value in expansions, averaged across across *all* investment managers (80% of the investor population), fall relative to the benchmark (from 9.9 to 5.8). Similarly, *Ftiming* increases in recessions for all funds and *Fpicking* decreases, but the changes are smaller than in the benchmark case. Likewise, our measures of portfolio dispersion continue to be higher in recessions than in expansions, but all dispersion levels are somewhat lower than before. The reason is that there is no dispersion among the unskilled, and there are more of them than in the benchmark. Finally, the performance results remain intact as well. The skilled investors make higher abnormal returns and alphas than in the benchmark, which means the unskilled loose more in total. However, they loose less per unskilled investor. As a result, alphas averaged across all funds are lower than in the benchmark: 18.6bp per month in recessions (versus 35.3bp) and 15.6bp in expansions (versus 30.7bp).

The opposite effects occur when we increase the fraction of skilled investors to 30 percent. The increase in *Ftiming* and *Timing* and the decrease in *Fpicking* and *Picking* in recessions are larger than those in the benchmark model. The same is true for portfolio dispersion and performance. For example, the average alpha is now 50.6bp per month in recessions and 45.4bp in expansions; the difference is slightly higher than in the benchmark. In expansions, all skilled investors continue to learn about the stock-specific information. In recessions, about 70% of attention is allocated to the aggregate shock in recessions and 15% to each of the stock-specific shocks. This 70% is lower than the 87% of skilled managers who learn about the aggregate shock in recessions in our benchmark parametrization. This is a general equilibrium effect, which we label *strategic substitutability*. When many informed investors learn about the aggregate shock, and buy assets that load heavily on that shock, they push up the price of these assets, making it less desirable to learn about for other informed investors *ceteris paribus*. This leads some to learn about the stock-specific shocks instead. Hence, the higher average *Fpicking* of the informed in recessions compared to the benchmark. Why is the reverse not happening in expansions? Because the volatility of the aggregate shock is low enough in expansions that it turns out not to be optimal for any of the 30% informed investors to deviate from the full attention allocation to the idiosyncratic shocks.

**Varying capacity  $K$**  The second variational experiment is to decrease and increase the amount of attention allocation capacity  $K$  that skilled investors have. In our benchmark,  $K = 1$ , which amounts to the ability to increase the precision on any one signal by 25% of the prior precision of the stock-specific information through learning. We now consider  $K = .5$  and  $K = 2$ . When the 20% of skilled have twice as much capacity, their *Ftiming* and *Timing* increase substantially in recessions (*Timing* goes up from 157 in the benchmark to 220), and their *Fpicking* and *Picking* increase in expansions (*Picking* goes up from 160 in the benchmark to 311). In contrast to the previous exercise, the *Timing* measure for the unskilled becomes more negative in recessions and their *Picking* more negative in expansions than in the benchmark. The reason is that there are as many unskilled as in the benchmark, but they are now at a larger informational disadvantage. The net effect of the skilled and the unskilled is an increase in *Timing* in recessions from 9.9 in the benchmark to 14.0. Likewise, *Picking* in expansions increases from 10.0 to 19.5. Giving 30% of investors  $K = 1$  has similar effects as giving 20% of investors  $K = 2$ . Portfolio dispersion increases substantially with higher  $K$ . The result is driven by the more concentrated portfolios of the skilled, which creates both more

dispersion among the skilled and a bigger difference with the unskilled. The skilled investors make abnormal returns and alphas that are about twice as high as those in the benchmark, and the unskilled lose about twice as much. The net effect are average fund alphas that are substantially higher than in the benchmark: 67.2bp per month in recessions (versus 35.3bp) and 59.3bp in expansions (versus 30.7bp). The opposite happens when we lower  $K$  to 0.5.

**Recessions are times with low returns.** We recall that recessions in the model are periods with not only a higher variance of the aggregate shock, but also with lower realized market returns. We implement the latter by first simulating the model in recessions for 3000 periods, then taking the bottom  $m\%$  of return realizations, and adding them to the 3000 draws when calculating the moments of interest. In our third robustness check we verify how robust our results are to different values for  $m$ . We explore  $m = 0$  and  $m = 0.08$ , while our benchmark is  $m = 0.025$ . When  $m = .08$ , realized market returns are 22 basis points per month lower in recessions than in expansions (0.54 versus 0.76% per month). This corresponds to the return difference in the data. The results for *Timing*, *Picking*, *Ftiming*, and *Fpicking* are slightly stronger, but the magnitudes are quite close to the benchmark. The same is true for all dispersion measures, except for the beta dispersion. The latter is quite a bit lower in recessions than in the benchmark (3.91 instead of 6.37), driven by a reduction in the beta dispersion of the skilled. Because of the lower returns in recessions, skilled managers have both lower betas and less differences in their betas compared to the unskilled in recessions. Finally, the performance results are similar to the benchmark. Alphas are slightly higher than in the benchmark: 38.5bp per month in recessions (versus 35.3bp) and 31.6bp in expansions (versus 30.7bp). The difference between recessions and expansions grows to 7bp per month.

The case of  $m = 0$  corresponds to a world in which assets have realized payoffs that are symmetrically distributed around the same mean in expansions and in recessions. However, because recessions are times in which returns are more volatile, *expected* (and unconditional average) returns must be higher to compensate the investors for bearing higher risk. In particular, the average market return is 1.30% in recessions and 0.95% in expansions. The results on the fund moments are opposite from the case with higher  $m$ , but still quantitatively similar to our benchmark case. For example, the difference in average alphas between recessions and expansions is 4.1bp per month compared to 4.6bp in the benchmark.

Table S.1: Robustness of Predictions of the Model

Panel A reports moments related to attention allocation, Panel B reports the moments related to dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text. The other pairs of columns report results for the benchmark parameters and for six robustness exercises. In each pair of columns, the first column reports the predictions for the model simulated in a recession and the second column for the model simulated in an expansion. All moments are generated from a simulation of 3000 draws and 750 investors. For both recessions and expansions, we list the equally weighted average across all investment managers (the 20% skilled and the 60% investment managers). The parameters are the same as in the benchmark model, except for the parameter listed in the first row.

	baseline		$\chi = .10$		$\chi = .30$		$K = 0.5$		$K = 2$		$m = 0$		$m = .08$	
	R	E	R	E	R	E	R	E	R	E	R	E	R	E
<b>Panel A: Attention Allocation</b>														
1. Ftiming	10.55	0.07	6.11	0.04	12.96	-0.08	6.23	0.01	14.53	-0.08	9.30	-0.04	11.01	-0.02
2. Timing	9.91	0.06	5.77	-0.09	11.56	0.12	5.93	-0.05	13.96	0.13	9.34	0.09	10.93	0.06
3. Fpicking	2.15	15.61	0.01	7.76	7.23	23.21	-0.03	7.89	12.66	30.14	2.08	16.16	2.14	15.76
4. Picking	1.66	10.02	0.13	5.04	5.10	14.83	0.18	5.04	8.19	19.45	1.68	9.31	1.65	10.28
<b>Panel B: Dispersion</b>														
5. Concentration	3.75	3.12	1.99	1.59	5.27	4.62	1.78	1.50	7.71	6.74	3.74	3.12	3.77	3.14
6. Idiosyncratic volatility	5.09	4.33	2.90	2.27	6.66	6.07	3.34	2.80	7.38	6.71	4.83	4.09	5.29	4.33
7. Dispersion in abnormal return	3.54	3.37	1.93	1.82	4.94	4.71	2.21	2.13	5.97	5.51	3.42	3.21	3.74	3.44
8. Dispersion in CAPM alpha	2.52	2.28	1.52	1.35	3.06	2.84	1.32	1.15	4.85	4.46	2.22	2.15	2.76	2.35
9. Dispersion in CAPM beta	6.37	1.46	4.70	1.03	5.76	2.45	3.45	0.98	10.84	2.67	21.03	2.08	3.91	1.38
<b>Panel C: Performance</b>														
10. Abnormal return	0.346	0.302	0.178	0.150	0.500	0.450	0.184	0.151	0.664	0.589	0.330	0.283	0.379	0.312
11. CAPM Alpha	0.353	0.307	0.186	0.156	0.506	0.454	0.192	0.156	0.672	0.593	0.329	0.288	0.385	0.316

## S.4 Endogenous Capacity Model

Finally, we consider an extended model in which skilled managers can freely choose not only how to allocate their information processing capacity, but also how much capacity to acquire. We let the cost of acquiring  $K$  units of capacity be  $\mathcal{C}(K)$ . Each skilled fund solves for the choice of signal precisions  $K_{aj} \geq 0$  and  $K_{1j} \geq 0$ , and capacity  $K$  that maximize time-1 expected utility, as in (9) but adjusted for a penalty term  $-\mathcal{C}(K)$ . In our numerical work, we assume that these choice variables lie on a  $25 \times 25$  grid in  $\mathbb{R}_+^3$ . The choice of signal precision  $K_{2j} \geq 0$  is implied by the capacity constraint (6).

In our numerical exercise, we consider two different functional forms for  $\mathcal{C}(K)$ . The first one is  $\mathcal{C}_1(K) = c_1 \exp(K)$  and the second one is  $\mathcal{C}_2(K) = c_2 K^\psi$ . For ease of comparison with our exogenous  $K$  results, we choose the scalars  $c_1$  and  $c_2$  such that the optimal capacity choice is  $K = 1$  on average across expansions and recessions. This is the same capacity choice we assume in our benchmark parametrization. Clearly, increasing (lowering) the scalars  $c_1$  and  $c_2$  will lead to lower (higher) optimal capacity choice. These scalars can be interpreted as (shadow) prices of capacity. All other parameters are the same as in our benchmark model.

More interesting than the level of  $K$  that is chosen is how that choice differs between recessions and expansions. We find that for both cost functions, investors acquire more capacity in recessions than in expansions. Nothing in the cost function makes it cheaper to acquire capacity in either expansions or recessions. This result is solely driven by the fact that the higher (aggregate) uncertainty in recessions makes it optimal to acquire more capacity and to allocate it to the aggregate shock. This extensive-margin effect acts as an amplification to our intensive-margin effect. How elastic capacity choice is to changes in prior aggregate uncertainty, and hence how large the amplification effect is, *does* depend on the functional form of the cost function. For cost function 1, we find that capacity choice is 1.02 in recessions and 0.97 in expansions. For cost function 2, the elasticity is much higher, with a capacity choice of 1.15 in recessions and 0.92 in expansions. The reason for the higher elasticity is that the marginal cost function 2 is less steep in capacity. As a result, a given change in the marginal benefit of acquiring information leads to larger equilibrium changes in capacity. Since we have no strong prior over the functional form, we conduct our numerical simulation for both cost functions.

Table S.4 summarizes the main moments of interest for the endogenous  $K$  model, alongside the benchmark, exogenous  $K$  results. For brevity, we only report the results averaged over all investment managers and omit the results broken out for skilled and unskilled managers, separately. Overall, we find that the results are very similar to those in our exogenous  $K$  model, not only qualitatively, but also quantitatively. The moments for cost function 2 (two most right columns) tend to be higher in recessions than do the benchmark numbers, and lower in expansions. Hence, there is amplification of the difference between recessions and expansions. For example, average fund alphas are somewhat higher than in the benchmark in recessions (40.7bp per month versus 35.3bp) and somewhat lower in expansions (27.7bp versus 30.7bp). The resulting difference between recessions and expansions grows substantially from 4.6bp to 13bp per month. For cost function 1 (two middle columns), the moments are slightly higher in recessions since the skilled investment managers choose to acquire slightly more capacity than what they are endowed with in the benchmark ( $K = 1.02$  versus 1). The moments are slightly lower in expansions, since they have slightly lower capacity ( $K = 0.97$  versus 1). Overall, the difference in our key variables between recessions and expansions is usually very similar to that in our benchmark model.



Table S.2: **Numerical Example**

The first column lists the parameter in question, the second column is its symbol, the third column lists its numerical value, and the last column briefly summarizes how we chose that value.

<i>Parameter</i>	<i>Symbol</i>	<i>Value</i>	<i>How Chosen?</i>
<i>CARA</i>	$\rho$	0.525	<i>Asset return mean</i>
<i>mean of payoffs 1,2,c</i>	$\mu_1, \mu_2, \mu_c$	10, 10, 10	<i>Asset return mean</i>
<i>variance aggr. payoff comp. a</i>	$\sigma_a$	0.1225 (E), 0.2625 (R)	<i>Market return vol in expansions vs. recessions</i>
<i>variance idio. payoff comp. s<sub>i</sub></i>	$\sigma_i$	0.25	<i>Asset return vol vs. market return vol</i>
<i>a-sensitivity of payoffs</i>	$b_1, b_2$	0.25, 0.50	<i>Asset beta level + dispersion</i>
<i>mean asset supply 1,2</i>	$\bar{x}_1 = \bar{x}_2$	1,1	<i>Normalization</i>
<i>mean asset supply 1,2</i>	$\bar{x}_c$	7	<i>Asset return volatility</i>
<i>variance asset supply</i>	$\sigma_x$	$(.05 * \bar{x})^2$	<i>Asset return idio vol</i>
<i>risk-free rate</i>	$r$	0.0022	<i>Average T-bill return</i>
<i>initial wealth</i>	$W_0$	90	<i>Average cash position</i>
<i>difficulty learning aggr. info</i>	$\psi$	1	<i>Simplicity</i>
<i>information capacity</i>	$K$	1	
<i>skilled fraction</i>	$\chi$	0.20	

Table S.3: **Benchmark Simulation Results from the Model**

This table provides the main statistics for a simulation of the model under the benchmark parameter values summarized in Table S.2. Panel A reports moments related to attention allocation, Panel B reports the moments related to portfolio dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text, the next three columns report the predictions for the model simulated in a recession, the last three columns report the results for the model simulated in an expansion. All moments are generated from a simulation of 3,000 draws and 800 investors. For both recessions and expansions, we list the equally-weighted average across *all* investment managers (the 20% skilled and the 60% investment managers), and separately for the *skilled* and the *unskilled* investment managers.

	<b>Recessions</b>			<b>Expansions</b>		
	All managers	Skilled	Unskilled	All managers	Skilled	Unskilled
<b>Panel A: Attention Allocation</b>						
1. Ftiming	10.55	162.54	-40.11	0.07	0.94	-0.22
2. Timing	9.91	155.78	-38.72	0.06	3.31	-1.02
3. Fpicking	2.15	33.46	-8.28	15.61	249.72	-62.42
4. Picking	1.66	25.84	-6.40	10.02	160.11	-40.01
<b>Panel B: Dispersion</b>						
5. Concentration	3.75	13.15	0.00	3.12	11.48	0.00
6. Idiosyncratic volatility	5.09	15.21	1.72	4.33	13.50	1.28
7. Dispersion in abnormal return	3.54	10.05	1.36	3.37	9.82	1.22
8. Dispersion in CAPM alpha	2.52	5.05	1.68	2.28	4.55	1.52
9. Dispersion in CAPM beta	6.37	13.20	4.09	1.46	4.30	0.51
<b>Panel C: Performance</b>						
10. Abnormal return	0.346	5.471	-1.363	0.302	4.867	-1.220
11. CAPM Alpha	0.353	5.401	-1.330	0.307	4.861	-1.211

Table S.4: **Endogenous Capacity Model**

This table provides the results from an extension of the model where skilled funds endogenously choose how much capacity to acquire. It reports on the main predictions of the model. Panel A reports moments related to attention allocation, Panel B reports the moments related to dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text. The other pairs of columns report results for the benchmark parameters and for two versions of the endogenous  $K$  model with different cost functions. The cost function in the first one is  $\mathcal{C}_1(K) = c_1 \exp(K)$ , while the cost function in the second one is  $\mathcal{C}_2(K) = c_2 K^\psi$ . We set  $c_1 = 1.057$ ,  $c_2 = 2.4$ , and  $\psi = 1.2$ . All other parameters are the same as in the benchmark model. In each pair of columns, the first column reports the predictions for the model simulated in a recession (R) and the second column for the model simulated in an expansion (E). All moments are generated from a simulation of 2000 draws and 100 investors. For both recessions and expansions, we list the equally weighted average across all investment managers (the 20% skilled and the 60% investment managers).

	Baseline		$\mathcal{C}_1(K) = c_1 \exp(K)$		$\mathcal{C}_2(K) = c_2 K^\psi$	
	R	E	R	E	R	E
<b>Panel A: Attention Allocation</b>						
1. Ftiming	10.55	0.07	10.53	-0.12	11.58	-0.01
2. Timing	9.91	0.06	9.83	0.10	10.65	0.13
3. Fpicking	2.15	15.61	2.35	15.43	3.69	14.37
4. Picking	1.66	10.02	1.76	9.91	2.65	8.87
<b>Panel B: Dispersion</b>						
5. Concentration	3.75	3.12	3.82	3.13	4.37	2.85
6. Idiosyncratic volatility	5.09	4.33	5.20	4.53	5.34	4.15
7. Dispersion in abnormal return	3.54	3.37	3.59	3.34	3.97	3.16
8. Dispersion in CAPM alpha	2.52	2.28	2.57	2.29	2.92	2.06
9. Dispersion in CAPM beta	6.37	1.46	5.18	3.01	5.28	1.94
<b>Panel C: Performance</b>						
10. Abnormal return	0.346	0.302	0.348	0.302	0.399	0.271
11. CAPM Alpha	0.353	0.307	0.355	0.307	0.407	0.277