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APPENDICES: Technology Capital and the U.S. Current Account*

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ABSTRACT

Appendix A provides details for the computation of our model's equilibrium paths, the construction of model national and international accounts, and the sensitivity of our main findings to alternative parameterizations of the model. We demonstrate that the main finding of our paper—namely, that the mismeasurement of capital accounts for roughly 60 percent of the gap in FDI returns—is robust to alternative choices of income shares, depreciation rates, and tax rates, assuming the same procedure is followed in setting exogenous parameters governing the model's current account. Appendix B demonstrates that adding technology capital and locations to an otherwise standard two-country general equilibrium model has a large impact on the predicted behavior of labor productivity and net exports.

*The paper, data, and codes are available at our website http://www.minneapolisfed.org/research/sr/sr406.html. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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Appendix A.

Computation, BEA Accounts, and Sensitivity

A.1. Introduction

In this appendix, we provide details for computing equilibrium paths of our model economy and constructing model accounts comparable to the national and international accounts of the Bureau of Economic Analysis (BEA). We also conduct sensitivity analysis with the model. We demonstrate that the main finding of our paper—namely that the mismeasurement of capital accounts for roughly 60 percent of the gap in FDI returns—is robust to alternative choices of income shares, depreciation rates, and tax rates assuming the same procedure is followed in setting exogenous parameters governing the model's current account.

A.2. Computation of Equilibrium Paths

We let i index countries and j index multinational companies. Assume that $j \in J^i$ are incorporated in country i (where the J^i sets are mutually exclusive).[1]

A.2.1. Multinational problem

Multinational j solves

$$\max \sum_t p_t \left(1 - \tau_{dt}\right) D_t^j,$$

where

$$D_t^j = \sum_i \Big\{ \left(1 - \tau_{p,it}\right) \left(Y_{it}^j - W_{it} L_{it}^j - \delta_T K_{T,it}^j - X_{I,it}^j - \chi_i^j X_{M,t}^j\right) - \left(K_{T,i,t+1}^j - K_{T,it}^j\right) \Big\}$$

[1] Without loss of generality, we will work with a representative multinational where the index j denotes the country of incorporation.

and $\sum_j \chi_i^j = 1$,

$$
\begin{aligned}
K^j_{T,i,t+1} &= (1-\delta_T)\, K^j_{T,it} + X^j_{T,it} \\
K^j_{I,i,t+1} &= (1-\delta_I)\, K^j_{T,it} + X^j_{I,it} \\
M^j_{t+1} &= (1-\delta_M)\, M^j_t + X^j_{M,t}.
\end{aligned}
$$

Outputs are given by

$$
Y^j_{it} = F\left(N_{it}, M^j_t, Z^j_{it}; A^j_{it}\right) = A^j_{it}\left(N_{it} M^j_t\right)^{\phi}\left(Z^j_{it}\right)^{1-\phi}
$$

$$
Z^j_{it} = G\left(K^j_{T,it}, K^j_{I,it}, L^j_{it}\right) = \left(K^j_{T,it}\right)^{\alpha_T}\left(K^j_{I,it}\right)^{\alpha_I}\left(L^j_{it}\right)^{1-\alpha_T-\alpha_I},
$$

where F and G are the same for all i and j.

The first-order conditions for multinational j with respect to L, K_T, K_I, and M are

$$
\begin{aligned}
W_{it} &= F^j_{3,it} G^j_{3,it} \\
&= (1-\phi)(1-\alpha_T-\alpha_I)\ Y^j_{it}/L^j_{it} \\
\frac{(1-\tau_{dt})\, p_t}{(1-\tau_{d,t+1})\, p_{t+1}} &= 1 + (1-\tau_{p,i,t+1})\left(F^j_{3,i,t+1} G^j_{1,i,t+1} - \delta_T\right) \\
&= 1 + (1-\tau_{p,i,t+1})\left((1-\phi)\,\alpha_T Y^j_{i,t+1}/K^j_{T,i,t+1} - \delta_T\right) \\
&\equiv 1 + (1-\tau_{p,i,t+1})\left(r^j_{T,i,t+1} - \delta_T\right) \\
\frac{(1-\tau_{dt})\, p_t}{(1-\tau_{d,t+1})\, p_{t+1}} &= \frac{(1-\tau_{p,i,t+1})}{(1-\tau_{p,it})}\left(F^j_{3,i,t+1} G^j_{2,i,t+1} + 1 - \delta_I\right) \\
&= \frac{(1-\tau_{p,i,t+1})}{(1-\tau_{p,it})}\left((1-\phi)\,\alpha_I Y^j_{i,t+1}/K^j_{I,i,t+1} + 1 - \delta_I\right) \\
&\equiv \frac{(1-\tau_{p,i,t+1})}{(1-\tau_{p,it})}\left(r^j_{T,i,t+1} + 1 - \delta_I\right) \\
\frac{(1-\tau_{dt})\, p_t}{(1-\tau_{d,t+1})\, p_{t+1}} &= \frac{\sum_i (1-\tau_{p,i,t+1})\left(F^j_{2,i,t+1} + \chi^j_i\,(1-\delta_M)\right)}{\sum_i (1-\tau_{p,it})\,\chi^j_i}
\end{aligned}
$$

$$= \frac{\sum_i \left(1-\tau_{p,i,t+1}\right)\left(\phi Y^j_{i,t+1}/M^j_{t+1} + \chi^j_i \left(1-\delta_{\scriptscriptstyle M}\right)\right)}{\sum_i \left(1-\tau_{p,it}\right)\chi^j_i}$$

$$\equiv \frac{\sum_i \left(1-\tau_{p,i,t+1}\right)\left(r^j_{\scriptscriptstyle M,i,t+1} + \chi^j_i \left(1-\delta_{\scriptscriptstyle M}\right)\right)}{\sum_i \left(1-\tau_{p,it}\right)\chi^j_i}.$$

A.2.2. Household problem

Households choose sequences of consumption C_{it}, labor L_{it}, shares in companies from j S^j_{it}, and bonds B_{it} to solve the following problem:

$$\max \sum_t \beta^t U\left(C_{it}/N_{it}, L_{it}/N_{it} + \bar{L}_{nb,it}/N_{it}\right) N_{it}$$

$$\text{subj. to} \quad \sum_t p_t \left[\left(1+\tau_{ci}\right) C_{it} + \sum_j V^j_t \left(S^j_{i,t+1} - S^j_{it}\right) + B_{i,t+1} - B_{it}\right]$$

$$\leq \sum_t p_t \left[\left(1-\tau_{li}\right) W_{it} L_{it} + \left(1-\tau_{dt}\right) \sum_j S^j_{it} D^j_t + r_{bt} B_{it} + \kappa_{it}\right],$$

where $\bar{L}_{nb,it}$ is exogenously determined labor in the nonbusiness sector, τ_{ci}, τ_{li}, and τ_{dt} are tax rates on consumption, labor, and company distributions, V^j_t is the price of a share in j, W_{it} is the wage rate in country i, and r_{bt} is the after-tax return on lending/borrowing. We assume that country i has a population of size $N_{it} = n_{it}(1+\gamma_{\scriptscriptstyle N})^t$, with common growth rate $\gamma_{\scriptscriptstyle N}$ and a country-specific shifter n_{it}. Note that the measure of a country's production locations is proportional to its population. Hence, we use the same notation for both variables and set the constant of proportionality equal to one (without loss of generality).

We have included nonbusiness hours (exogenously) in total hours and will include nonbusiness income less investment in κ_i. The nonbusiness sector is added in order to ensure that the NIPA aggregates are of the right order of magnitude. Because our focus is on returns to capital, we also assume that taxes on consumption and labor are constant over time while technology parameters and tax rates on dividends and profits vary over time.

If $U(c,l) = \log c + \psi \log(1-l)$, then the first-order conditions with respect to C_i, L_i, B_i, and S_i for the household in country i are

$$\lambda\left(1+\tau_{ci}\right)p_t = \beta^t U_{c,it} = \beta^t N_{it}/C_{it}$$

$$\lambda\left(1-\tau_{li}\right)W_{it}p_t = \beta^t U_{l,it} = \psi\beta^t / \left(1 - L_{it}/N_{it} - \bar{L}_{nb,it}/N_{it}\right)$$

$$\frac{p_t}{p_{t+1}} = \left(1 + r_{b,t+1}\right)$$

$$\frac{p_t}{p_{t+1}} = \left(\frac{V_{t+1}^j + \left(1-\tau_{d,t+1}\right)D_{t+1}^j}{V_t^j}\right), \quad \forall j,$$

where λ is the Lagrange multiplier associated with the household budget constraint.

A.2.3. Resource constraint

The worldwide resource constraint is

$$\sum_i C_{it} + \sum_{i,j}\left[X_{T,it}^j + X_{I,it}^j\right] + \sum_j X_{M,t}^j + \sum_i \bar{X}_{nb,it} = \sum_{i,j} Y_{it}^j + \sum_i \bar{Y}_{nb,it}.$$

Here, we have explicitly included (exogenous) nonbusiness investment $\bar{X}_{nb,i}$ and output $\bar{Y}_{nb,i}$.

A.2.4. Detrended first-order conditions

We'll use small letters for growth-detrended variables. Specifically, let

$$c_{it} = \frac{C_{it}}{N_{it}\left(1+\gamma_y\right)^t} = \frac{C_{it}}{n_{it}\left(1+\gamma_Y\right)^t}$$

$$y_{it}^j = \frac{Y_{it}^j}{N_{it}\left(1+\gamma_y\right)^t} = \frac{Y_{it}^j}{n_{it}\left(1+\gamma_Y\right)^t}$$

$$l_{it} = \frac{L_{it}}{N_{it}},\ l_{it}^j = \frac{L_{it}^j}{N_{it}}$$

$$w_{it} = \frac{W_{it}}{\left(1+\gamma_y\right)^t}$$

$$k^j_{\cdot,it} = \frac{K^j_{\cdot,it}}{N_{it}\left(1+\gamma_y\right)^t} = \frac{K^j_{\cdot,it}}{n_{it}\left(1+\gamma_Y\right)^t}$$

$$x^j_{\cdot,it} = \frac{X^j_{\cdot,it}}{N_{it}\left(1+\gamma_y\right)^t} = \frac{X^j_{\cdot,it}}{n_{it}\left(1+\gamma_Y\right)^t}$$

$$x^j_{M,t} = \frac{X^j_{M,t}}{\left(1+\gamma_Y\right)^t}$$

$$m^j_t = \frac{M^j_t}{\left(1+\gamma_Y\right)^t}$$

$$d^j_t = \frac{D^j_t}{\left(1+\gamma_Y\right)^t}$$

$$a^j_{it} = \frac{A^j_{it}}{\left(1+\gamma_A\right)^t},$$

where γ_Y is the growth rate of output, γ_y is the growth rate of per capita output, and γ_A is the growth rate of TFP. Using the production technology, we can determine the growth rate of total output on the balanced growth trend:

$$\begin{aligned}
\left(1+\gamma_Y\right) &= \left(1+\gamma_A\right)\left(1+\gamma_N\right)^{\phi}\left(1+\gamma_Y\right)^{\phi}\left(1+\gamma_Y\right)^{\alpha_T(1-\phi)} \\
&\qquad \cdot \left(1+\gamma_Y\right)^{\alpha_I(1-\phi)}\left(1+\gamma_N\right)^{(1-\alpha_T-\alpha_I)(1-\phi)} \\
&= \left(1+\gamma_A\right)^{\frac{1}{(1-\alpha_T-\alpha_I)(1-\phi)}}\left(1+\gamma_N\right)^{\frac{1-\left(\alpha_T+\alpha_I\right)(1-\phi)}{(1-\alpha_T-\alpha_I)(1-\phi)}},
\end{aligned}$$

where recall that γ_N is the growth rate of the population (and locations).

A.2.5. Equilibrium paths

Substituting detrended variables into first-order conditions implies

$$\begin{aligned}
\left(1+\gamma_Y\right)k^j_{T,i,t+1} &= \left[\left(1-\delta_T\right)k^j_{T,it} + x^j_{T,it}\right]n_{it}/n_{i,t+1} \\
\left(1+\gamma_Y\right)k^j_{I,i,t+1} &= \left[\left(1-\delta_I\right)k^j_{I,it} + x^j_{I,it}\right]n_{it}/n_{i,t+1} \\
\left(1+\gamma_Y\right)m^j_{t+1} &= \left[\left(1-\delta_M\right)m^j_t + x^j_{M,t}\right]
\end{aligned}$$

$$d_t^j = \sum_i \left\{ (1-\tau_{p,it})\, n_{it} \left(y_{it}^j - w_{it} l_{it}^j - \delta_T k_{T,it}^j - x_{I,it}^j \right) \right\}$$
$$- x_{M,t}^j \sum_i (1-\tau_{p,it})\, \chi_i^j - \sum_i \left\{ n_{i,t+1} (1+\gamma_Y)\, k_{T,i,t+1}^j - n_{it} k_{T,it}^j \right\}$$

$$y_{it}^j = a_{it}^j \left(m_t^j \right)^\phi \left(\left(k_{T,it}^j \right)^{\alpha_T} \left(k_{I,it}^j \right)^{\alpha_I} \left(l_{it}^j \right)^{1-\alpha_T-\alpha_I} \right)^{1-\phi}$$

$$\sum_{i,j} n_{it} y_{it}^j = \sum_i n_{it} \Big(c_{it} + \bar{x}_{nb,it} - \bar{y}_{nb,it} + \sum_j x_{T,it}^j + \sum_j x_{I,it}^j \Big) + \sum_j x_{M,t}^j$$

$$p_t/p_{t+1} = (1+\gamma_y)\, c_{i,t+1} / (\beta c_{it})$$

$$p_t/p_{t+1} = 1 + r_{b,t+1}$$

$$(1-\tau_{li})\, w_{it} = \psi (1+\tau_{ci})\, c_{it} / (1 - l_{it} - l_{nb,it})$$

$$w_{it} = (1-\phi)(1-\alpha_T-\alpha_I)\, y_{it}^j / l_{it}^j, \quad \text{all} j$$

$$r_{T,it}^j = (1-\phi)\, \alpha_T y_{it}^j / k_{T,it}^j$$

$$r_{T,it}^j = (1-\phi)\, \alpha_I y_{it}^j / k_{I,it}^j$$

$$r_{M,it}^j = \phi n_{it} y_{it}^j / m_t^j,$$

where $\sum_i \chi_i^j = 1$. In equilibrium, $\sum_i S_i^j = 1$ and $\sum_i B_i = 0$.

In the case of two countries, computing equilibrium paths involves solving a fixed point problem of size $T \times 11$, where T is the length of the time series and 11 is the number of unknowns. The unknowns are as follows: c_{it}, l_{it}, $k_{T,it}^j$, m_t^j, for all i and j, and one asset holding. Because asset returns are equated deterministically, we pre-set b_{it} and S_{it}^j for one j and i and include S_{-it}^{-j} in the list of unknowns, where superscript $-j$ means "not j" and subscript $-i$ means "not i". The fact that bond holdings sum to zero and share holdings sum to 1 imply all other asset holdings.

Given values for consumption, labor, tangible capital, technology capital, and asset holdings, we use a subset of first-order conditions to infer all remaining variables, and then we check that the remaining 11 first-order conditions hold—namely, the two budget constraints of the households, the two labor market clearing conditions, four Euler equations for tangible capital stocks, two Euler equations for technology capital stocks, and one Euler equation for foreign bonds. It turns out that the plant-specific intangible stocks are proportional to tangible stocks.

We choose initial capital stocks to ensure that investments do not jump at the start of our sample. Specifically, we add constraints that the growth in detrended investment between the first and second period is equal to the growth in detrended investment between the second and third period. We also set initial U.S. GDP to 1 and initial rest of world GDP to 2.2, which determines the scale for initial U.S. TFP and a scale for the ratio of intangible to tangible capital.

A.2.6. Steady state

We do not linearize around a steady state when computing equilbria, but the steady state is useful for gaining intuition about our solutions.

The steady state in this economy can be computed as follows. Given parameters, guess c_i, l_i, and m^j and compute

$$
\begin{aligned}
r_b &= (1+\gamma_y)/\beta - 1 \\
w_i &= \psi(1+\tau_{ci}) / \left[(1-\tau_{li})\left(1-l_i-\bar{l}_{nb,i}\right)\right], \text{ all } i \\
y_i^1/l_i^1 &= w_i/\left((1-\phi)(1-\alpha_T-\alpha_I)\right), \text{ all } i \\
y_i^j/l_i^j &= y_i^1/l_i^1, \text{ all } i, j=2,\ldots,J \\
k_{T,i}^j/y_i^j &= ((1-\phi)\alpha_T)/\left(r_b/(1-\tau_{pi})+\delta_T\right) \text{ all } i,j \\
k_{I,i}^j/y_i^j &= ((1-\phi)\alpha_I)/(r_b+\delta_I), \quad \text{all } ,i,j \\
y_i^j &= \left[a_i^j\left(k_{T,i}^j/y_i^j\right)^{(1-\phi)\alpha_T}\left(k_{I,i}^j/y_i^j\right)^{(1-\phi)\alpha_I}\left(l_i^j/y_i^j\right)^{(1-\phi)(1-\alpha_T-\alpha_I)}\right]^{1/\phi} m^j, \text{ all } i,j \\
&\Rightarrow \text{values for } k_{T,i}^j, k_{I,i}^j, l_i^j \text{ given ratios above}
\end{aligned}
$$

$$
\begin{aligned}
d^j =& \sum_i n_i \{ (1-\tau_{pi}) \left(y_i^j - w_i l_i^j - \delta_T k_{T,i}^j - (\gamma_Y + \delta_I) k_{I,i}^j \right) \\
& - \gamma_Y k_{T,i}^j \} - (\gamma_Y + \delta_M) m^j \sum_i (1-\tau_{pi}) \chi_i^j \text{all } j \\
\kappa_i =& \tau_{ci} c_i + \tau_{li} w_i l_i + \tau_d \sum_j s_i^j d^j / n_i \\
& + \tau_{pi} \sum_j (y_i^j - w_i l_i^j - \delta_T k_{T,i}^j \\
& - (\gamma_Y + \delta_I) k_{I,i}^j - (\gamma_Y + \delta_M) \chi_i^j m^j / n_i) + \bar{y}_{nb,i} - \bar{x}_{nb,i}.
\end{aligned}
$$

With these intermediate results, we check that

$$
\begin{aligned}
l_i &= \sum_j l_i^j, \text{ all } i \\
(r_b - \gamma_Y) b_i &= (1+\tau_{ci}) c_i - (1-\tau_{li}) w_i l_i - (1-\tau_d) \sum_j s_i^j d^j / n_i - \kappa_i, \text{ all } i \\
r_b - \delta_M &= \sum_i (1-\tau_{pi}) \left(\phi n_i y_i^j / m^j \right) / \sum_i (1-\tau_{pi}) \chi_i^j, \text{ all } j.
\end{aligned}
$$

If it does not, we update the guess and continue.

To make sure all is adding up, we also double-check the global resource constraint:

$$
\sum_i n_i c_i + \sum_{i,j} n_i x_{T,i}^j + \sum_{i,j} n_i x_{I,i}^j + \sum_j x_M^j + \sum_i n_i \bar{x}_{nb,i} = \sum_{i,j} n_i y_i^j + \sum_i n_i \bar{y}_{nb,i}.
$$

A.2.7. International equity values

Assuming the total shares of multinational j are normalized to 1, the market value of j is V_t^j. We next guess and verify that

$$
V_t^j = (1-\tau_{dt}) \Big(\sum_i K_{T,i,t+1}^j + \sum_i (1-\tau_{p,it}) K_{I,i,t+1}^j + \sum_i (1-\tau_{p,it}) \chi_i^j M_{t+1}^j \Big).
$$

Using this guess, we have

$$
V_{t+1}^j + (1-\tau_{d,t+1}) D_{t+1}^j
$$

$$
\begin{aligned}
&= (1-\tau_{d,t+1})\left(\sum_i K^j_{T,i,t+2} + \sum_i (1-\tau_{p,i,t+1}) K^j_{I,i,t+2} + \sum_i (1-\tau_{p,i,t+1}) \chi^j_i M^j_{t+2}\right) \\
&\qquad + (1-\tau_{d,t+1}) \sum_i \Big\{ (1-\tau_{p,i,t+1}) \Big(Y^j_{i,t+1} - W_{i,t+1} L^j_{i,t+1} - \delta_T K^j_{T,i,t+1} \\
&\qquad\qquad\qquad - \left[K^j_{I,i,t+2} - (1-\delta_I) K^j_{I,i,t+1} \right] \\
&\qquad\qquad\qquad - \chi^j_i [M^j_{t+2} - (1-\delta_M) M^j_{t+1}] \Big) \Big\} \\
&\qquad - (1-\tau_{d,t+1}) \sum_i \left(K^j_{T,i,t+2} - K^j_{T,i,t+1} \right) \\
\\
&= (1-\tau_{d,t+1}) \sum_i \{ \left[(1-\tau_{p,i,t+1}) \left(r^j_{T,i,t+1} - \delta_T \right) + 1 \right] K^j_{T,i,t+1} \\
&\qquad\qquad\qquad + \left[(1-\tau_{p,i,t+1}) \left(r^j_{T,i,t+1} + 1 - \delta_I \right) \right] K^j_{I,i,t+1} \\
&\qquad\qquad\qquad + \left[(1-\tau_{p,i,t+1}) \left(r^j_{M,i,t+1} + \chi^j_i (1-\delta_M) \right) \right] M^j_{t+1} \} \\
\\
&= (1-\tau_{dt}) \sum_i \{ p_t/p_{t+1} K^j_{T,i,t+1} \\
&\qquad\qquad\qquad + p_t/p_{t+1} (1-\tau_{p,it}) K^j_{I,i,t+1} \\
&\qquad\qquad\qquad + p_t/p_{t+1} \left(\sum_i (1-\tau_{p,i,t+1}) \chi^j_i \right) M^j_{t+1} \} \\
\\
&= p_t/p_{t+1} V^j_t,
\end{aligned}
$$

which verifies the guess because it is consistent with the household's first-order condition derived above.

A.3. BEA Accounts

Before comparing the model accounts to the BEA accounts for the United States, we make three adjustments to U.S. GNP and its components. First, we subtract consumption taxes from NIPA Table 3.5. Second, we subtract personal business expenses for handling life insurance and pension funds (found in NIPA Table 2.5.5) and treat them as intermediate financial services. Third, we add consumer durable depreciation (in Flow of Funds Table F10) and capital services for consumer durables and government capital services. The capital stocks for consumer durables and government capital are found in the BEA's Fixed Asset Table 1.1.[2]

We now apply the BEA's procedure to set up the national and international accounts for our economy. This implies the following for GDP and GNP and their components:

- $\text{GDP}_{it} = \sum_j (Y^j_{it} - X^j_{I,it} - \chi^j_i X^j_{M,t}) + \bar{Y}_{nb,it}$

  Income

  Depreciation: $\delta_T \sum_j K^j_{T,it}$

  Compensation: $W_{it} \sum_j L^j_{it} = W_{it} L_{it}$

  Profits:

  Tax liability: $\tau_{p,it} \sum_j (Y^j_{it} - W_{it} L^j_{it} - \delta_T K^j_{T,it} - X^j_{I,it} - \chi^j_i X^j_{M,t})$

  Dividends: $\sum_j \{(1-\tau_{p,it})(Y^j_{it} - W_{it} L^j_{it} - \delta_T K^j_{T,it} - X^j_{I,it} - \chi^j_i X^j_{M,t})$
  $-(K^j_{T,i,t+1} - K^j_{T,it})\}$

  Retained earnings: $\sum_j (K^j_{T,i,t+1} - K^j_{T,it})$

  Nonbusiness income: $\bar{Y}_{nb,it}$

  Product

  Consumption: C_{it}

  Measured investment: $\sum_j X^j_{T,it} + \bar{X}_{nb,it}$

  Net exports: $\sum_j (Y^j_{it} - X^j_{I,it} - \chi^j_i X^j_{M,t} - X^j_{T,it}) - C_{it} + \bar{Y}_{nb,it} - \bar{X}_{nb,it}$

- $\text{GNP}_{it} = \text{GDP}_{it}$ + Net factor receipts less payments

  Net factor receipts (from $l \neq i$)

  Direct investment: $\sum_{l\neq i}(1-\tau_{p,lt}) \sum_{j\in J^i} (Y^j_{lt} - W_{lt} L^j_{lt} - \delta_T K^j_{T,lt} - X^j_{I,lt} - \chi^j_l X^j_{M,t})$

  Portfolio equity: $\sum_{j\in J^l} S^j_{it} D^j_t$

  Portfolio interest: $r_{bt} B_{it}$ if $B_{it} \geq 0$

  Net factor payments (to $l \neq i$)

  Direct investment: $(1-\tau_{p,it}) \sum_{j\in J^l} (Y^j_{it} - W_{it} L^j_{it} - \delta_T K^j_{T,it} - X^j_{I,it} - \chi^j_i X^j_{M,t})$

  Portfolio equity: $\sum_{l\neq i} \sum_{j\in J^i} S^j_{lt} D^j_t$

[2] See more details in the Matlab code accounts.m. This program loads in BEA and Flow of Funds original data files and writes out Table 4 of the main paper.

Portfolio interest: $r_{bt}B_{it}$ if $B_{it} \le 0$

- Balance of Payments: Current account = Financial account

Current account

Net exports

Net factor receipts less payments

Financial account

Direct investment: $\sum_{l\neq i}\sum_{j\in J^i}(K^j_{T,l,t+1} - K^j_{T,lt}) - \sum_{j\in J^l}(K^j_{T,i,t+1} - K^j_{T,it})$

Portfolio equity: $\sum_{j\in J^l} V^j_t (S^j_{i,t+1} - S^j_{it}) - \sum_{l\neq i}\sum_{j\in J^i} V^j_t (S^j_{l,t+1} - S^j_{lt})$

Portfolio debt: $B_{i,t+1} - B_{it}$

It is useful to examine the current account and financial account for a two-country case, since we can relate it to the household budget constraints. Let u be the United States and r be the rest of world. We'll index companies in the United States by d, which we'll refer to as "Dell" (or alternatively "Domestic"). We'll index rest-of-world companies by f, which we'll refer to as "Fujitsu" (or alternatively "Foreign"). We'll assume full expensing at home.

In this case, the current account can be written as net exports (NX) plus net factor receipts (NFR) less net factor payments (NFP):

$$\begin{aligned}
\mathrm{CA}_{ut} &= NX_{ut} + \mathrm{NFR}_{ut} - \mathrm{NFP}_{ut} \\
&= \left[Y^d_{ut} + Y^f_{ut} - X^d_{I,ut} - X^f_{I,ut} - X^d_{T,ut} - X^f_{T,ut} - X^d_{M,t} + \bar{Y}_{nb,ut} - \bar{X}_{nb,ut} - C_{ut}\right] \\
&+ \left[(1-\tau_{p,rt})\left(Y^d_{rt} - W_{rt}L^d_{rt} - \delta_T K^d_{T,rt} - X^d_{I,rt}\right) + S^f_{ut}D^f_t\right] \\
&- \left[(1-\tau_{p,ut})\left(Y^f_{ut} - W_{ut}L^f_{ut} - \delta_T K^f_{T,ut} - X^f_{I,ut}\right) + S^d_{rt}D^d_t - r_b B_{ut}\right] \\
&= (1-\tau_{lu})\,W_{ut}L_{ut} + (1-\tau_{dt})\sum_j S^j_{ut}D^j_t + r_{bt}B_{ut} + \kappa_{ut} - (1-\tau_{cu})\,C_{ut} \\
&\qquad + K^d_{T,r,t+1} - K^d_{T,rt} - K^f_{T,u,t+1} + K^f_{T,ut}, \qquad (A.3.1)
\end{aligned}$$

where

$$\kappa_{ut} = \tau_{cu} C_{ut} + \tau_{lu} W_{ut} L_{ut} + \tau_{dt} \left(S^d_{ut} D^d_t + S^f_{ut} D^f_t \right)$$
$$+ \tau_{p,ut} \left(Y_{ut} - W_{ut} L_{ut} - \delta_T K_{T,ut} - X_{I,ut} \right) + \bar{Y}_{nb,ut} - \bar{X}_{nb,ut},$$

Y_{it} is total production in country i, $K_{T,it} = \sum_j K^j_{T,it}$ the total tangible capital stock in country i, and $X_{I,it} = \sum_j X^j_{I,it}$ is total plant-specific investment in country i. In writing net factor payments, we assume that $B_{ut} < 0$ and therefore net factor interest is paid by the United States to rest of world.

Next, consider the financial account (FA) which is the change in assets and given by

$$\begin{aligned}
\mathrm{FA}_{ut} &= \left[K^d_{T,r,t+1} - K^d_{T,rt} - V^d_t \left(S^d_{r,t+1} - S^d_{rt} \right) \right] \\
&\quad - \left[K^f_{T,u,t+1} - K^f_{T,ut} - V^f_t \left(S^f_{u,t+1} - S^f_{ut} \right) \right] + B_{u,t+1} - B_{ut} \\
&= \sum_j V^j_t \left(S^j_{u,t+1} - S^j_{ut} \right) + B_{u,t+1} - B_{ut} \\
&\quad + K^d_{T,r,t+1} - K^d_{T,rt} - K^f_{T,u,t+1} + K^f_{T,ut},
\end{aligned} \tag{A.3.2}$$

where we use the fact that $\sum_i S^j_{it} = 1$ for all t. By the balance of payments, FA less CA is equal to zero and therefore

$$(1 - \tau_{cu}) C_{ut} + \sum_j V^j_t \left(S^j_{u,t+1} - S^j_{ut} \right) + B_{u,t+1} - B_{ut}$$
$$= (1 - \tau_{lu}) W_{ut} L_{ut} + (1 - \tau_{dt}) \sum_j S^j_{ut} D^j_t + r_{bt} B_{ut} + \kappa_{ut},$$

which in turn implies that the household period t budget holds each period.

Net foreign asset positions in the BEA's international accounts are based on the flows from the financial account, with adjustments made for capital gains. Unfortunately, because of several unavoidable measurement problems, the foreign net asset position concept is flawed. First, in a world with intangible capital that is expensed, part of the FA earnings are not counted. Even if they could be estimated, the part of intangible capital that is technology capital is neither domestic nor foreign. Finally, without decent transaction prices for capital stock abroad, an inevitable mismatch occurs when we add portfolio incomes and direct investment retained earnings.

We asked ourselves, is there a natural alternative to the BEA's net foreign asset position measure? Unfortunately, the answer is no for our economy.

A.4. Sensitivity of Main Results

In this section we perform sensitivity analyses. Specifically, we rerun the exercise described in the main paper for alternative parameterizations of the model economy, varying parameters for which we have little independent information. For convenience, we report the benchmark constants in Table A.1 of this appendix. In Table A.2 we report the benchmark time-varying inputs.

The experiments are conducted as follows. For each alternative set of model constants, we choose the path for the openness parameters and the relative size so as to mimic trends in the U.S. current account.[3] We set the initial capital stocks so that initial U.S. GDP is 31 percent of initial world GDP and so that there are no jumps in initial investments. The initial U.S. TFP is set so that initial U.S. GDP is normalized to 1.

We also investigate the impact of the openness parameters and the (residual) choice of the weight on foreign stocks in U.S. portfolios. The benchmark inputs are shown in Table A.2. In the first experiment, we fix the openness parameters at the benchmark 1960 level throughout the sample. In the second experiment, we fix the U.S. share of foreign equities. In both experiments, we adjust the relative size of the rest of world to the United

[3] We also set portfolio weights so that the model generates the right split of debt and equity net factor incomes. Later, we show that the impact of this choice is negligible for our main findings.

States to fit the trend in U.S. net exports relative to GNP. As in the benchmark economy, this is done by adjusting the relative TFPs.

Our results are reported in Tables A.3 and A.4. Table A.3 reports the model's predictions for average investment shares and capital to output ratios for the period 1960 to 2006 and business valuations for U.S. companies relative to GNP in the 1960s. These estimates were used when choosing the benchmark parameters. In Table A.4 we report the predicted returns on foreign direct investment for U.S. companies and rest of world companies. The prediction for these returns is the central finding of the paper. For comparison, we include predictions of the benchmark model in both tables and returns based on BEA data in Table A.4. All returns are constructed using the same procedure as the BEA for their current-cost measures.

The sensitivity analysis summarized in Tables A.3 and A.4 highlights the role that rents from technology capital and plant-specific intangible capital play in raising measured FDI returns and the role that investment in plant-specific capital plays in lowering measured foreign returns. As we discussed in the main text, the return on foreign direct investment in country i made by companies from j, $r^j_{FDI,it}$ relative to the true return r_t to capital (of all types) is given by

$$r^j_{FDI,it} - r_t = (1-\tau_{p,it})\left[\phi + (1-\phi)\,\alpha_I\right]\frac{Y^j_{it}}{K^j_{T,it}} - (1-\tau_{p,it})\,\frac{X^j_{I,it}}{K^j_{T,it}}. \tag{A.4.1}$$

The first term is the excess return due to profits on technology capital and plant-specific capital. The second term is the discount in return due to expensed investment in plant-specific intangible capital.

In our sensitivity analysis, as we vary the depreciation rate of technology capital δ_M we find significant changes in predicted investments, stocks, and valuations, but negligible changes in the returns to FDI. The results of these experiments are shown in rows 1 and 2 of Tables A.3 and A.4. Returns are little changed because the technology capital depreciation rate has a negligible impact on the tangible capital to output ratio and a negligible impact on the share of plant-specific investment. The choice of δ_M equal to 8 percent implies a technology capital to GNP ratio in the range of 5 to 6 percent and a U.S. business value to GNP ratio between 1.5 and 1.6 in the 1960s. These were the targets used when parameterizing the benchmark model.

When we vary intangible income shares and the depreciation rate of plant-specific intangible capital, we find a nonnegligible effect on FDI returns. Consider first the income share on technology capital ϕ. The benchmark value is 7 percent. We experimented with $\phi = 8$ percent and $\phi = 6$ percent and, as before, changed the exogenous inputs to ensure that the model generates the same trends in current account flows. The results show that these alternate specifications have a nonnegligible effect on both the macro quantities in Table A.3 and on FDI returns in Table A.4.

Interestingly, in Table A.3, we see that the investment share for plant-specific capital rises with ϕ while the ratio of plant-specific capital to output falls. This finding is due to the fact that the initial capital stocks are also changed in each experiment to ensure that the auxiliary constraints on initial investments and initial GDPs hold for each experiment. The magnitude of the capital stocks in turn affects the business valuations. A value of ϕ =7 percent implies that the model's prediction for the 1960s U.S. business value to GNP is in the range of 1.5 to 1.6.

In rows 3 and 4 of Table A.4, we report the predicted returns on FDI. As is evident from $(A.4.1)$, there is a direct effect of changing ϕ through the first term and indirect effects through changes in investment shares and capital to output ratios. With $\phi = 8$ percent, we find an increase in both the return on U.S. direct investment abroad and the return on FDI in the United States. The former increases by 60 basis points and the latter by 66 basis points. Thus, there is a slight narrowing of the return gap. With $\phi = 6$ percent, the opposite occurs: both the return on U.S. direct investment abroad and the return on direct investment in the United States are lower. The impact is nonlinear, however, since the gap widens by more than 6 basis points. In fact, with $\phi = 6$ percent, the return gap is 426 basis points, which is close to 70 percent of the actual gap.

Like ϕ, the income share α_I has a direct effect on the excess return in $(A.4.1)$. However, technology capital and plant-specific intangible capital affect the FDI return differently because one is expensed at home and the other abroad. In the case of foreign plant-specific intangible capital, what matters is the timing of expensing, since it directly lowers the return in $(A.4.1)$. Therefore, what matters is not the choice of the income share α_I alone or the the choice of the depreciation rate δ_I alone, but rather the pair.

In rows 5 and 6 of Tables A.3 and A.4, we show the results as we vary δ_I and α_I. We first increased δ_I from 0 in the benchmark to 6 percent, which is equal to the rate used for tangible capital. This change has the effect of cutting the average plant-specific intangible capital to output ratio in half, from 1.2 times GNP to 0.6 times GNP, and the average ratio of plant-specific intangible capital to tangible capital by even more, from 0.91 to 0.39. The lower intangible capital stock implies a lower 1960s business value to GNP ratio, although the impact is partially offset by the fact that companies substitute across types of capital. The effect on FDI returns shows up in a higher predicted return on FDI in the United States. Less expensed investment implies a smaller negative term in $(A.4.1)$. The predicted return for FDI in the United States, then, is 4.3 percent, which is higher than the roughly 3.1 percent return in the benchmark economy and the U.S. data. Interestingly, even with a ratio of plant-specific intangible capital to tangible capital of less than 40 percent, the return gap is still 270 basis points.

We find a much wider FDI return gap when we increase α_I from 7 percent in the benchmark economy to 10 percent. In this case, expensing of plant-specific intangible capital plays a much bigger role and the predicted return on FDI in the United States is only 2.54 percent, which is lower than the roughly 3.1 percent return in the benchmark economy and the U.S. data.

In rows 7 and 8 of Tables A.3 and A.4, we show that varying tax rates on consumption and labor have almost no effect on investments and returns.

In the last three rows of Tables A.3 and A.4, we report results for alternative specifications of time-varying inputs. Row 9 has results for an alternative projection of rest of world population. The benchmark economy (in Table A.2) has the ratio of relative populations falling after 2010 at the same rate as the most recent decade. In the alternative specification, we assume the ratio of populations does not fall further after 2010. The results show that the predictions in this case are very close to the benchmark. The FDI return gap increases, but only slightly.

In row 10 are results for a constant U.S. share of foreign equity, S_{ut}^{f}, equal to the initial level of 1 percent. Recall that, in the benchmark economy, we needed to assume a large shift in shares in 2000 to get the timing in the difference between receipts and payments of equity portfolio income to match the U.S. time series. This seems implausible and is likely due to our strong assumption that there are no differences in returns due to risk. In Tables A.3 and A.4, we show that the choice of path for S_{ut}^{f} does not affect our main findings. For the case of a constant share, the average return gap is different from the benchmark economy by only 4 basis points.

In the final experiment, we investigate the model's predictions if the U.S. and rest of world economies had not opened up further relative to where they were in 1960. This is clearly counterfactual given the large rise in FDI incomes, but we are interested in investigating the impact of our choice of openness parameters. In this experiment, we only adjust the relative TFPs to ensure that the trend in net exports to GNP is the same in this case as in the benchmark.

With no change in openness, both measured returns on FDI are high—roughly 7.9 percent per year—relative to the actual annual return, which is roughly 4.6 percent per year on all types of capital.[4] The gap is approximately zero because foreign companies do not significantly increase their investments in their U.S. subsidiaries with openness parameters expected to be constant.

[4] Since we model trends, we set the period equal to five years when computing equilibria. In our experiments, the actual arithmetic return is 4.6 percent per year and the actual geometric return is 4.2 percent per year.

TABLE A.1. Model Constants at Annual Rates

Parameter	Expression	Value
Growth Rates (%)		
Population	γ_N	1.0
Technology	γ_A	1.2
Preferences		
Discount factor	β	.98
Leisure weight	ψ	1.32
Nonbusiness Sector (%)		
Fraction of time at work, $i = u, r$	$\bar{L}_{nb,i}/N_i$	6.0
Nonbusiness investment, $i = u, r$	$\bar{X}_{nb,i}/\text{GDP}_i$	15.4
Nonbusiness value added, $i = u, r$	$\bar{Y}_{nb,i}/\text{GDP}_i$	31.2
Fixed Tax Rates (%)		
Tax rates on labor $i = u, r$	$\tau_{l,i}$	29.0
Tax rate on consumptions, $i = u, r$	$\tau_{c,i}$	7.3
Income Shares (%)		
Technology capital	ϕ	7.0
Tangible capital	$(1-\phi)\alpha_T$	21.4
Plant-specific intangible capital	$(1-\phi)\alpha_I$	6.5
Labor	$(1-\phi)(1-\alpha_T-\alpha_I)$	65.1
Depreciation Rates (%)		
Technology capital	δ_M	8.0
Tangible capital	δ_T	6.0
Plant-specific intangible capital	δ_I	0

TABLE A.2. Model Time-Varying Inputs

Year	Relative[a] Populations	Tax Rates Dividends	Tax Rates Profits	Openness ROW	Openness U.S.	Relative[a] TFPs	Per Capita U.S. Debt	U.S. Foreign Shares
1960	8.20	.400	.408	.8350	.6900	.3730	0	.010
1965	8.42	.400	.403	.8397	.6942	.3727	0	.032
1970	8.64	.400	.396	.8443	.7003	.3725	0	.050
1975	8.86	.397	.386	.8490	.7090	.3722	0	.070
1980	9.08	.370	.375	.8537	.7207	.3719	0	.113
1985	9.30	.246	.361	.8583	.7357	.3714	−.049	.178
1990	9.37	.164	.348	.8630	.7531	.3717	−.098	.220
1995	9.28	.153	.336	.8677	.7718	.3731	−.146	.260
2000	9.16	.152	.327	.8723	.7899	.3743	−.195	.300
2005	9.04	.152	.320	.8770	.8058	.3751	−.244	−.050
2010	8.91	.152	.315	.8817	.8186	.3743	−.270	.000
2015	8.79	.152	.312	.8863	.8283	.3732	−.293	.000
2020	8.67	.152	.310	.8910	.8352	.3723	−.293	.000
2025	8.55	.152	.309	.8957	.8399	.3721	−.293	.000
2030	8.42	.152	.308	.9003	.8431	.3731	−.293	.000
2035	8.30	.152	.307	.9050	.8452	.3745	−.293	.000

[a] Note: "Relative" implies rest of world relative to the United States.

TABLE A.3. Alternative Model Predictions for Investments and Stocks[a]

Model	Averages, 1960-2006: Intangible Investment Divided by GNP (%) $\frac{X^d_{M,t}}{GNP_{ut}}$	$\frac{\sum_j X^j_{I,ut}}{GNP_{ut}}$	Intangible Stocks Divided by GNP $\frac{M^d_t}{GNP_{ut}}$	$\frac{\sum_j K^j_{I,ut}}{GNP_{ut}}$	Ratio of Intangible to Tangible $\frac{K^j_{I,it}}{K^j_{T,it}}$	1960s Business Value to GNP $\frac{V^d_t}{GNP_{ut}}$
Alternatives:						
$\delta_M = 0\%$	4.3	3.7	1.39	1.20	0.91	1.82
$\delta_M = 16\%$	5.5	3.9	0.37	1.20	0.91	1.45
$\phi = 8\%$	6.1	4.1	0.61	1.17	0.90	1.49
$\phi = 6\%$	4.4	2.9	0.47	1.34	0.96	1.61
$\delta_I = 6\%$	5.2	4.2	0.59	0.60	0.39	1.47
$\alpha_I = 10\%$	5.6	7.0	0.52	1.54	1.22	1.56
$\tau_c = 40\%$	5.3	3.9	0.53	1.21	0.91	1.51
$\tau_l = 40\%$	5.3	3.9	0.53	1.21	0.91	1.51
$\frac{n_{rt}}{n_{ut}} = 8.8, t > 2010$	5.3	3.6	0.54	1.24	0.92	1.54
S^f_{ut} constant[b]	5.3	4.1	0.53	1.16	0.89	1.47
σ_{it} constant[b]	5.3	4.0	0.52	1.19	0.90	1.47
Benchmark	5.3	3.9	0.53	1.20	0.91	1.51

[a] Parameters and results are in annual units.

[b] Model FDI incomes are not matched to U.S. FDI incomes.

TABLE A.4. Alternative Model Predictions for FDI Returns, 1982–2006[a]

Model	% Return on U.S. DI Abroad	% Return on FDI in U.S.	Difference
Alternatives:			
$\delta_M = 0\%$	7.03	3.12	3.91
$\delta_M = 16\%$	7.09	3.12	3.97
$\phi = 8\%$	7.63	3.78	3.85
$\phi = 6\%$	6.59	2.33	4.26
$\delta_I = 6\%$	7.00	4.30	2.70
$\alpha_I = 10\%$	7.05	2.54	4.51
$\tau_c = 40\%$	7.07	3.11	3.96
$\tau_l = 40\%$	7.07	3.11	3.96
$\frac{n_{rt}}{n_{ut}} = 8.8, t > 2010$	7.06	3.07	3.99
S^f_{ut} constant[b]	7.10	3.15	3.95
σ_{it} constant[b]	7.90	7.93	−.03
Benchmark	7.08	3.12	3.96
U.S. Data	9.40	3.15	6.25

[a] Parameters and results are in annual units.

[b] Model FDI incomes are not matched to U.S. FDI incomes.

Appendix B.

The Impact of Technology Capital on Productivity and Net Exports

B.1. Introduction

In this appendix, we work with a simple version of the model presented in the main paper to gain intuition for some of the results. We refer to this simple model as the *stripped-down model* because we "strip out" taxation, plant-specific intangible capital, nonbusiness activities, and equities from our general model to make our analysis tractable. In the stripped-down version of the model, the only recorded transactions in the current account are net shipments of goods and net borrowing or lending.[5]

We use the stripped-down model to analytically characterize and contrast equilibria in economies with and without technology capital. We demonstrate that including technology capital in our stripped-down model—which is an otherwise standard two-country growth model—has an important impact on its predictions for relative labor productivities and net exports. In a standard model without technology capital, relative productivities and the level of borrowing and lending across countries depend only on countries' relative TFPs. When we include technology capital, we find that relative populations and the degree of countries' openness also matter. We demonstrate this in several propositions and then show equilibrium paths for several empirically motivated numerical examples.

The main lesson that we draw from the results is that the change in the relative

[5] There are two Matlab codes that generate equilibrium paths shown in the figures of this appendix. The code nx_tcap.m generates results for the model with technology capital included, and nx_std.m generates results for the standard model without technology capital.

populations was an important factor in the recent decline of the trade balance and this fact is not captured by standard international models that abstract from technology capital.

B.2. Stripped-Down Model

We start with a stripped-down model. In order to make it easier to follow, we adopt the following notation: u stands for United States, r stands for rest of world, d stands for Dell (a U.S. company), and f stands for Fujitsu (a non-U.S. company).

We'll consider both a planning problem (with utility weights λ and $1-\lambda$) and a decentralized economy with borrowing and lending and some initial outstanding debt. The allocations for the planner's problem and the decentralized economy are the same for a particular λ in the planner's problem and initial debt in the decentralized problem.

The planner solves the following problem:

$$\max_{\left\{\substack{C_{ut},C_{rt},L_{ut},L_{rt}\\X_{ut},X_{rt}}\right\}} E\sum_{t=0} \beta^t \lambda\{\log\left(C_{ut}/N_{ut}\right)+\psi\log\left(1-L_{ut}/N_{ut}\right)\}N_{ut} \\ +\left(1-\lambda\right)\{\log\left(C_{rt}/N_{rt}\right)+\psi\log\left(1-L_{rt}/N_{rt}\right)\}N_{rt},$$

subject to the global resource constraint and the capital accumulation equations

$$C_{ut}+C_{rt}+X_{K,ut}+X_{K,rt}+X^d_{M,t}+X^f_{M,t}=Y_{ut}+Y_{rt}$$

$$K_{u,t+1}=\left(1-\delta\right)K_{ut}+X_{K,ut}$$

$$K_{r,t+1}=\left(1-\delta\right)K_{rt}+X_{K,rt}$$

$$M^d_{t+1}=\left(1-\delta\right)M^d_t+X^d_{M,t}$$

$$M^f_{t+1}=\left(1-\delta\right)M^f_t+X^f_{M,t},$$

with initial stocks K_{i0}, $i=u,r$, and M^j_0, $j=d,f$ given.

The technologies available to the planner are given by

$$Y^d_{ut}=A_{ut}\left(N_{ut}M^d_t\right)^{\phi}\left(\left(K^d_{ut}\right)^{\alpha}\left(L^d_{ut}\right)^{1-\alpha}\right)^{1-\phi}$$

$$
\begin{aligned}
Y_{ut}^{f} &= \sigma_{ut} A_{ut} \left(N_{ut} M_t^f\right)^{\phi} \left(\left(K_{ut}^f\right)^{\alpha} \left(L_{ut}^f\right)^{1-\alpha}\right)^{1-\phi} \\
Y_{rt}^{d} &= \sigma_{rt} A_{rt} \left(N_{rt} M_t^d\right)^{\phi} \left(\left(K_{rt}^d\right)^{\alpha} \left(L_{rt}^d\right)^{1-\alpha}\right)^{1-\phi} \\
Y_{rt}^{f} &= A_{rt} \left(N_{rt} M_t^f\right)^{\phi} \left(\left(K_{rt}^f\right)^{\alpha} \left(L_{rt}^f\right)^{1-\alpha}\right)^{1-\phi},
\end{aligned} \tag{B.2.1}
$$

and aggregate output in country i is $Y_{it} = Y_{it}^d + Y_{it}^f$, $i = u, r$. In the standard model, $\phi = 0$ and $\sigma_{it} = 0$, $i = u, r$. Note that the aggregate capital stocks and labor inputs in country i are given by

$$
\begin{aligned}
K_{it} &= K_{it}^d + K_{it}^f \\
L_{it} &= L_{it}^d + L_{it}^f,
\end{aligned}
$$

where $i = u$ or r.

The allocations for the planner's problem are equivalent to those of the following decentralized economy with borrowing and lending conditional on a particular value for initial debt. Here, households in country u solve

$$
\max_{\{C_{ut}, L_{ut}\}} E \sum_{t=0}^{\infty} \beta^t \left\{\log\left(C_{ut}/N_{ut}\right) + \psi \log\left(1 - L_{ut}/N_{ut}\right)\right\} N_{ut}
$$

subject to the period budget constraints and the capital accumulation equations,

$$
\begin{aligned}
& C_{ut} + X_{K,ut} + X_{M,t}^d + B_{t+1} \leq W_{ut} L_{ut} + r_{ut}^k K_{ut} + r_{dt}^m M_t^d + \left(1 + r_t^b\right) B_t \\
& K_{u,t+1} = (1-\delta) K_{ut} + X_{ut} \\
& M_{t+1}^d = (1-\delta) M_t^d + X_{M,t}^d,
\end{aligned}
$$

and initial conditions K_{u0}, M_0^d, B_0. The household takes as given TFP, population and all prices, $\{A_{ut}, N_{ut}, W_{ut}, r_{ut}^k, r_{dt}^m, r_t^b\}$ given. The rest-of-world households solve a similar problem, specified by replacing u with r and d with f.

If we solve the planner's problem for a particular value of λ, we can construct the associated initial debt level for the decentralized economy as follows:

$$B_0 = \phi \sum_{t=0}^{\infty} \frac{\left(Y_{rt}^{d*} - Y_{ut}^{f*}\right)}{\Pi_{s=0}^{t}\left(1 + r_s^b\right)}$$

where $r_s^b = c_{us}^*/(\beta c_{u,s-1}^*) - 1$, c_u^* is per capita U.S. consumption, and the asterisk denotes allocations of the planner's problem. Alternatively, if we solve the decentralized problem for a particular initial debt level, then the associated utility weight is

$$\lambda = \frac{c_{ut}}{c_{ut} + c_{rt}}$$

for any period t allocation from the decentralized economy, where c_r is per capita rest of world (ROW) consumption.

B.3. Relative Labor Productivities

In this section, we derive an expression for the relative labor productivities when there is no uncertainty and countries are not yet fully open.[6] Of particular interest is a comparison of results for cases without technology capital ($\phi = 0$) and cases with technology capital ($\phi > 0$). The main results are summarized in Proposition 1.

Proposition 1. Assume $\sigma_{it} \in [0, 1)$, $i = u, r$ and parameters are chosen so that companies in both countries have positive technology capital stocks.[7] If $\phi = 0$, then there is no foreign production, $Y_{ut}^f = Y_{rt}^d = 0$ for all t, as long as the openness parameters are strictly less than 1. In this case, the relative labor productivities depend only on the relative TFPs:

$$\frac{Y_{ut}/L_{ut}}{Y_{rt}/L_{rt}} = \left(\frac{A_{ut}}{A_{rt}}\right)^{\frac{1}{1-\alpha}}.$$

[6] In the case that countries are fully open, the country technology capital stocks are indeterminate.

[7] In our companion paper, we consider examples where some countries are on corners and do not accumulate technology capital.

If $\phi > 0$, then the ratio of foreign to domestic outputs in the two countries is given by

$$\frac{Y_{rt}^d}{Y_{ut}^d} = \sigma_{rt}^{\frac{1}{\phi}} \left(\frac{1-\sigma_{ut}^{1/\phi}}{1-\sigma_{rt}^{1/\phi}} \right), \quad \frac{Y_{ut}^f}{Y_{rt}^f} = \sigma_{ut}^{\frac{1}{\phi}} \left(\frac{1-\sigma_{rt}^{1/\phi}}{1-\sigma_{ut}^{1/\phi}} \right),$$

and the ratio of labor productivities is given by

$$\frac{Y_{ut}/L_{ut}}{Y_{rt}/L_{rt}} = \left(\frac{\left(1-\sigma_{ut}^{1/\phi}\right) A_{ut}^{1/\phi} N_{ut}}{\left(1-\sigma_{rt}^{1/\phi}\right) A_{rt}^{1/\phi} N_{rt}} \right)^{\frac{\phi}{(1-\alpha)(1-\phi)}}$$

which depends on the relative degrees of openness, the relative TFPs, and the relative populations. The more closed the country is, the higher is its TFP, and the more populous it is, the higher is its labor productivity relative to the other country.

Proof. The first order conditions for the planner's problem, assuming no uncertainty, are given as follows:

$$\mu_t = \lambda \frac{N_{ut}}{C_{ut}}$$

$$\mu_t = (1-\lambda) \frac{N_{rt}}{C_{rt}}$$

$$\mu_t (1-\alpha)(1-\phi) \frac{Y_{ut}^d}{L_{ut}^d} = \frac{\lambda \psi N_{ut}}{N_{ut} - L_{ut}}$$

$$\mu_t (1-\alpha)(1-\phi) \frac{Y_{ut}^f}{L_{ut}^f} = \frac{\lambda \psi N_{ut}}{N_{ut} - L_{ut}}$$

$$\mu_t (1-\alpha)(1-\phi) \frac{Y_{rt}^d}{L_{rt}^d} = \frac{(1-\lambda) \psi N_{rt}}{N_{rt} - L_{rt}}$$

$$\mu_t (1-\alpha)(1-\phi) \frac{Y_{rt}^f}{L_{rt}^f} = \frac{(1-\lambda) \psi N_{rt}}{N_{rt} - L_{rt}}$$

$$\mu_t = \beta \mu_{t+1} \left[1 - \delta + \alpha (1-\phi) Y_{u,t+1}^d / K_{u,t+1}^d\right]$$

$$\mu_t = \beta \mu_{t+1} \left[1 - \delta + \alpha (1-\phi) Y_{u,t+1}^f / K_{u,t+1}^f\right]$$

$$\mu_t = \beta \mu_{t+1} \left[1 - \delta + \alpha (1-\phi) Y_{r,t+1}^d / K_{r,t+1}^d\right]$$

$$\mu_t = \beta\mu_{t+1}\left[1-\delta+\alpha\left(1-\phi\right)Y^f_{r,t+1}/K^f_{r,t+1}\right]$$

$$\mu_t = \beta\mu_{t+1}\left[1-\delta+\phi\left(Y^d_{u,t+1}+Y^d_{r,t+1}\right)/M^d_{t+1}\right]$$

$$\mu_t = \beta\mu_{t+1}\left[1-\delta+\phi\left(Y^f_{r,t+1}+Y^f_{u,t+1}\right)/M^f_{t+1}\right]$$

for $t \geq 0$, where μ_t is the multiplier on the global resource constraint.

Capital-output ratios are equated within and across countries, and labor productivities are equated within countries. The capital-output ratios are equal to

$$\kappa_t = \frac{\beta\alpha\left(1-\phi\right)}{c_{ut}/c_{u,t-1}-\beta\left(1-\delta\right)},$$

where $c_{ut} = C_{ut}/N_{ut}$. Since labor productivities are equated within countries, it follows that

$$\begin{aligned}
\frac{Y^f_{ut}}{Y^d_{ut}} &= \sigma_{ut}\left(\frac{M^f_t}{M^d_t}\right)^{\phi}\left(\frac{K^f_{ut}}{K^d_{ut}}\right)^{\alpha(1-\phi)}\left(\frac{L^f_{ut}}{L^d_{ut}}\right)^{(1-\alpha)(1-\phi)} \\
&= \sigma_{ut}\left(\frac{M^f_t}{M^d_t}\right)^{\phi}\left(\frac{\kappa_t Y^f_{ut}}{\kappa_t Y^d_{ut}}\right)^{\alpha(1-\phi)}\left(\frac{Y^f_{ut}}{Y^d_{ut}}\right)^{(1-\alpha)(1-\phi)} \\
&= \sigma_{ut}^{\frac{1}{\phi}}M^f_t/M^d_t \qquad (B.3.1)
\end{aligned}$$

if both regions are investing a positive amount in technology capital. The ratio $(B.3.1)$ tells us that the ratio of foreign to domestic production in the United States increases with the ratio of technology capital stocks and the degree of U.S. openness. Here, we used the fact that labor productivities are equated when substituting out the ratio of labor inputs, $L^f_{ut}/L^d_{ut} = Y^f_{ut}/Y^d_{ut}$.

The ratio of foreign to domestic output in the rest of world is found similarly and is given by

$$\frac{Y^d_{rt}}{Y^f_{rt}} = \sigma_{rt}^{\frac{1}{\phi}}M^d_t/M^f_t. \qquad (B.3.2)$$

Again, it must be the case that $L_{rt}^d/L_{rt}^f = Y_{rt}^d/Y_{rt}^f$.

Next, we use the fact that returns to technology capital are equated across companies,

$$\frac{Y_{ut}^d + Y_{rt}^d}{M_t^d} = \frac{Y_{rt}^f + Y_{ut}^f}{M_t^f}. \tag{B.3.3}$$

Into $(B.3.3)$, we substitute the expressions $(B.2.1)$ and $K_{ijt} = \kappa_t Y_{ijt}$ and simplify to get

$$\begin{aligned}
&\left(M_t^d\right)^{\phi-1}\left\{A_{ut}N_{ut}^{\phi}\left(\left(Y_{ut}^d\right)^{\alpha}\left(L_{ut}^d\right)^{1-\alpha}\right)^{1-\phi} + \sigma_{rt}A_{rt}N_{rt}^{\phi}\left(\left(Y_{rt}^d\right)^{\alpha}\left(L_{rt}^d\right)^{1-\alpha}\right)^{1-\phi}\right\}\\
&\qquad = \left(M_t^f\right)^{\phi-1}\left\{\sigma_{ut}A_{ut}N_{ut}^{\phi}\left(\left(Y_{ut}^f\right)^{\alpha}\left(L_{ut}^f\right)^{1-\alpha}\right)^{1-\phi}\right.\\
&\qquad\qquad\left. + A_{rt}N_{rt}^{\phi}\left(\left(Y_{rt}^f\right)^{\alpha}\left(L_{rt}^f\right)^{1-\alpha}\right)^{1-\phi}\right\}.
\end{aligned} \tag{B.3.4}$$

Several more substitutions are needed before we can write an expression for the ratio of foreign to domestic output in each country.

Next, we use $(B.3.1)$ and $(B.3.2)$ and the fact that the ratios of labor inputs have the same expressions to eliminate Y_u^f, Y_r^f, L_u^f, and L_r^f in the right-hand side of $(B.3.4)$. The result is

$$\begin{aligned}
&A_{ut}N_{ut}^{\phi}\left(\left(Y_{ut}^d\right)^{\alpha}\left(L_{ut}^d\right)^{1-\alpha}\right)^{1-\phi} + \sigma_{rt}A_{rt}N_{rt}^{\phi}\left(\left(Y_{rt}^d\right)^{\alpha}\left(L_{rt}^d\right)^{1-\alpha}\right)^{1-\phi}\\
&\qquad = \sigma_{ut}^{\frac{1}{\phi}}A_{ut}N_{ut}^{\phi}\left(\left(Y_{ut}^d\right)^{\alpha}\left(L_{ut}^d\right)^{1-\alpha}\right)^{1-\phi} + \sigma_{rt}^{\frac{-(1-\phi)}{\phi}}A_{rt}N_{rt}^{\phi}\left(\left(Y_{rt}^d\right)^{\alpha}\left(L_{rt}^d\right)^{1-\alpha}\right)^{1-\phi}
\end{aligned} \tag{B.3.5}$$

Dividing all terms in $(B.3.5)$ by $((Y_{ut}^d)^{\alpha}(L_{ut}^d)^{1-\alpha})^{1-\phi}$ and combining terms results in the following relation:

$$A_{ut}N_{ut}^{\phi}\left(1-\sigma_{ut}^{\frac{1}{\phi}}\right) = \left(\sigma_{rt}^{\frac{-(1-\phi)}{\phi}} - \sigma_{rt}\right)A_{rt}N_{rt}^{\phi}\left(\left(Y_{rt}^d/Y_{ut}^d\right)^{\alpha}\left(L_{rt}^d/L_{ut}^d\right)^{1-\alpha}\right)^{1-\phi}. \tag{B.3.6}$$

This gives us one equation in the two ratios we are deriving: Y_{rt}^d/Y_{ut}^d and L_{rt}^d/L_{ut}^d.

A second equation relating these ratios comes from the production technologies in $(B.2.1)$ and the fact that all nontechnology capital-output ratios are equated. Using these facts, we get

$$\frac{Y_{rt}^d}{Y_{ut}^d} = \left(\frac{\sigma_{rt} A_{rt} N_{rt}^{\phi}}{A_{ut} N_{ut}^{\phi}} \left(\frac{L_{rt}^d}{L_{ut}^d}\right)^{(1-\alpha)(1-\phi)}\right)^{\frac{1}{1-\alpha(1-\phi)}}. \qquad (B.3.7)$$

With $(B.3.6)$ and $(B.3.7)$, we now can write simple expressions for the ratios of outputs and labor inputs of U.S. companies abroad relative to home:

$$\frac{Y_{rt}^d}{Y_{ut}^d} = \sigma_{rt}^{\frac{1}{\phi}} \left(\frac{1-\sigma_{ut}^{\frac{1}{\phi}}}{1-\sigma_{rt}^{\frac{1}{\phi}}}\right) \qquad (B.3.8)$$

$$\frac{L_{rt}^d}{L_{ut}^d} = \sigma_{rt}^{\frac{1}{\phi}} \left(\frac{1-\sigma_{ut}^{\frac{1}{\phi}}}{1-\sigma_{rt}^{\frac{1}{\phi}}}\right)^{\frac{1-\alpha(1-\phi)}{(1-\alpha)(1-\phi)}} \left(\frac{A_{ut} N_{ut}^{\phi}}{A_{rt} N_{rt}^{\phi}}\right)^{\frac{1}{(1-\alpha)(1-\phi)}} \qquad (B.3.9)$$

The same steps are followed to derive the analogous relations for Y_{ut}^f / Y_{rt}^f and L_{ut}^f / L_{rt}^f.

Finally, we can use the ratios of outputs and hours in $(B.3.8)$ and $(B.3.9)$ to express the ratio of labor productivities:

$$\begin{aligned}\frac{Y_{ut}/L_{ut}}{Y_{rt}/L_{rt}} &= \frac{Y_{ut}^d / L_{ut}^d}{Y_{rt}^d / L_{rt}^d} \\ &= \left(\frac{1-\sigma_{ut}^{\frac{1}{\phi}}}{1-\sigma_{rt}^{\frac{1}{\phi}}}\right)^{\frac{\phi}{(1-\alpha)(1-\phi)}} \left(\frac{A_{ut} N_{ut}^{\phi}}{A_{rt} N_{rt}^{\phi}}\right)^{\frac{1}{(1-\alpha)(1-\phi)}}. \qquad (B.3.10)\end{aligned}$$

If $\phi = 0$, then $\sigma_{it}^{1/\phi} = 0$, $N_{it}^{\phi} = 1$ and the ratio of labor productivities is equal to the ratio of TFPs raised to the power $1/(1-\alpha)$. If $\phi > 0$, then the relative productivities depend not only on the relative TFPs, but also on the relative degrees of openness and the relative populations. ▮

Next, we report on the model's predictions for components of the U.S. net exports.

B.4. Net Exports

In this section, we investigate the model's predictions as we change the relative TFPs, the relative population sizes, and the relative degrees of openness. We first consider perturbations around a steady state and derive analytical results, and then we run numerical simulations and report transition paths. In both, we demonstrate that the experiments look very different for cases with and without technology capital.

B.4.1. Steady state results

The goal of this section is to derive analytical expressions for changes in U.S. net exports and net factor incomes relative to U.S. output as we change the rest-of-world TFP, population, and degree of openness. The main results are summarized in Proposition 2. After deriving the results, we consider cases with and without technology capital.

Proposition 2. Changes in the ratio of net exports to output are given by

$$d\left(NX_u/Y_u\right) = -d\left(C_u/Y_u\right) - \delta\ d\left(M_d/Y_u\right) \tag{B.4.1}$$

where the percentage change in the consumption share is equal to

$$\begin{aligned}\frac{d\left(C_u/Y_u\right)}{C_u/Y_u} = &\left\{1 - \frac{\Delta_u}{C_u} + \frac{h_u}{h_r}\left(\frac{\Delta_u + C_r}{C_u}\right)\right\}^{-1} \\ &\cdot\left\{\left(\frac{\Delta_u}{C_u}\right)\frac{dN_r}{N_r} + \frac{1}{h_r}\left(\frac{\Delta_u + C_r}{C_u}\right)\left[\frac{d\left(Y_r/L_r\right)}{Y_r/L_r} - \frac{d\left(Y_u/L_u\right)}{Y_u/L_u}\right]\right\}\end{aligned} \tag{B.4.2}$$

and the changes in labor productivities equal to

$$\frac{d\left(Y_u/L_u\right)}{Y_u/L_u} = \frac{1}{\left(1-\alpha\right)\left(1-\phi\right)}\left(\frac{\sigma_r^{\frac{1}{\phi}}}{1-\sigma_r^{\frac{1}{\phi}}\sigma_u^{\frac{1}{\phi}}}\right)\left(\frac{1-\sigma_u^{\frac{1}{\phi}}}{1-\sigma_r^{\frac{1}{\phi}}}\right)\frac{d\sigma_r}{\sigma_r} \tag{B.4.3}$$

$$\frac{d\left(Y_r/L_r\right)}{Y_r/L_r} = \frac{1}{\left(1-\alpha\right)\left(1-\phi\right)}\left\{\left(\frac{-\sigma_r^{\frac{1}{\phi}}\sigma_u^{\frac{1}{\phi}}}{1-\sigma_r^{\frac{1}{\phi}}\sigma_u^{\frac{1}{\phi}}}\right)\frac{d\sigma_r}{\sigma_r} + \frac{dA_r}{A_r} + \phi\frac{dN_r}{N_r}\right\}. \tag{B.4.4}$$

All variables and derivatives are evaluated at the steady state, $l_i = L_i/N_i$, and

$$\Delta_u = (1 - \delta\kappa - \delta\mu)\, Y_r - C_r \qquad (B.4.5)$$

$$\kappa = \beta\alpha\,(1-\phi)\,/\,(1-\beta\,(1-\delta)) \qquad (B.4.6)$$

$$\mu = \beta\phi/\,(1-\beta\,(1-\delta))\,. \qquad (B.4.7)$$

The change in M_d/Y_u is

$$d\,(M_d/Y_u) = \frac{1}{1-\sigma_u^{1/\phi}\sigma_r^{1/\phi}}\left(\frac{\sigma_r^{1/\phi}}{\phi(1-\sigma_r^{1/\phi})}\frac{M_u}{Y_u}\frac{d\sigma_r}{\sigma_r} - \sigma_u^{1/\phi}\frac{M_r}{Y_r}d\,(Y_r/Y_u)\right), \qquad (B.4.8)$$

where $M_u = M_d + \sigma_u^{1/\phi}M_f$, $M_r = M_f + \sigma_r^{1/\phi}M_d$, and

$$\frac{d\,(Y_r/Y_u)}{Y_r/Y_u} = \left\{1 - \frac{\Delta_u}{C_u} + \frac{h_u}{h_r}\left(\frac{\Delta_u + C_r}{C_u}\right)\right\}^{-1}$$
$$\cdot\left\{\left(1+\frac{h_u}{h_r}\frac{C_r}{C_u}\right)\frac{dN_r}{N_r} + \frac{1}{h_r}\left(1+\frac{C_r}{C_u}\right)\left[\frac{d\,(Y_r/L_r)}{Y_r/L_r} - \frac{d\,(Y_u/L_u)}{Y_u/L_u}\right]\right\} \qquad (B.4.9)$$

Proof. In a steady state, the ratio of net exports to output for country u is given by

$$\begin{aligned}\frac{NX_u}{Y_u} &= (Y_u - X_d - X_u - C_u)\,/Y_u \\ &= 1 - \delta M_d/Y_u - \delta\kappa - C_u/Y_u,\end{aligned}$$

where $\kappa = K_{ij}/Y_{ij}$ is equal to $(B.4.6)$ above. Fully differentiating this ratio, we get

$$d\,(NX_u/Y_u) = -\delta d\,(M_d/Y_u) - d\,(C_u/Y_u)\,. \qquad (B.4.10)$$

Note that the variations we consider do not affect κ.

Since both derivatives in $(B.4.10)$ depend on derivatives of relative productivities, we derive them first. Using $(B.3.10)$, simple algebra yields

$$\frac{d\,(Y_r/L_r)}{Y_r/L_r} = -\frac{1}{(1-\alpha)\,(1-\phi)}\left\{\frac{\sigma_r^{1/\phi}}{1-\sigma_r^{1/\phi}}\frac{d\sigma_r}{\sigma_r} - \frac{dA_r}{A_r} - \phi\frac{dN_r}{N_r}\right\} + \frac{d\,(Y_u/L_u)}{Y_u/L_u}, \quad (B.4.11)$$

where it is implicitly assumed that variation is due only to changes in N_r, A_r, or σ_r. A second relation involving $d(Y_r/L_r)$ and $d(Y_u/L_u)$ is derived using the fact that the return on U.S. technology capital, $\phi(Y_u^d + Y_r^d)/M^d$, does not depend on the magnitudes of A_r, N_r, or σ_r, so that

$$\begin{aligned}
\frac{dM_d}{M_d} &= \left(\frac{Y_u^d}{Y_u^d + Y_r^d}\right)\frac{dY_u^d}{Y_u^d} + \left(\frac{Y_r^d}{Y_u^d + Y_r^d}\right)\frac{dY_{rd}}{Y_{rd}} \\
&= \left(\frac{Y_u^d}{Y_u^d + Y_d^r}\right)\left(\frac{dM^d}{M^d} - \frac{(1-\alpha)(1-\phi)}{\phi}\frac{d(Y_u/L_u)}{Y_u/L_u}\right) \\
&+ \left(\frac{Y_r^d}{Y_u^d + Y_r^d}\right)\Bigg(\frac{dM^d}{M^d} - \frac{(1-\alpha)(1-\phi)}{\phi}\frac{d(Y_r/L_r)}{Y_r/L_r} \\
&\qquad + \frac{1}{\phi}\frac{d\sigma_r}{\sigma_r} + \frac{1}{\phi}\frac{dA_r}{A_r} + \frac{dN_r}{N_r}\Bigg). \qquad (B.4.12)
\end{aligned}$$

The second equation in $(B.4.12)$ is derived by fully differentiating the equations in $(B.2.1)$ and using the fact that labor productivities are equated within countries and capital-output ratios are equated across countries. The term dM_d/M_d cancels on both sides of equation $(B.4.12)$, and after simplifying, we are left with

$$\frac{Y_u^d}{Y_r^d}\frac{d(Y_u/L_u)}{Y_u/L_u} + \frac{d(Y_r/L_r)}{Y_r/L_r} = \frac{1}{(1-\alpha)(1-\phi)}\left\{\frac{d\sigma_r}{\sigma_r} + \frac{dA_r}{A_r} + \phi\frac{dN_r}{N_r}\right\}. \qquad (B.4.13)$$

Solving $(B.4.11)$ and $(B.4.13)$ for $d(L_u/Y_u)$ and $d(L_r/Y_r)$ yields the expressions $(B.4.3)$ and $(B.4.4)$ given in the statement of the proposition.

The next step is to use the intratemporal conditions and the global resource constraint to jointly determine dL_u, dL_r, and dC_u/Y_u. In particular, we fully differentiate the following three equations:

$$\frac{N_u - L_u}{N_r - L_r} = \frac{\lambda}{1-\lambda}\left(\frac{N_u}{N_r}\right)\left(\frac{Y_r/L_r}{Y_u/L_u}\right) \qquad (B.4.14)$$

$$\frac{C_u}{Y_u} = \frac{(1-\alpha)(1-\phi)}{\psi}\left(\frac{N_u - L_u}{L_u}\right) \qquad (B.4.15)$$

$$\frac{C_u}{Y_u}\left(1 + \frac{(1-\lambda)N_r}{\lambda N_u}\right) = (1 - \delta\kappa - \delta\mu)\left(1 + \frac{L_r}{L_u}\frac{Y_r/L_r}{Y_u/L_u}\right), \qquad (B.4.16)$$

with the derivative of the ratio of productivities given as above. In writing the last equations, we made two simple substitutions for C_r/Y_r and Y_r/Y_u so as to make the algebra more manageable.

We spare the reader details of the tedious algebra but describe the exact steps to get the expression $(B.4.2)$. We first replace C_u/Y_u in $(B.4.16)$ using $(B.4.15)$, and then we fully differentiate $(B.4.14)$ and $(B.4.16)$. Once differentiated, the two equations can be simplified and written as two equations with unknowns dL_u and dL_r that are written in terms of dN_r, $d(Y_r/L_r)$, and $d(Y_u/L_u)$. As shown above, derivatives of labor productivities can be written in terms of changes of exogenous variables. See, in particular, equations $(B.4.3)$ and $(B.4.4)$. Thus, the changes in hours found by differentiating $(B.4.14)$ and $(B.4.16)$ can be written explicitly in terms of changes of exogenous variables, namely dN_r, dA_r, and $d\sigma_r$. The final step is to differentiate $(B.4.15)$, thus expressing dC_u/Y_u in terms of dL_u. Substituting in the expression for dL_u, we have $(B.4.2)$.

Next, we need M_d/Y_u. Since it is easier to work with effective technology capital stocks, we define $M_u = M_d + \sigma_u^{1/\phi} M_f$ and $M_r = M_f + \sigma_r^{1/\phi} M_d$ as the effective stocks used in the United States and the ROW, respectively. Using the production technologies, we can write total outputs in terms of M_u and M_r as follows:

$$Y_u = A_u \left(N_u M_u\right)^{\phi} K_u^{\alpha(1-\phi)} L_u^{(1-\alpha)(1-\phi)}$$

$$Y_r = A_r \left(N_r M_r\right)^{\phi} K_r^{\alpha(1-\phi)} L_r^{(1-\alpha)(1-\phi)}.$$

If we replace K_i with κY_i and simplify, we get

$$\frac{M_u}{Y_u} = A_u^{-1/\phi} N_u^{-1} \kappa^{\frac{-\alpha(1-\phi)}{\phi}} \left(Y_u/L_u\right)^{\frac{(1-\alpha)(1-\phi)}{\phi}} \quad (B.4.17)$$

$$\frac{M_r}{Y_r} = A_r^{-1/\phi} N_r^{-1} \kappa^{\frac{-\alpha(1-\phi)}{\phi}} \left(Y_r/L_r\right)^{\frac{(1-\alpha)(1-\phi)}{\phi}}. \quad (B.4.18)$$

Next, express M_d/Y_u in terms of M_u/Y_u and M_r/Y_r:

$$\frac{M_d}{Y_u} = \frac{1}{1-\sigma_u^{1/\phi}\sigma_r^{1/\phi}} \left(\frac{M_u}{Y_u} - \sigma_u^{\frac{1}{\phi}} \frac{M_r}{Y_r} \frac{Y_r}{Y_u}\right).$$

Differentiating this, we get

$$\frac{d\left(M_d/Y_u\right)}{M_d/Y_u} = \frac{1}{1-\sigma_u^{1/\phi}\sigma_r^{1/\phi}}\left(\frac{M_u}{M_d}\frac{d\left(M_u/Y_u\right)}{M_u/Y_u} - \sigma_u^{\frac{1}{\phi}}\frac{M_r}{M_d}\left\{\frac{d\left(M_r/Y_r\right)}{M_r/Y_r} + \frac{d\left(Y_r/Y_u\right)}{Y_r/Y_u}\right\}\right)$$
$$+\frac{\sigma_u^{\frac{1}{\phi}}\sigma_r^{\frac{1}{\phi}}}{\phi\left(1-\sigma_u^{\frac{1}{\phi}}\sigma_r^{\frac{1}{\phi}}\right)}\frac{d\sigma_r}{\sigma_r}, \qquad (B.4.19)$$

where $M_u = M_d + \sigma_u^{1/\phi}M_f$ and $M_r = M_f + \sigma_r^{1/\phi}M_d$.

Equation $(B.4.19)$ requires the derivatives of the technology capital stocks relative to output shown in $(B.4.17)$ and $(B.4.18)$:

$$\frac{d\left(M_u/Y_u\right)}{M_u/Y_u} = \frac{(1-\alpha)(1-\phi)}{\phi}\frac{d\left(Y_u/L_u\right)}{Y_u/L_u} \qquad (B.4.20)$$

$$\frac{d\left(M_r/Y_r\right)}{M_r/Y_r} = \frac{(1-\alpha)(1-\phi)}{\phi}\frac{d\left(Y_r/L_r\right)}{Y_r/L_r} - \frac{1}{\phi}\frac{dA_r}{A_r} - \frac{dN_r}{N_r}, \qquad (B.4.21)$$

which in turn requires knowing $d(Y_u/L_u)$, $d(Y_r/L_r)$, and $d(Y_r/Y_u)$. Derivatives for the labor productivities were derived above and are given in equations $(B.4.3)$ and $(B.4.4)$. Substituting for these in $(B.4.19)$ and simplifying yields

$$d\left(M_d/Y_u\right) = \frac{1}{1-\sigma_u^{1/\phi}\sigma_r^{1/\phi}}\left(\frac{\sigma_r^{1/\phi}}{\phi(1-\sigma_r^{1/\phi})}\frac{M_u}{Y_u}\frac{d\sigma_r}{\sigma_r} - \sigma_u^{1/\phi}\frac{M_r}{Y_r}d\left(Y_r/Y_u\right)\right),$$

where

$$\frac{d\left(Y_r/Y_u\right)}{Y_r/Y_u} = \frac{C_u}{\Delta_u}\left(\frac{d\left(C_u/Y_u\right)}{C_u/Y_u}\right) + \frac{C_r}{\Delta_u}\left(\frac{d\left(C_r/Y_r\right)}{C_r/Y_r}\right) \qquad (B.4.22)$$

$$\frac{d\left(C_r/Y_r\right)}{C_r/Y_r} = \frac{h_u}{h_r}\left(\frac{d\left(C_u/Y_u\right)}{C_u/Y_u}\right) - \frac{1}{h_r}\left[\frac{dY_r/L_r}{Y_r/L_r} - \frac{dY_u/L_u}{Y_u/L_u}\right]. \qquad (B.4.23)$$

The derivative for the relative outputs was found by differentiating the global resource constraint,

$$\frac{C_u}{Y_u} + \frac{C_r}{Y_r}\frac{Y_r}{Y_u} = (1-\delta\kappa-\delta\mu)\left(1+\frac{Y_r}{Y_u}\right),$$

which yields $(B.4.22)$ as a function of the derivatives of the two consumption shares, C_u/Y_u and C_r/Y_r. The rest of world consumption share is found by differentiating:

$$\frac{C_r}{Y_r} = \frac{(1-\alpha)(1-\phi)}{\psi}\left(\frac{N_r - L_r}{L_r}\right)$$

and using the solution to dL_r found above.

The expression for $d(Y_r/Y_u)$ can be further simplified by replacing $d(C_u/Y_u)$ in $(B.4.22)$ and $(B.4.23)$ with $(B.4.2)$. $\blacksquare$

We are now ready to compare changes in the ratio of net exports share in economies with technology capital to changes in economies without technology capital.

If $\phi = 0$, then productivity in the economy that does not experience any changes in TFP, population, or openness does not change, that is $d(Y_u/L_u) = 0$. In ROW, the only thing that matters is TFP:

$$\frac{d\left(Y_r/L_r\right)}{Y_r/L_r} = \frac{1}{1-\alpha}\frac{dA_r}{A_r}.$$

As an example, assume that $\alpha = .3$. Then, a 1 percent increase in ROW TFP leads to a 1.43 percent increase in ROW productivity. But there are no spillovers to productivity in country u, the United States.

When $\phi = 0$, a rise in TFP in ROW impacts net exports in the United States only through changes in consumption because neither country invests in technology capital. In this case,

$$d\left(NX_u/Y_u\right)\Big|_{\phi=0} = -\left\{1 - \frac{\Delta_u}{C_u} + \frac{h_u}{h_r}\left(\frac{\Delta_u + C_r}{C_u}\right)\right\}^{-1}$$

$$\cdot\left\{\left(\frac{\Delta_u}{Y_u}\right)\frac{dN_r}{N_r} + \frac{1}{h_r\left(1-\alpha\right)}\left(\frac{\Delta_u + C_r}{Y_u}\right)\frac{dA_r}{A_r}\right\}, \quad (B.4.24)$$

where, in this case with $\phi = 0$, $\Delta_u = NX_r = -NX_u$.

If we choose empirically plausible parameters, the steady state level of Δ_u would be close to zero. With this further simplification the ratio of net exports to output in the standard model without technology capital is given by

$$d\left(NX_u/Y_u\right)\Big|_{\phi=0,\Delta_u=0} = -\left\{1+\frac{h_u}{h_r}\frac{C_r}{C_u}\right\}^{-1}\left(\frac{C_r}{h_r\left(1-\alpha\right)Y_u}\right)\frac{dA_r}{A_r}. \tag{$B.4.25$}$$

Compare ($B.4.25$) with the same expression for the case with $\phi > 0$ and $\Delta_u = 0$, which is given by

$$\begin{aligned} d\left(NX_u/Y_u\right)\Big|_{\phi>0,\Delta_u=0} = -\left\{1+\frac{h_u}{h_r}\frac{C_r}{C_u}\right\}^{-1}\left(\frac{C_r}{h_r\left(1-\alpha\right)Y_u}\right)&\left\{\frac{1}{1-\phi}\frac{dA_r}{A_r}\right. \\ &+\frac{\phi}{1-\phi}\frac{dN_r}{N_r} \\ &\left.-\frac{\sigma_r^{1/\phi}}{\left(1-\sigma_r^{1/\phi}\right)}\frac{1}{1-\phi}\frac{d\sigma_r}{\sigma_r}\right\} \\ -\,\delta\, d\left(M_d/Y_u\right). \end{aligned} \tag{$B.4.26$}$$

We laid out equation ($B.4.26$) in such a way as to make the comparison with ($B.4.25$) simple. These relations share the first term, although the impact of TFP is larger when $\phi > 0$ because $1/(1-\phi) > 1$. In the case of $\phi = 0$, there are no additional effects due to dN_r or $d\sigma_r$. As we show next, this abstraction can be quantitatively important.

We now consider a simple numerical exercise to illustrate that adding $\phi > 0$ can have a quantitatively important impact on the productivities, consumption shares, and net exports. In Table B.1, we report parameter values that we use in this simple model for this exercise. These are not the same parameters used in our paper, since the model of this section is a stripped-down version of the model in the main paper.

We choose $\beta = 0.96$, since we abstracted from trend growth. This implies an annual interest rate of around 4 percent. We choose the leisure weight so that time at work is between 25 percent and 30 percent of total time. We set income shares so that the split of non-technology capital income is 30/70 for capital and labor. The common depreciation rate is set at 5 percent. The ratio of ROW population to the U.S. population is 8. Regardless of ϕ we set initial debt equal to 0. This implies $\lambda = .79$ in both the model with and the model without technology capital. The relative technology levels are set so that U.S. GDP is equal to 32 percent of world GDP. Finally, in the case with technology capital, we need to specify openness parameters. For the steady state calculations we use .75 for both.

Let's start with the model with technology capital and $\phi > 0$. With the parameters set as in Table B.1, h_u and h_r are both around 0.287, the consumption ratios are both around 0.81, and $C_r/C_u = 2.13$. The parameters also imply that the residual Δ_u is approximately equal to 0.

Using these values in the formulas above, we consider changes in the three exogenous ROW variables: A_r, N_r, and σ_r. The U.S. labor productivity does not change with either A_r or N_r, but ROW labor productivity does. The formulas yield $d(Y_r/L_r)/(Y_r/L_r) = 4.21 dA_r$ and $d(Y_r/L_r)/(Y_r/L_r) = .0134 dN_r$. The openness parameter affects both labor productivities, with the largest impact on U.S. productivity: $d(Y_u/L_u)/(Y_u/L_u) = .0336 d\sigma_r$ and $d(Y_r/L_r)/(Y_r/L_r) = -.0006 d\sigma_r$.

In terms of changes in the U.S. consumption share, we find $d(C_u/Y_u)/(C_u/Y_u) = -9.98 dA_r$, $d(C_u/Y_u)/(C_u/Y_u) = .0318 dN_r$, and $d(C_u/Y_u)/(C_u/Y_u) = -.081 d\sigma_r$. These effects are much larger than those on the technology capital investment: $d(\delta M_d/Y_u) = .02 dA_r$, $d(\delta M_d/Y_u) = -.0002 dN_r$, and $d(\delta M_d/Y_u) = .013 d\sigma_r$. Thus, most of the change in the U.S. net exports share comes from the change in the U.S. consumption share term in $(B.4.1)$.

Adding the derivatives of the consumption share and the investment share together yields the following results for the net exports share: $d(NX_u/Y_u) = -8.10dA_r$, $d(NX_u/Y_u) = -.026dN_r$, and $d(NX_u/Y_u) = .053d\sigma_r$. With $A_r = .365$, a 1 percent change in the ROW TFP implies a drop in net exports from 0.13 percent of output to -3 percent of output. With $N_r = 8$, a 1 percent increase in the ROW population implies a drop in net exports from 0.13 percent of output to roughly $-.2$ percent of output. With a 20 percent increase in the ROW population, which is more empirically plausible for the post–World War II period, there is a drop in net exports from 0.13 percent to about -4 percent of output. With $\sigma_r = .75$, a 1 percent increase in the ROW degree of openness implies an increase in net exports from 0.13 percent of output to 0.17 percent of output.

Now, suppose that technology capital is not included in the model and $\phi = 0$. With the parameters set as in Table B.1, h_u and h_r are both around 0.295, the consumption ratios are both around 0.836, and $C_r/C_u = 2.13$. In this case, the parameters again imply that the residual Δ_u is approximately equal to 0. Using the formula in $(B.4.25)$, we get $d(NX_u/Y_u) = -6.96dA_r$. In the case with $\phi = .07$, we found $d(NX_u/Y_u) = -8.10dA_r$, which implies that technology amplifies the impact of changes in TFP.

B.4.2. Transition results

In this section, we analyze equilibrium paths for the stripped-down model economy that we analyzed above.

The main point of this section, as in the earlier section, is to demonstrate that adding technology capital leads to very different predictions. Here, we focus on equilibrium paths. Since changes in population and TFP have different impacts on equilibrium paths, we consider changing each, one at a time. The time series of these inputs are displayed in Figures B.1 and B.2.

The experiments use the same constants as in Table B.1 and initial conditions from

Tables B.2 and B.3. In Table B.2, we list the initial capital stocks for experiments with fixed degrees of openness. In Table B.3, we list the initial capital stocks for an experiment with increasing degrees of openness.

If there are no changes in any exogenous parameters, the capital stocks remain at the levels shown in Tables B.2 and B.3. If households do expect changes in the exogenous parameters and B_0 is set equal to zero, then the equilibrium paths display initial jumps or declines in investments in anticipation of shifting production. For this reason, we adjusted the initial debt level in each experiment so that investment would adjust smoothly. These necessary adjustments are reported in Tables B.2 and B.3. If we instead hold $B_0 = 0$, the model predicts initial adjustments in investments, but the equilibrium paths that we display would have exactly the same patterns after year 1.[8]

Increased ROW Population

We first consider the model's time series predictions when the population of the rest of world increases relative to that of the United States, as shown in Figure B.1. The main findings for this experiment are shown in Figures B.3–B.6.

In Figure B.1, we display the time series of the ROW population N_{rt} for our first experiment, with N_{ut} set equal to one in all periods. The ROW population starts at 8, rises almost 20 percent, and then returns. Here, we assume that TFPs are fixed.[9]

As we discussed in the main paper, the path of the relative size of countries is an important determinant of the path of net exports. In Figure B.3, we show the actual U.S. net exports relative to GDP along with model predictions in the case with technology capital included and in the case without. We plot the U.S. data to show that the model

[8] The computer codes at http://www.minneapolisfed.org offer the user the choice of adjusting or not adjusting the debt level.

[9] We also ran cases for permanent increases after 2003. The results over the period 1960–2000 are so close that we do not report them here. Interested readers can see results of these experiments at our website.

with technology capital can generate a dramatic drop in net exports relative to GDP, such as that experienced recently in the United States. To emphasize the difference between the models with and without technology capital, we show both in Figure B.3. If there is no technology capital, the net exports share remains at about 1 percent until the late 1990s and then declines to roughly -1 percent of output.

If there is no technology capital, the stripped-down model predicts an increase in investment abroad, which is becoming more populous, and then a shift back when the rest-of-world population reverts to its balanced growth path.[10] Let $\mathcal{A} = A^{1/(1-\alpha)}$. We can write net exports in the United States and ROW relative to size as follows:

$$\frac{NX_{ut}}{\mathcal{A}_{ut}N_{ut}} = \frac{Y_{ut} - C_{ut} - X_{ut}}{\mathcal{A}_{ut}N_{ut}}$$

$$-\frac{NX_{ut}}{\mathcal{A}_{rt}N_{rt}} = \frac{Y_{rt} - C_{rt} - X_{rt}}{\mathcal{A}_{rt}N_{rt}},$$

where we have used the fact that $NX_{rt} = -NX_{ut}$. If we sum these, we get

$$\begin{aligned}
&\left(\frac{1}{\mathcal{A}_{ut}N_{ut}} + \frac{1}{\mathcal{A}_{rt}N_{rt}}\right) NX_{ut} \\
&\quad = \left(\frac{Y_{ut}}{\mathcal{A}_{u}N_{ut}} - \frac{Y_{rt}}{\mathcal{A}_{r}N_{rt}}\right) - \left(\frac{C_{ut}}{\mathcal{A}_{u}N_{ut}} - \frac{C_{rt}}{\mathcal{A}_{r}N_{rt}}\right) - \left(\frac{X_{ut}}{\mathcal{A}_{u}N_{ut}} - \frac{X_{rt}}{\mathcal{A}_{r}N_{rt}}\right) \\
&\quad = -\left(\frac{X_{ut}}{\mathcal{A}_{u}N_{ut}} - \frac{X_{rt}}{\mathcal{A}_{r}N_{rt}}\right), \quad \text{if } \phi = 0.
\end{aligned} \tag{B.4.27}$$

The equality in $(B.4.27)$ only holds when $\phi = 0$ and follows from the fact that the parameters in Tables B.1 and B.2 imply that the ratio of per capita consumption in the United States relative to the ROW is equal to $\mathcal{A}_u/\mathcal{A}_r$.[11] Since relative labor productivities are also equal to this ratio (as seen in $(B.3.10)$), it follows from the intratemporal conditions

[10] If B_0 is set equal to zero, then there is an initial jump in U.S. net exports and an initial drop in U.S. investment.

[11] The allocations for the decentralized economy with $B_0 = -.081$ are the same as the planner's problem with $\lambda = .79$. The ratios of per capita consumptions in both cases are equal to 3.76.

that outputs per effective person, $Y_{it}/\mathcal{A}_i N_{it}$, are also equal. Thus, in the case of $\phi = 0$, the only borrowing and lending that goes on (assuming no initial jumps) is done so that capital-output ratios can be equalized.

In Figures B.4–B.6, we show the paths of the consumption shares, labor productivities, and per capita GDPs for the models with and without technology capital. In all three, the paths for the two countries lie on top of each other when technology capital is excluded ($\phi = 0$). When the model includes technology capital ($\phi > 0$), the patterns for the United States and ROW are completely different. There are two reasons for this. The first reason is that GDP does not include all output produced in a country. True output includes investment in technology capital. Therefore, although capital-output ratios are equated, both within countries (across domestic and foreign firms) and across countries, these relations do not imply that measured capital to GDP ratios are equated.

A second reason for the different patterns in equilibrium paths for U.S. and ROW series is that size has a positive scale effect on GDP per capita and productivity. This scale effect arises because of the fact that technology capital can be used simultaneously at multiple locations and the measure of locations is proportional to a country's population. As the ROW population increases, its output increases by more than the rise in population. With greater world production, per capita consumptions rise in both countries, but the share of consumption in GDP rises in the United States as more production is being done abroad and falls in the ROW where GDP is rising. This is shown in Figure B.4.

Figure B.5 shows that the ROW labor productivity increases relative to U.S. labor productivity as N_{rt}/N_{ut} increases, which is consistent with (B.3.10). Because relative per capita hours are changing, we also predict deviations in per capita GDPs, with the ROW increasing relative to the United States.

In Figures B.7–B.8, we show how the prediction for the U.S. trade deficit changes if we allow for different patterns of openness. In Figure B.7, we use the same series as in the main paper. Figure B.8 shows the result and compares it to the case with the degrees of openness fixed. In the case of the trade deficit, the main difference is that the predicted decline in the net export share is not as large if we assume that countries are opening to FDI as the ROW population increases.

Here, we are working with a stripped-down version of our model with technology capital, but there are some lessons that are common in the two exercises. First, with an empirically plausible rise in the population of the ROW, the model generates a large and empirically plausible decline in net exports relative to GDP, which is much larger than standard theory predicts. Second, the model generates a plausible increase in the U.S. consumption share of GDP and a plausible decline in the U.S. share of world GDP.

Increased ROW TFP

We turn next to the experiment of increasing ROW size by increasing total factor productivity. In Figure B.2, we display the time series of the ROW total factor productivity A_{rt} relative to A_{r0}. We chose an increase of 1.2 percent for ROW TFP in order to generate an empirically plausible decline in the share of net exports in GDP. As before, we compare the predictions of the models with and without technology capital. The results are shown in Figures B.9–B.12.

In Figure B.9, we plot the ratio of U.S. net exports relative to GDP for the stripped-down model with and without technology. For the sake of comparison, we also plot the actual U.S. share. Interestingly, the pattern looks very different from that of Figure B.3, which shows the change in U.S. net exports in the case of higher ROW population. The primary reason for the difference is that GDP rather than output is in the denominator. Recall that our formulas above used output rather than GDP, which is equal to output less investment in technology capital. We did this to make the analytical results more tractable.

In Figures B.10–B.12, we show the consumption shares, labor productivities, and per capita GDP for the models with and without technology capital. As we demonstrated earlier, changes in the endogenous variables are similar in the two models, except that there is some amplification of the impact in the case of the model with technology capital, since the term $1/(1-\phi)$, which multiplies A_r, is greater than 1.

TABLE B.1. Parameter Values for Steady State Analysis in the Stripped-Down Model

Parameters	Expression	Value
Common parameters		
Discount factor	β	.96
Leisure weight in utility	ψ	2
Capital share of other income	α	.3
Depreciation rate	δ	.05
Relative populations	N_r/N_u	8
Initial debt	B_0	0
With technology capital		
Technology capital share	ϕ	.07
Relative technology level	A_r/A_u	.365
U.S. openness	σ_u	.75
ROW openness	σ_r	.75
Without technology capital		
Technology capital share	ϕ	0
Relative technology level	A_r/A_u	.396

TABLE B.2. Initial Conditions for Transition Analysis in the Stripped-Down Model, Openness parameters fixed

Description	Expression	Value
With technology capital, $\phi > 0$		
Tangible capital of Dell in U.S.	K_{u0}^d	1.32
Tangible capital of Fujitsu in U.S.	K_{u0}^f	.047
Tangible capital of Dell in ROW	K_{r0}^d	.022
Tangible capital of Fujitsu in ROW	K_{r0}^f	2.89
Technology capital of Dell	M_0^d	.337
Technology capital of Fujitsu	M_0^f	.737
Initial debt		
No changes in exogenous variables	B_0	0
ROW population as in Figure B.1	B_0	.047
ROW TFP as in Figure B.2	B_0	.130
Without technology capital, $\phi = 0$		
Tangible capital of U.S.	K_{u0}	1.60
Tangible capital of ROW	K_{r0}	3.42
Initial debt		
No changes in exogenous variables	B_0	0
ROW population as in Figure B.1	B_0	-.081
ROW TFP as in Figure B.2	B_0	.139

TABLE B.3. Initial Conditions for Transition Analysis in the Stripped-Down Model, Openness parameters varying[a]

Description	Expression	Value
Tangible capital of Dell in U.S.	K_{u0}^{d}	1.36
Tangible capital of Fujitsu in U.S.	K_{u0}^{f}	.013
Tangible capital of Dell in ROW	K_{r0}^{d}	.123
Tangible capital of Fujitsu in ROW	K_{r0}^{f}	2.79
Technology capital of Dell	M_{0}^{d}	.373
Technology capital of Fujitsu	M_{0}^{f}	.703
Initial debt	B_0	.047

[a] For this experiment, $\phi = .07$, $A_{rt} = .363$ for all t, the path $\{N_{rt}\}$ is shown in Figure B.1, and the paths for $\{\sigma_{it}\}$, $i = u, r$ are shown in Figure B.11.

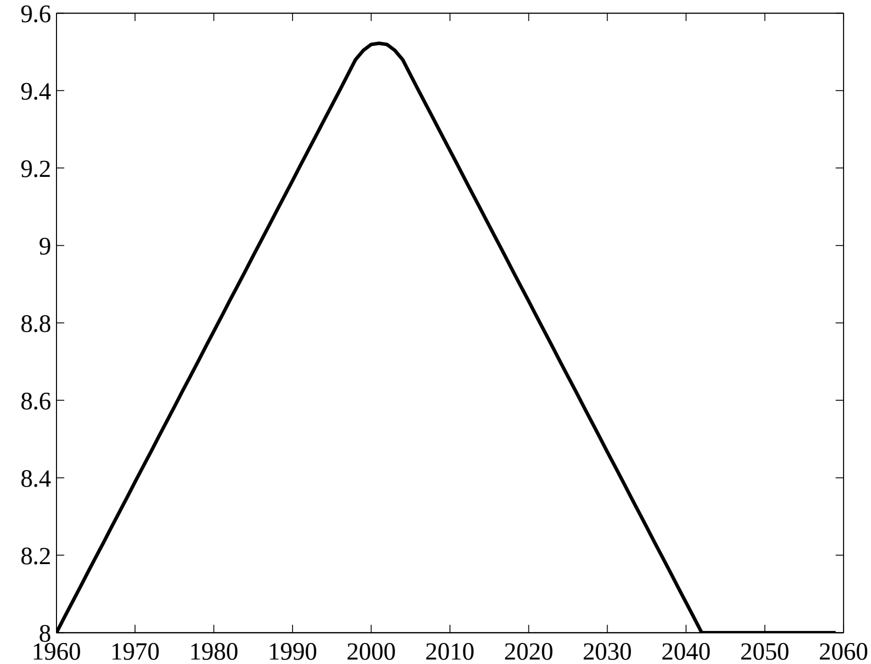

Figure B.1. Temporary Increase in ROW Population

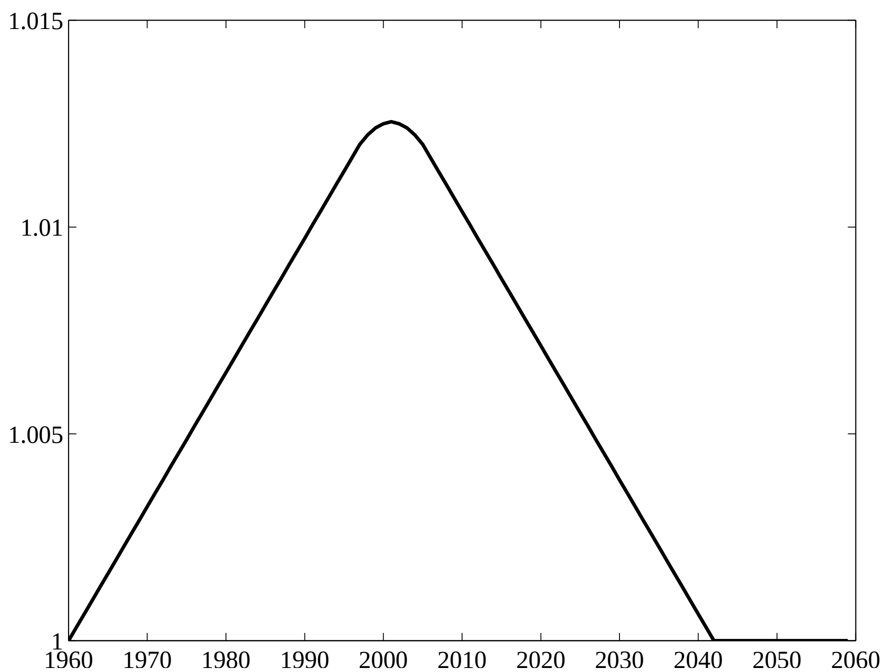

Figure B.2. Temporary Increase in ROW TFP

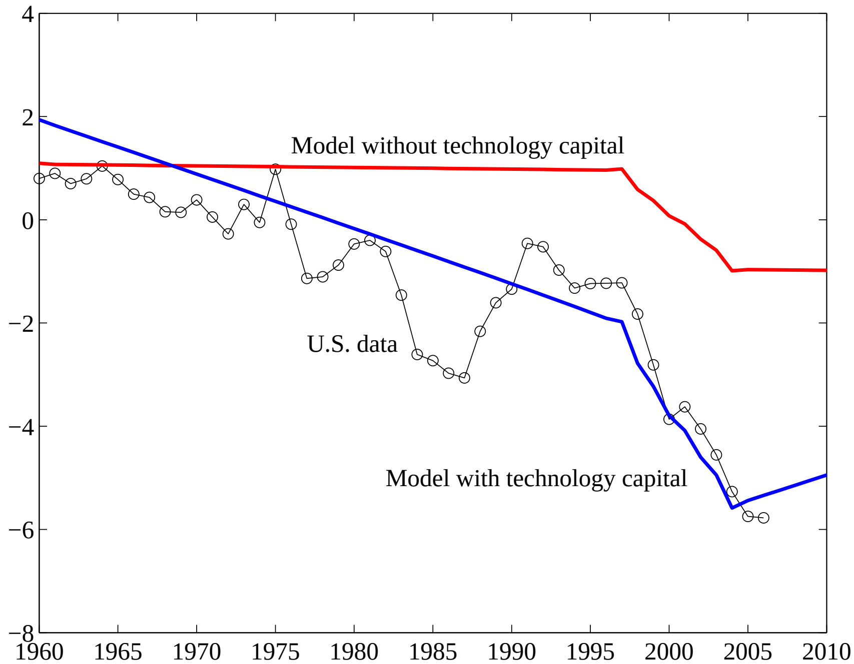

FIGURE B.3. U.S. NET EXPORTS TO GDP AND PREDICTIONS IN THE STRIPPED-DOWN MODEL WITH A TEMPORARY INCREASE IN ROW POPULATION

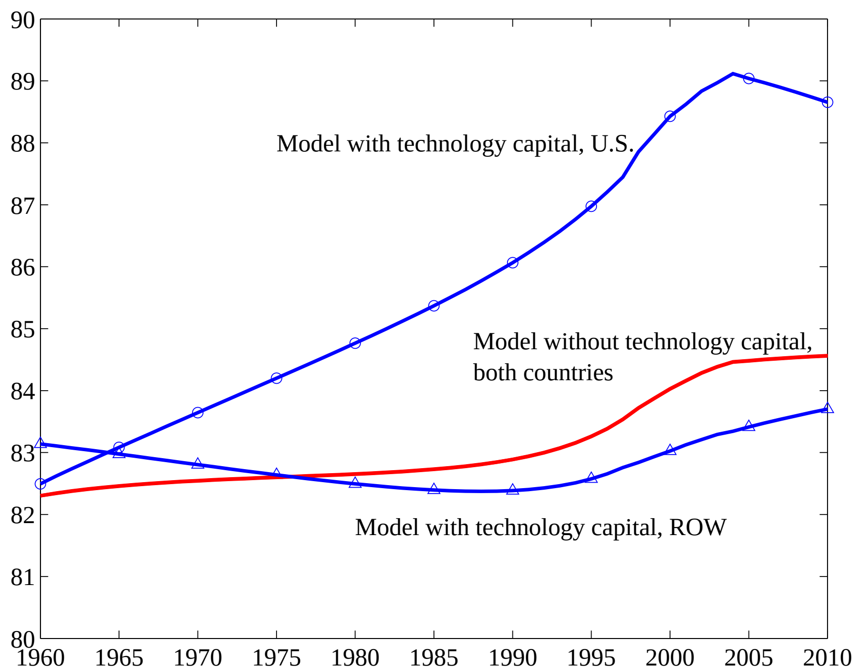

FIGURE B.4. PREDICTIONS OF CONSUMPTION TO GDP RATIO IN THE STRIPPED-DOWN MODEL WITH A TEMPORARY INCREASE IN ROW POPULATION

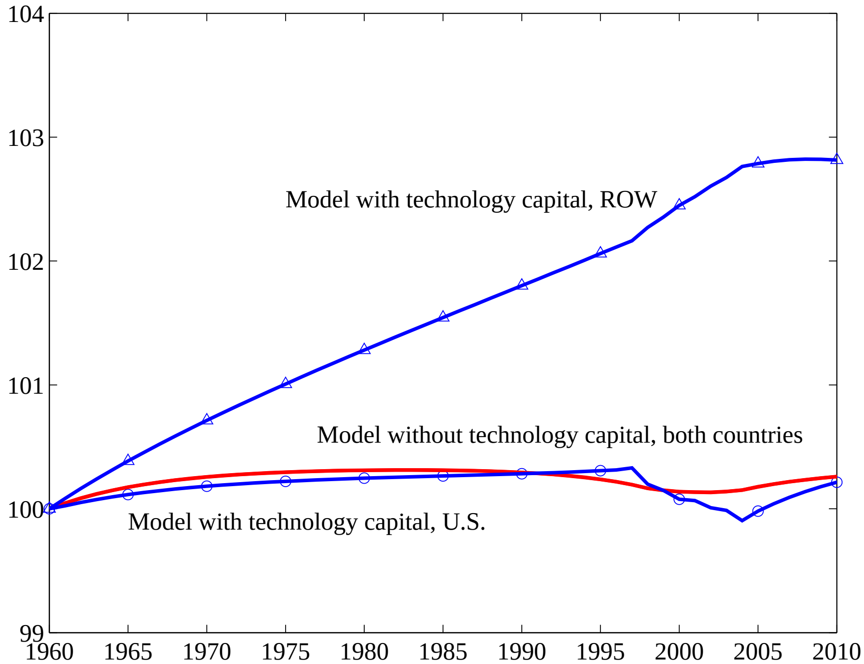

Figure B.5. Predictions of Labor Productivity in the Stripped-Down Model with a Temporary Increase in ROW Population

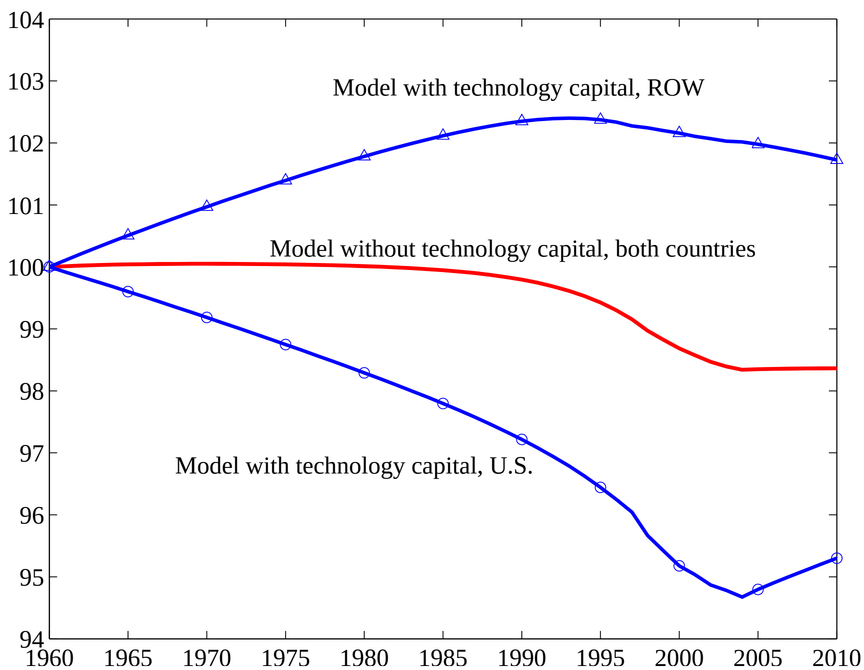

Figure B.6. Model Predictions of Per Capita GDP in the Stripped-Down Model with a Temporary Increase in ROW Population

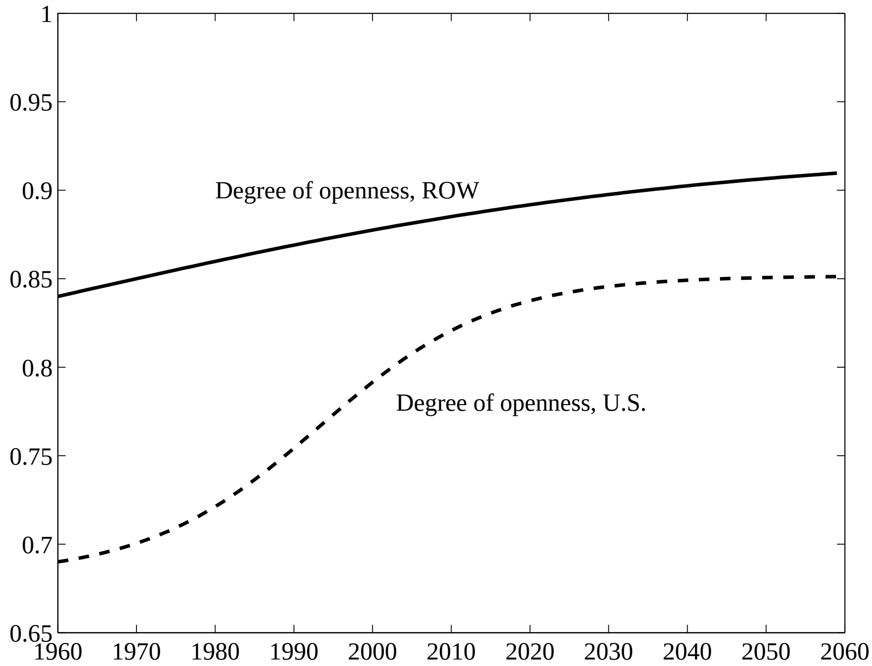

FIGURE B.7. INCREASING DEGREES OF OPENNESS

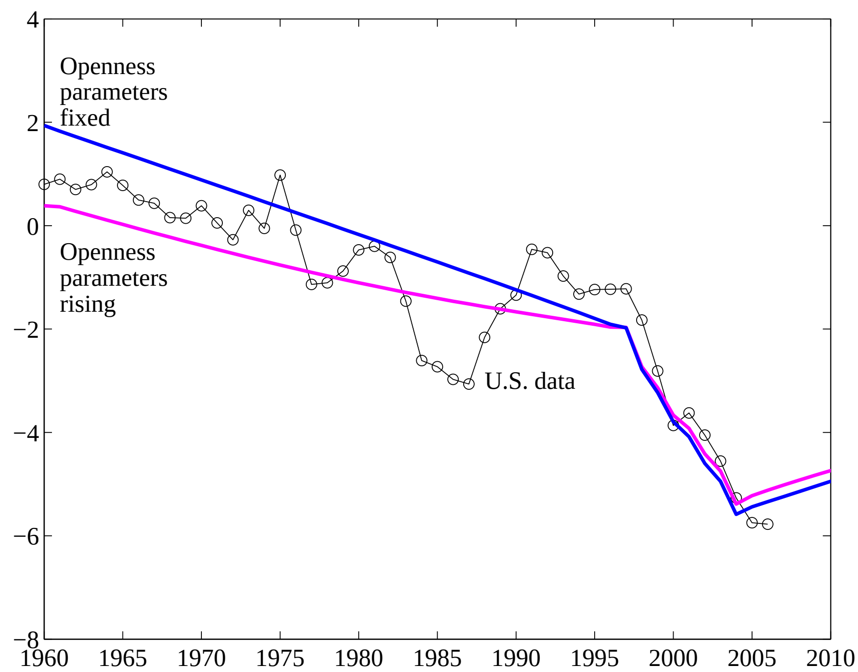

FIGURE B.8. U.S. NET EXPORTS TO GDP AND PREDICTIONS IN THE STRIPPED-DOWN MODEL WITH A TEMPORARY INCREASE IN ROW POPULATION: A COMPARISON OF FIXED AND INCREASING OPENNESS

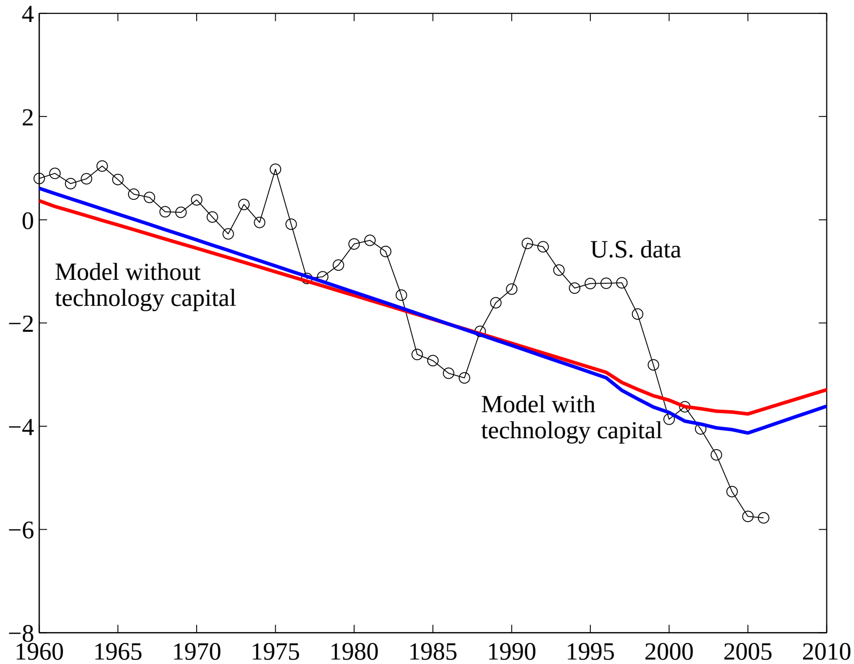

FIGURE B.9. U.S. NET EXPORTS TO GDP AND PREDICTIONS IN THE STRIPPED-DOWN MODEL WITH A TEMPORARY INCREASE IN ROW TFP

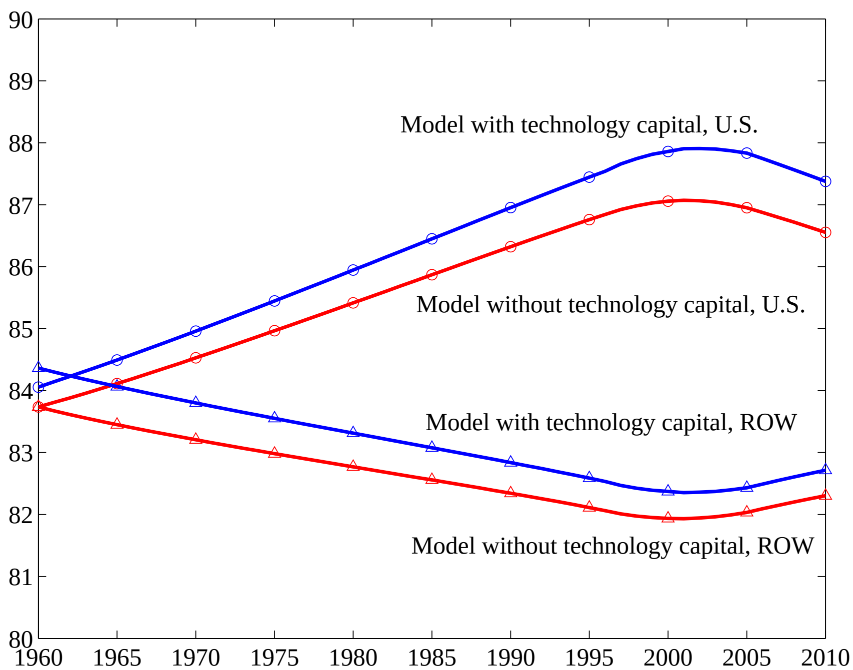

FIGURE B.10. PREDICTIONS OF CONSUMPTION TO GDP RATIO IN THE STRIPPED-DOWN MODEL WITH A TEMPORARY INCREASE IN ROW TFP

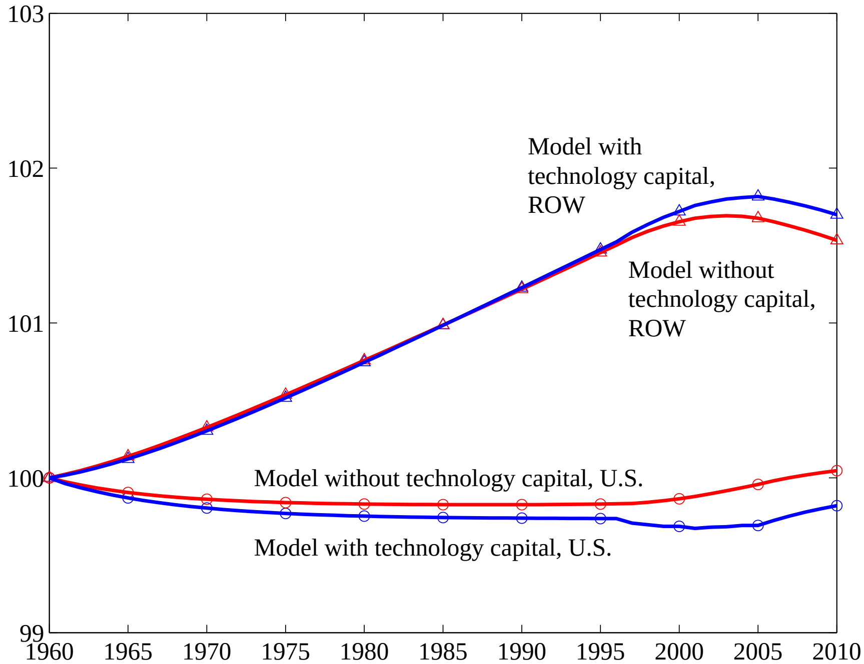

Figure B.11. Predictions of Labor Productivity in the Stripped-Down Model with a Temporary Increase in ROW TFP

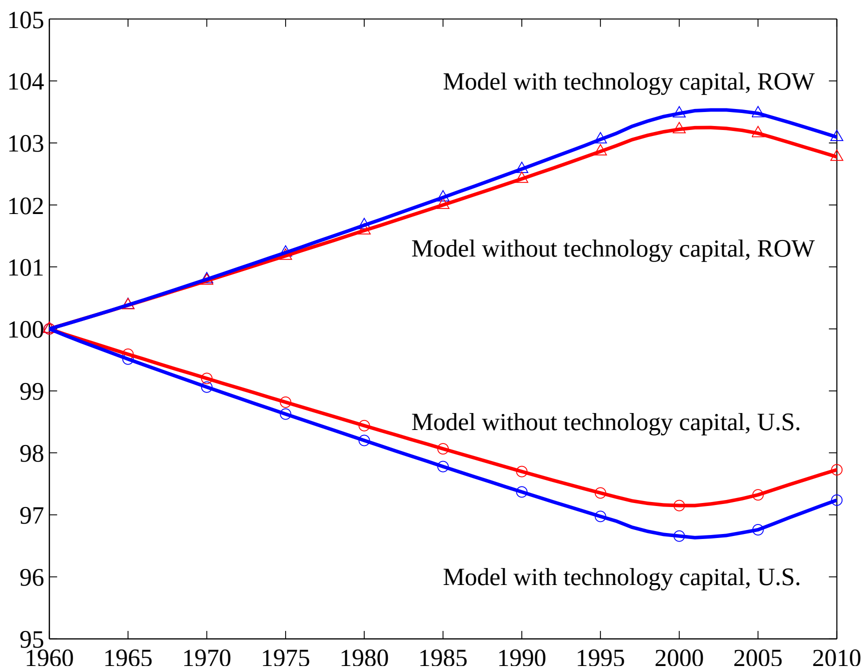

Figure B.12. Model Predictions of Per-Capita GDP in the Stripped-Down Model with a Temporary Increase in ROW TFP