Oil Shocks in A DSGE Model for the Korean Economy

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Abstract

This paper evaluates the relevance and the importance of oil-price shocks to each part of the economy. We augment a DSGE model for a small open economy with oil imports and assess its performance using DSGE-VAR method developed by Del Negro and Schorfheide (2004). The model economy uses oil imports either as direct consumption or an input of production. The empirical analysis with Korean aggregate data reveals that production motive of oil usage is important in improving the fit and that the pass-through of oil prices becomes stronger as oil prices increase.

Keywords: DSGE-VAR, Oil Price Shocks, Misspecification, Pass-Through

JEL Classifications: C52, E52, F41, Q43

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1 Introduction

The WTI crude oil price was 29.19 US dollars per barrel at the third quarter of 2003 and it peaked at 139.96 dollars by the third quarter of 2008. This rapid and continual rise in oil prices over recent years posed many questions among the general public as well as economists. Since the Korean economy depends entirely on imports for its acquisition of crude oil, households, entrepreneurs, and policy makers are interested in knowing to what extent the rise in oil prices affects each part of the economy. Especially, the most sought-after question is whether the effect on direct oil consumption is more harmful than that on production. What would be the most effective policy in either case? In this paper we evaluate the relevance and the importance of oil-price shocks to consumption and production. For this, we employ a DSGE model for a small open economy with oil imports and assess its performance using DSGE-VAR method developed by Del Negro and Schorfheide (2004).

In our baseline model economy, oil imports are used both for direct consumption and production. To find out the importance of each component, we consider two alternative model economies by restricting the baseline economy: "No Oil Consumption" and "No Oil in Production." Within Bayesian estimation framework including DSGE-VARs, we bring aforementioned various DSGE models to the Korean aggregate data. The main findings of the paper are the following. First, we argue that the production component of oil usage is more important than the consumption component in fitting the data from the Korean economy. Marginal likelihood functions across DSGE prior weights are similar for the baseline and No Oil Consumption economies. When we prevent oil usage from producing goods, however, the fit changes dramatically. Second, impulse response comparisons between the DSGE model and the DSGE-VAR($\hat{\lambda}$) show that the baseline model behaves well qualitatively, but quantitative results are not satisfactory. We could improve the model performance by changing the wage setting behavior. Third, from investigation of the smoothed underlying states we can conclude that the pass-through of oil prices in Korean economy moves towards a perfect one as oil prices continue to rise. The pre-existing incomplete pass-through can be characterized by a high tax on oil consumption. When oil shocks are present, the government responds to a shock by alleviate the gasoline tax.

The rest of the paper is organized as follows. Section 2 sets up a small open economy model with oil that is used either as direct consumption or an input of production. Section 3 describes DSGE-VARs, the main tool for empirical analysis used in this paper. Section 4 discusses empirical findings, and Section 5 concludes.

2 The Model

We use a simplified version of Medina and Soto (2005) to model how oil imports affect the economy. Imported oil is either directly consumed by households or used as an input of production. Most common source of direct consumption is fuel for heating and transportation. It is also obvious that oil is used in the production.

Households are heterogeneous in the sense that they are monopsonistic labor suppliers, but wage setting by each household is limited by reoptimization probability that is governed by an i.i.d. process. Each household's consumption basket consists of Home and Foreign goods and oil. Firms are monopolistically competitive firms that produce differentiated goods. Just like the wage setting of households, the price setting behavior is characterized as á la Calvo that introduces nominal stickiness of output price of the economy. The government plays a passive role in this model where it runs a balanced budget without any government spending. Monetary authority plays monetary policy based on the interest rate feedback rule.

As an open economy, imports consist of consumption-dedicated Foreign goods and oil while only Home goods that are produced with oil and labor are exported. Exchange rate pass through is perfect for import and export prices except oil prices. Since we treat the Korean economy as a small open economy, foreign sectors are modeled as a set of exogenous processes.

2.1 Households

The domestic economy is populated by a continuum of monopolistically competitive households indexed by $j \in [0, 1]$. Each household supplies a differentiated labor services to firms. There exists a set of perfectly competitive employment agencies that combine the different labor services from households into an aggregate labor index H_t, defined as

$$H_t = \left(\int_0^1 H_t(j)^{\frac{\nu_L - 1}{\nu_L}} \, dj \right)^{\frac{\nu_L}{\nu_L - 1}}$$

where ν_L is the elasticity of substitution across different labor services. Let $W_t(j)$ denote the nominal wage set by household j. Then demand for this household's labor is

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\nu_L} H_t \tag{1}$$

where the aggregate wage index W_t is given by

$$W_t = \left(\int_0^1 W_t(j)^{1-\nu_L} \, dj \right)^{\frac{1}{1-\nu_L}}$$

Household j maximizes its expected lifetime utility drawn from consumption $C_t(j)$ relative to a habit stock, real money balances $M_t(j)/P_t$, and leisure:

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} \beta^k \left(\log \left(C_{t+k}(j) - \gamma h C_{t+k-1} \right) + \frac{\chi_M}{\mu} \left(\frac{M_{t+k}(j)}{\gamma^{t+k} P_{t+k}} \right)^{\mu} - \chi_H \frac{H_{t+k}(j)^{1+\tau}}{1+\tau} \right) \right] \tag{2}$$

where β is the discount factor, τ is the inverse of the intertemporal substitution elasticity of hours. The habit persistence in consumption is governed by h while γ denotes the growth of the aggregate output by which it is ensured that the economy evolves along a balanced growth path. Note here that the habit stock refers to the entire economy's habit consumption rather that individual habit consumption.

The consumption bundle of household j is given as a CES aggregate of oil (fuel) $O_{C,t}(j)$ consumption and non-fuel core consumption $Z_t(j)$:

$$C_t(j) = \left[\omega_o^{\frac{1}{\phi_c}} O_{C,t}(j)^{\frac{\phi_c-1}{\phi_c}} + (1-\omega_o)^{\frac{1}{\phi_c}} Z_t(j)^{\frac{\phi_c-1}{\phi_c}}\right]^{\frac{\phi_c}{\phi_c-1}} \quad (3)$$

where ϕ_c is the intratemporal elasticity of substitution between oil and core consumption and ω_o denotes the share of oil consumption. Oil is directly consumed as fuel for heating and transportation. The core consumption is again defined as a CES aggregate of domestically produced goods (Home goods) $C_{H,t}(j)$ and imported goods (Foreign goods) $C_{F,t}(j)$:

$$Z_t(j) = \left[(1-\omega_F)^{\frac{1}{\phi_z}} C_{H,t}(j)^{\frac{\phi_z-1}{\phi_z}} + \omega_F^{\frac{1}{\phi_z}} C_{F,t}(j)^{\frac{\phi_z-1}{\phi_z}}\right]^{\frac{\phi_z}{\phi_z-1}}$$

where ϕ_z denotes the intratemporal elasticity of substitution between Home and Foreign goods, and ω_F is the import share. For any given level of consumption bundle $C_t(j)$ as a result of household utility maximization behavior, household j tries to maximize the profit in purchasing such a consumption bundle. Let $P_{o,t}$ and $P_{Z,t}$ denote the prices of oil and core consumption goods, respectively. We further define P_t as the price of the composite consumption good. Then the demand for oil and core consumption are given by

$$O_{C,t}(j) = \omega_o \left(\frac{P_{o,t}}{P_t}\right)^{-\phi_c} C_t(j), \qquad Z_t(j) = (1-\omega_o)\left(\frac{P_{Z,t}}{P_t}\right)^{-\phi_c} C_t(j) \quad (4)$$

The core consumption goods basket $Z_t(j)$ is purchased in a similar fashion:

$$C_{H,t}(j) = (1-\omega_F)\left(\frac{P_{H,t}}{P_{Z,t}}\right)^{-\phi_z} Z_t(j), \qquad C_{F,t}(j) = \omega_F \left(\frac{P_{F,t}}{P_{Z,t}}\right)^{-\phi_z} Z_t(j)$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of Home and Foreign goods, respectively. From (3) and (4) the price of the composite consumption good P_t, namely, the consumption-based price index (CPI), can be written as

$$P_t = \left[\omega_o P_{o,t}^{1-\phi_c} + (1-\omega_o) P_{Z,t}^{1-\phi_c}\right]^{\frac{1}{1-\phi_c}} \quad (5)$$

where

$$P_{Z,t} = \left[(1-\omega_F) P_{H,t}^{1-\phi_z} + \omega_F P_{F,t}^{1-\phi_z}\right]^{\frac{1}{1-\phi_z}} \quad (6)$$

Household j enters period t with domestic portfolio $D_t(j)$ that pays out one unit of domestic currency in a particular state, foreign-currency bond $B_{t-1}^*(j)$ that pays one unit for sure, and nominal money balances $M_{t-1}(j)$.[1] In period t, the household pays a lump-sum tax $T_t(j)$, receives

[1] As usual, 'starred' variables refer to foreign economy.

labor income and profits (dividends) $\Pi_t(j)$ from monopolistic firms, and adjusts the balances on domestic portfolio, foreign-currency bond, and nominal money balances. In particular, acquiring the position on foreign-currency bond entails the premium, that is, households need to pay more than the international price to purchase bonds. Now we can write the budget constraints that domestic households face each period as

$$\begin{aligned} & P_t C_t(j) + \mathbf{E}_t\left[Q_{t,t+1} D_{t+1}(j)\right] + M_t(j) + \frac{e_t B_t^*(j)}{R_t^* \Theta\left(\frac{e_t B_t^*}{P_{X,t} X_t}\right)} \\ \leq \; & W_t(j) H_t(j) + D_t(j) + M_{t-1}(j) + e_t B_{t-1}^*(j) + \Pi_t(j) - T_t(j) \end{aligned}$$

where $Q_{t,t+1}$ is the stochastic discount factor used for evaluating consumption streams, e_t is the nominal exchange rate, and $P_{F,t}^*$ is the price index of the foreign country. Had it not been for the foreign bond premium, household j would have paid $1/R_t^*$ as the price of the foreign bond. In reality, however, this household should pay the premium $\Theta\left(\frac{e_t B_t^*}{P_{X,t} X_t}\right)$ to purchase the foreign bond. The functional form suggests that the premium is related to the ratio of the outstanding foreign debt to nominal value of exports, a measure for healthiness of the economy. That is, the premium increases as foreign debt ratio increases. For simplicity, we further assume that $\Theta(\cdot)$ show constant elasticity κ.[2] In this case, the premium of foreign bond prices changes κ percent when the foreign debt ratio changes by 1 percent. The international interest rate, inverse of the foreign bond price, is assumed to follow a stochastic process.

Under the assumption of the complete domestic asset market, households entertains the perfect risk-sharing, which implies the same level of consumption across household regardless of the labor income they receive each period. With stochastic Lagrangean multiplier $\lambda_{t+k}(j)$ the first order conditions are given as

$$C_t(j): \qquad \frac{1}{C_t(j) - \gamma h C_{t-1}} = \lambda_t(j) P_t$$

$$D_{t+1}(j): \qquad \lambda_t(j) Q_{t,t+1} = \beta \lambda_{t+1}(j)$$

$$B_t^*(j): \qquad \lambda_t(j) e_t = \beta \mathbf{E}_t\left[\lambda_{t+1}(j) e_{t+1} R_t^* \Theta\left(\frac{e_t B_t^*}{P_{X,t} X_t}\right)\right]$$

$$W_t(j): \qquad \mathbf{E}_t\left[\sum_{k=0}^{\infty} \beta^k \left(-\chi_H H_{t+k}(j)^{\tau} \frac{\partial H_{t+k}(j)}{\partial W_t(j)} + \lambda_{t+k}(j) \frac{\partial W_{t+k}(j) H_{t+k}(j)}{\partial W_t(j)}\right)\right] = 0$$

Given the equal consumption across all household from perfect risk-sharing, we have

$$\mathbf{E}_t\left[\beta R_t \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - \gamma h C_t}{C_t - \gamma h C_{t-1}}\right)^{-1}\right] = 1$$

[2] Let $y_t = \Theta(x_t)$. Then

$$\frac{\partial \ln y_t}{\partial \ln x_t} = \frac{\Theta'(x_t)}{\Theta(x_t)} x_t = \kappa \tag{7}$$

where $R_t = \mathbf{E}_t[Q_{t,t+1}]^{-1}$. Also we have

$$\mathbf{E}_t\left[\beta R_t^* \frac{e_{t+1}}{e_t}\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}-\gamma h C_t}{C_t - \gamma h C_{t-1}}\right)^{-1}\Theta\left(\frac{e_t B_t^*}{P_{X,t}X_t}\right)\right] = 1$$

As in Erceg, Henderson, and Levin (2000) we assume that wage setting is subject to a nominal rigidity à la Calvo (1983) and Yun (1996). While each household can set the wage $W_t(j)$ of its own labor service by entertaining its monopoly power, only a fraction $(1-\theta_L)$ of households are entitled chances for full optimization at any given period, independent of the time elapsed since the last adjustment. Thus, in each period a measure $(1-\theta_L)$ of households reoptimizes its wage, while a fraction θ_L adjusts its wage according to a partial indexation rule:

$$W_{t+k}(j) = \Gamma_{W,t}^k W_t(j) \tag{8}$$

where

$$\Gamma_{W,t}^k = \gamma^k \bar{\pi}^{k(1-\xi_L)} \prod_{s=1}^{k} \pi_{t+s-1}^{\xi_L}$$

That is, households who cannot reoptimize wages update them by considering a weighted average of past CPI inflation π_{t-1} and the inflation target $\bar{\pi}$ set by the monetary authority.

Household j who has the chance to reoptimize its wage at period t chooses $\widetilde{W}_t(j)$ to maximize the lifetime utility (2) subject to the labor demand (1) and the updating rule for the nominal wage (8). The first order condition can be written as

$$\mathbf{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta_L)^k\left(\left(1-\frac{1}{\nu_L}\right)\frac{\widetilde{W}_t(j)\Gamma_{W,t}^k}{P_{t+k}}\left(C_{t+k}-\gamma h C_{t+k-1}\right)^{-1} - \chi_H \widetilde{H}_{t+k}(j)^{\tau}\right)\widetilde{H}_{t+k}(j)\right] = 0$$

2.2 Domestic Firms

There is a continuum of monopolistically competitive Home goods producing firms indexed by $i \in [0,1]$. Home goods producers have identical CES production functions that use labor service and oil as inputs:

$$Y_{H,t}(i) = \zeta_{A,t}\left[(1-\alpha)^{\frac{1}{\phi_y}}\left(\gamma^t N_{H,t}(i)\right)^{1-\frac{1}{\phi_y}} + \alpha^{\frac{1}{\phi_y}} O_{H,t}(i)^{1-\frac{1}{\phi_y}}\right]^{\frac{\phi_y}{\phi_y - 1}} \tag{9}$$

where $N_{H,t}(i)$ is the labor input hired by firm i, $O_{H,t}(i)$ is oil used in production of the variety i, and $\zeta_{A,t}$ represents a stationary productivity shock in the Home goods sector that is common to all firms. ϕ_y governs the elasticity of substitution between labor and oil in production and α denotes the share of oil. While firms behave monopolistically in the goods market, they buy inputs competitively in the factor market. Given input prices W_t and $P_{o,t}$, the cost minimization gives us

$$\left(\frac{W_t}{\gamma^t P_{o,t}}\right)^{\phi_y} = \frac{1-\alpha}{\alpha}\frac{O_{H,t}(i)}{\gamma^t N_{H,t}(i)} \tag{10}$$

Furthermore, from (9) and (10) we can derive the nominal marginal cost of production

$$MC_t = \frac{1}{\zeta_{A,t}} \left[(1-\alpha) \left(\frac{W_t}{\gamma^t} \right)^{1-\phi_y} + \alpha P_{o,t}^{1-\phi_y} \right]^{\frac{1}{1-\phi_y}} \tag{11}$$

which implies that the marginal cost of production is constant and the same across all firms.

Price setting is again subject to a nominal rigidity à la Calvo (1983) and Yun (1996). In each period only a fraction $(1-\theta_H)$ of firms can fully optimize their output prices. The remaining firms of fraction θ_H can only adjust the price according to a partial indexation scheme:

$$P_{H,t+k}(i) = \Gamma_{H,t}^k P_{H,t}(i)$$

where

$$\Gamma_{H,t}^k = \bar{\pi}^{k(1-\xi_H)} \prod_{s=1}^{k} \pi_{H,t+s-1}^{\xi_H}$$

where $\pi_{H,t} = P_{H,t}/P_{H,t-1}$. For firms who do not have chances to reoptimize prices, the price adjustment factor is a weighted average between the past inflation of Home goods $\pi_{H,t-1}$ and the target inflation rate $\bar{\pi}$. ξ_H captures the degree of indexation in the economy.

For firm i who has opportunity to reoptimize the output price, it chooses $\widetilde{P}_{H,t}(i)$ to maximize the expected profit

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta_H^k \Lambda_{t,t+k} \left(\Gamma_{H,t}^k \widetilde{P}_{H,t}(i) - MC_{H,t+k} \right) Y_{H,t+k}(i) \right]$$

subject to the demand function:[3]

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu_H} Y_{H,t}$$

The first order condition is

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta_H^k \Lambda_{t,t+k} Y_{H,t+k}(i) \left(\Gamma_{H,t}^k \widetilde{P}_{H,t}(i) - \frac{\nu_H}{\nu_H - 1} MC_{t+k} \right) \right] = 0$$

Note that $\Lambda_{t,t+k}$ is the marginal value of a unit of the consumption good to households, which is treated as exogenous by the firm. Hence, we have

$$\Lambda_{t,t+k} = \frac{\beta^k \lambda_{t+k}}{\lambda_t} = \beta^k \frac{P_t}{P_{t+k}} \left(\frac{C_t - \gamma h C_{t-1}}{C_{t+k} - \gamma h C_{t+k-1}} \right)$$

[3]Different varieties of Home goods are aggregated by CES technology. That is,

$$Y_{H,t} = \left(\int_0^1 Y_{H,t}(i)^{\frac{\nu_H - 1}{\nu_H}} di \right)^{\frac{\nu_H}{\nu_H - 1}}$$

The aggregated Home goods are either consumed domestically $C_{H,t}$ or exported to foreign economy $C_{H,t}^*$. Moreover, the aggregate price of Home goods is derived as

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\nu_H} di \right)^{\frac{1}{1-\nu_H}}$$

Given the price charged by a firm i, its profit is given by

$$\Pi_t(i) = P_{H,t}(i)Y_{H,t}(i) - W_t N_{H,t}(i) - P_{o,t}O_{H,t}(i)$$

2.3 The Foreign Economy

The foreign demand for Home goods is given by

$$C^*_{H,t} = \omega^*_H \left(\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\phi^*} C^*_t$$

where ω^*_H denotes the import share in the consumption basket of foreign agents and ϕ^* captures the intratemporal elasticity of substitution between Foreign and Home goods in the foreign economy.[4] The foreign consumption C^*_t is exogenously given and follows a stochastic process.

We assume the law of one price (LOP) holds for Home goods. That is, the domestic firms cannot discriminate across markets in terms of prices. This also holds for imported Foreign goods except oil.

$$P^*_{H,t} = \frac{P_{H,t}}{e_t}, \qquad P_{F,t} = e_t P^*_{F,t}$$

We can define the real exchange rate as:

$$s_t = \frac{e_t P^*_{F,t}}{P_t} \tag{12}$$

Note that the price of consumption bundle of foreign agents is dominated by $P^*_{F,t}$ rather than P^*_t because home country is assumed to be a small open economy; therefore the import share of the foreign economy ω^*_H is negligible.

The domestic real price of oil is given by

$$\frac{P_{o,t}}{P_t} = s_t \frac{P^*_{o,t}}{P^*_{F,t}} \zeta_{o,t} \tag{13}$$

where $P^*_{o,t}$ is the foreign currency price of oil abroad. The pass-through of oil prices is not perfect in the sense that $\zeta_{o,t}$ signifies the deviations from the law of one price in the oil price. This deviation $\zeta_{o,t}$ is assumed to follow a stochastic process. The international oil price $P^*_{o,t}$ also follows a stochastic process.

[4] We implicitly assume that the representative household in the foreign economy consumes a basket of Home and Foreign goods via a CES aggregator, but oil is excluded from the consumption bundle.

$$C^*_t = \left[(\omega^*_H)^{\frac{1}{\phi^*}} \left(C^*_{H,t}\right)^{\frac{\phi^*-1}{\phi^*}} + (1-\omega^*_H)^{\frac{1}{\phi^*}} \left(C^*_{F,t}\right)^{\frac{\phi^*-1}{\phi^*}}\right]^{\frac{\phi^*}{\phi^*-1}}$$

2.4 Monetary Authority

Monetary policy is described by an interest rate feedback rule of the form

$$R_t = R_{t-1}^{\rho_R} \overline{R}_t^{1-\rho_R} \exp(\epsilon_{R,t}) \tag{14}$$

where $\epsilon_{R,t}$ is a monetary policy shock and $\overline{R}_t$ is the nominal target interest rate. Monetary authority sets its target in responding to inflation and deviations of output growth rate from its trend:

$$\overline{R}_t = \bar{r}\bar{\pi} \left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_\pi} \left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_y}$$

where $\bar{r}$ is real interest rate at the steady state.

2.5 Aggregation and Equilibrium

We abstract from the government spending. We further assume that the government runs a balanced budget every period passively:

$$\int_0^1 \Big(M_t(j) - M_{t-1}(j)\Big)\, dj + \int_0^1 T_t(j)\, dj = 0$$

The goods market and the labor market clears

$$Y_{H,t} = C_{H,t} + C_{H,t}^* \tag{15}$$

$$H_t = \int_0^1 N_t(i)\, di \tag{16}$$

Combining equilibrium conditions, the budget constraint of the government and the aggregate budget constraint of households, we get the following dynamics of foreign bond holdings:

$$\frac{e_t B_t^*}{R_t^* \Theta\left(\frac{e_t B_t^*}{P_{X,t} X_t}\right)} = e_t B_{t-1}^* + P_{X,t} X_t - P_{M,t} M_t$$

As noted before, imports consist of oil and consumption-dedicated Foreign goods while domestically produced goods are only export of the economy. Therefore, the aggregate nominal value of exports and imports are defined as

$$P_{X,t} X_t = P_{H,t} C_{H,t}^*$$

$$P_{M,t} M_t = s_t P_t C_{F,t} + e_t P_{o,t}^* O_t$$

where X_t and M_t denote exports and imports, respectively. Total oil imports are the sum of oil for direct consumption and that for production, $O_t = O_{C,t} + O_{H,t}$. We can also write the nominal GDP as

$$P_{Y,t} Y_t = P_t C_t + P_{X,t} X_t - P_{M,t} M_t$$

where $P_{Y,t}$ denotes the implicit output deflator.

2.6 Steady State and Log-linearization

The model is equipped with deterministic trend. Hence, we first detrend variables to define the steady state. All price and wage variables are written as relative prices to the Home CPI P_t. Real variables with trend are to be divided by γ^t. At the steady state after detrending, all relative prices and the real wage are normalized to one for computational convenience. The details of steady state and log-linearization are given in Appendix.

3 Estimation Methods

This section consists of three parts. First, we briefly discuss how to cast the linearized model into an estimable representation. With the state space representation, we can estimate the model within Bayesian estimation frameworks, so called, Metropolis-Hastings algorithm with Kalman filter. See An and Schorfheide (2007) for a review. Next, we introduce the DSGE-VAR framework developed by Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007). DSGE-VARs are useful to check how DSGE models are misspecified. This framework tries to find out the optimal weight between two approaches, DSGEs and VARs, that fit data best. And we explain the data used in our analysis.

3.1 Representations

The log-linearized model given in Appendix contains the rational expectation terms, and it is not easy to deal with these terms directly. Several solution algorithms of the linearized rational expectations system are available, for instance, Blanchard and Kahn (1980) , Uhlig (1999), and Sims (2002). With the help of the solution algorithm, the log-linearized system can be written as autoregressive model in a vector of variables:

$$\mathbf{s}_t = \Phi^{(s)}(\theta)\mathbf{s}_{t-1} + \Phi^{(\varepsilon)}(\theta)\varepsilon_t \tag{17}$$

where

$$\mathbf{s}_t = \left(\hat{b}_t, \hat{c}_t, \hat{y}_t, \hat{x}_t, \hat{o}_t, \widehat{pr}_{H,t}, \hat{w}_t, \hat{\pi}_t, \hat{\pi}_{H,t}, \hat{\pi}_{Z,t}, \Delta\hat{e}_t, \hat{s}_t, \hat{R}_t, \hat{R}^*_t, \widehat{pr}^*_{o,t}, \hat{\pi}^*_t, \hat{c}^*_t, \hat{\zeta}_{A,t}, \hat{\zeta}_{o,t}, \epsilon_{R,t}, \epsilon^*_{R,t}\right)$$
$$\varepsilon_t = (\epsilon_{R,t}, \epsilon_{R^*,t}, \epsilon_{o^*,t}, \epsilon_{\pi^*,t}, \epsilon_{C^*,t}, \epsilon_{A,t}, \epsilon_{o,t})$$

and $\Phi^{(s)}(\theta)$ and $\Phi^{(\varepsilon)}(\theta)$ are conformable matrices whose values are dependent on the values of DSGE model parameters θ. Given that some of variables in $\mathbf{s}_t$ is not observable, we can treat (17) as the transition equation of a state space representation. Once we define a vector of observables $\mathbf{y}_t$ we can set up measurement equations:

$$\mathbf{y}_t = \Theta^{(0)}(\theta) + \Theta^{(s)}(\theta)\mathbf{s}_t \tag{18}$$

More specifically, we assume that the time period t in the model corresponds to one quarter and that the following observations are available for estimation: quarter-to-quarter per capita GDP growth rate (YGR), annualized nominal interest rate (INT), annualized quarter-to-quarter core CPI inflation rate ($CoreINFL$), annualized quarter-to-quarter hourly wage inflation ($WageINFL$), quarter-to-quarter nominal exchange rate depreciation ($FXGR$), international oil prices relative to domestic price level (RPO), and quarter-to-quarter growth rate of oil imports (OGR). Then we can write

$$\begin{aligned}
YGR_t &= \gamma^{(Q)} + 100\,(\hat{y}_t - \hat{y}_{t-1}) \\
INT_t &= \pi^{(A)} + r^{(A)} + 4\gamma^{(Q)} + 400\hat{R}_t \\
CoreINFL_t &= \pi^{(A)} + 400\hat{\pi}_{Z,t} \\
WageINFL_t &= \pi^{(A)} + 4\gamma^{(Q)} + 400\,(\hat{\pi}_t + \hat{w}_t - \hat{w}_{t-1}) \\
FXGR_t &= 100 * \Delta\hat{e}_t \\
RPO_t &= \widehat{pr}^*_{o,t} \\
OGR_t &= \gamma^{(Q)} + 100\,(\hat{o}_t - \hat{o}_{t-1})
\end{aligned}$$

where $\gamma^{(Q)}$, $\pi^{(A)}$, and $r^{(A)}$ are related to the steady states of the model economy as follows:

$$\gamma = \exp\left(\frac{\gamma^{(Q)}}{100}\right), \quad \beta = \exp\left(-\frac{r^{(A)}}{400}\right), \quad \bar{\pi} = \exp\left(\frac{\pi^{(A)}}{400}\right)$$

Now define the parameter of the linearized model economy

$$\theta = \Big(\alpha, \tau, h, \kappa, \phi_Z, \phi^*, \phi_C, \phi_Y, \theta_H, \theta_L, \xi_H, \xi_L, \psi_\pi, \psi_Y, \rho_R, \gamma^{(Q)}, r^{(A)}, \pi^{(A)}, \omega_F, \omega_o,$$
$$\rho_A, \rho_o, \rho_{o^*}, \rho_{R^*}, \rho_{\pi^*}, \rho_{C^*}, \sigma_R, \sigma_A, \sigma_o, \sigma_{o^*}, \sigma_{R^*}, \sigma_{\pi^*}, \sigma_{C^*}, \nu_L, C/Y, X/Y\Big)$$

then the system matrices, $\Phi^{(s)}$, $\Phi^{(\varepsilon)}$, $\Theta^{(0)}$, and $\Theta^{(\varepsilon)}$, in the state space representation, (17) and (18), are given as highly nonlinear functions of the DSGE model parameters θ.[5]

3.2 Evaluation of DSGE Models: DSGE-VARs

While DSGE models are popular among the economists because of their microfoundations, the empirical performance is not so successful until Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). On the contrary, VARs are widely used in empirical macroeconomics and considered as benchmarks for evaluating dynamic economies due to better fit of the data and forecasting power. Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007) investigate possible connections between DSGE models and VARs. There are two approaches for this connection.

The first approach targets to improve the performance of VARs. Even though empirical performances of VARs are fair enough, a practical question remains when they are brought to the impulse response analysis, that is, the lack of identification. Many identification schemes have

[5]Note that ν_L, C/Y, X/Y are not identified in this log-linearized economy.

been suggested in the literature, for example, Sims (1980), Blanchard and Quah (1989), Faust (1998), and Uhlig (1998) among others, but most of them are still controversial. Del Negro and Schorfheide (2004) focuses on DSGE models that can provide structural restrictions on VAR parameters. Rather than fully restricting VAR parameters as in the structural VARs (SVARs), they use information from DSGE models to construct the prior distributions for VAR parameters. To see this, let us consider a VAR representation of a linearized DSGE model. For most of cases, VARs can only approximate DSGE models with appropriate truncation of lags at order p:

$$\mathbf{y}_t = \Phi_0^*(\theta) + \Phi_1^*(\theta)\mathbf{y}_{t-1} + \cdots + \Phi_p^*(\theta)\mathbf{y}_{t-p} + u_t \tag{19}$$

with $\mathbf{E}_{t-1}(u_t) = 0$ and $\text{var}_{t-1}(u_t) = \Sigma^*(\theta)$. We denote the dimension of $\mathbf{y}_t$ by m and define a $k \times 1$ vector $x_t = \left(1, \mathbf{y}_{t-1}, \cdots, \mathbf{y}_{t-p}\right)'$ and the coefficient matrix $\Phi^* = \left(\Phi_0^*(\theta), \Phi_1^*(\theta), \cdots, \Phi_p^*(\theta)\right)'$. Furthermore, define the $T \times m$ matrices Y and U composed of rows y_t' and u_t', and the $T \times k$ matrix X with rows of x_t'. Then restricted VAR parameters $\Phi^*(\theta)$ and $\Sigma^*(\theta)$ are derived from DSGE parameters θ as

$$\begin{aligned} \Phi^*(\theta) &= \Gamma_{xx}^{-1}(\theta)\Gamma_{xy}(\theta) \\ \Sigma^*(\theta) &= \Gamma_{yy}(\theta) - \Gamma_{yx}(\theta)\Gamma_{xx}^{-1}(\theta)\Gamma_{xy}(\theta) \end{aligned}$$

where $\Gamma_{xx}(\theta) = \mathbf{E}^{DSGE}(x_t x_t')$, $\Gamma_{xy}(\theta) = \mathbf{E}^{DSGE}(x_t y_t')$, and $\Gamma_{yy}(\theta) = \mathbf{E}^{DSGE}(y_t y_t')$ are (approximately) theoretical covariances of observables, all of which can easily be derived from the state space representation of linearized DSGE model (17) and (18). Now consider a VAR(p) model of the observables $\mathbf{y}_t$:

$$\mathbf{y}_t = \Phi_0 + \Phi_1\mathbf{y}_{t-1} + \cdots + \Phi_p\mathbf{y}_{t-p} + u_t \tag{20}$$

with $\mathbf{E}_{t-1}(u_t) = 0$ and $\text{var}_{t-1}(u_t) = \Sigma$. We assume the normal likelihood of VAR parameters $\mathcal{L}(\Phi, \Sigma)$, that is, the error terms u_t are normally distributed. In this case, the conjugate prior for VAR parameters from the linearized DSGE model would be normal-inverted Wishart family:

$$\Sigma|\theta, \lambda \sim IW\left(\lambda T \Sigma^*(\theta), \lambda T - k, n\right) \tag{21}$$

$$\text{vec}(\Phi)|\Sigma, \theta, \lambda \sim N\left(\text{vec}\left(\Phi^*(\theta)\right), \frac{1}{\lambda T}\left[\Sigma^{-1} \otimes \Gamma_{xx}(\theta)\right]^{-1}\right) \tag{22}$$

This prior distribution can be interpreted as a posterior calculated from a sample of λT observations generated from the DSGE model with parameter θ. This interpretation stems from the Bayesian linear regression with normal-inverted Wishart conjugate framework (See Zeller, 1971, for details). Note that the posterior distribution of VAR parameters Φ and Σ is

$$\Sigma|Y, \theta, \lambda \sim IW\left((1+\lambda)T\widetilde{\Sigma}(\theta), (1+\lambda)T - k, n\right) \tag{23}$$

$$\text{vec}(\Phi)|Y, \Sigma, \theta, \lambda \sim N\left(\text{vec}\left(\widetilde{\Phi}(\theta)\right), \Sigma \otimes \left(\lambda T \Gamma_{xx}(\theta) + X'X\right)^{-1}\right) \tag{24}$$

where

$$\widetilde{\Phi}(\theta) = \widetilde{\Gamma}_{xx}^{-1}(\theta)\widetilde{\Gamma}_{xy}(\theta), \qquad \widetilde{\Sigma}(\theta) = \widetilde{\Gamma}_{yy}(\theta) - \widetilde{\Gamma}_{yx}(\theta)\widetilde{\Gamma}_{xx}^{-1}(\theta)\widetilde{\Gamma}_{xy}(\theta)$$

and

$$\widetilde{\Gamma}_{xx} = \frac{\lambda}{1+\lambda}\Gamma_{xx}(\theta) + \frac{1}{1+\lambda}\frac{X'X}{T}$$
$$\widetilde{\Gamma}_{xy} = \frac{\lambda}{1+\lambda}\Gamma_{xy}(\theta) + \frac{1}{1+\lambda}\frac{X'Y}{T}$$
$$\widetilde{\Gamma}_{yy} = \frac{\lambda}{1+\lambda}\Gamma_{yy}(\theta) + \frac{1}{1+\lambda}\frac{Y'Y}{T}$$

Hence, the larger the weight λ of the prior, the close the posterior mean of the VAR parameters is to $\Phi^*(\theta)$ and $\Sigma^*(\theta)$, the values that respect the cross-equation restrictions of the DSGE model. As λ shrinks down to $(k+m)/T$, the posterior mean is close to the OLS estimate. The marginal posterior density of θ given the prior tightness λ can be obtained through the marginal likelihood $p(Y|\theta,\lambda)$ whose expression can be found in Del Negro and Schorfheide (2004):

$$p(\theta|Y,\lambda) \propto p(Y|\theta,\lambda)p(\theta) \tag{25}$$

Given that the joint posterior distribution of VAR and DSGE model parameters can be factorized as

$$p(\Phi,\Sigma,\theta|Y,\lambda) = p(\Phi,\Sigma|Y,\theta,\lambda)\, p(\theta|Y,\lambda) \tag{26}$$

we can draw from the joint posterior (26) using Metropolis-Hastings algorithm with (23), (24), and (25).

The second approach of DSGE-VARs tackles misspecification issues of DSGE models. As noted before, DSGE models are well accepted among the economists since their modeling is based on economic theory and impulse response analysis is straightforward. However, restrictions derived from DSGE models are often too tight to match the observables, and hence, the empirical performance is usually far from satisfactory. Del Negro, Schorfheide, Smets, and Wouters (2007) point out that the data generating process of a VAR is decomposed into the DSGE model part and its possible misspecifications, and this misspefication can be modeled in a Bayesian framework. Let $\Phi^{\triangle}$ and $\Sigma^{\triangle}$ capture deviations from the restrictions $\Phi^*(\theta)$ and $\Sigma^*(\theta)$. Then we can write

$$\Phi = \Phi^*(\theta) + \Phi^{\triangle}, \qquad \Sigma = \Sigma^*(\theta) + \Sigma^{\triangle}$$

For Bayesian analysis, we need to specify the prior distribution for DSGE model parameters θ and the misspecification matrices, $\Phi^{\triangle}$ and $\Sigma^{\triangle}$. It is rather convenient to specify priors in terms of Φ and Σ conditional on θ. Using the information-theoretic criterion, we can show that these prior densities are well approximated by normal-inverted Wishart conjugate as (21) and (22). Once we have the priors, the rest of analysis should not be different from the previous discussion. In this approach, we should note that the prior tightness of the DSGE model, $\lambda \in ((k+m)/T, \infty)$, signifies the degrees of correct specification. If we can find out the "optimal" value, namely $\hat{\lambda}$, which will be discussed later, it can be used to evaluate the specification of the DSGE model. In short, the larger $\hat{\lambda}$ is, the smaller is the misspecification of the DSGE model and a lot of weight should be placed on its implied restrictions.

In Bayesian analysis, the posterior odds ratio plays a key role in model selection. We can assign prior probabilities on competing models and assess alternative specifications based on their posterior odds. We note that DSGE-VARs are indexed by the prior weight on the DSGE model. Hence, the model selection in this framework is to choose an optimal value of λ in a certain metric. For simplicity, we employ a grid $\Lambda = \{\lambda_1, \ldots, \lambda_q\}$, rather than a continuous parameter space. If we assign equally weighted prior probabilities on each λ, the posterior odds are simply written as the ratio of marginal likelihood functions of λ, $p(Y|\lambda)$. Therefore, the model selection problem in this framework equals to maximizing the marginal likelihood function of λ:

$$\hat{\lambda} = \underset{\lambda \in \Lambda}{\operatorname{argmax}}\ p(Y|\lambda)$$

The marginal likelihood of λ is obtained from the following relationship

$$p(Y|\lambda) = \int p(Y|\theta, \lambda) p(\theta)\, d\theta$$

and the numerical calculation is performed using Geweke's (1999) modified harmonic mean estimator.

When $\hat{\lambda}$ is chosen according to the posterior odds criterion, a comparison between DSGE-VAR($\hat{\lambda}$) and DSGE model impulse responses can reveal important insights about the misspecification of the DSGE model. While DSGE model impulse response is well defined, impulse responses of DSGE-VAR($\hat{\lambda}$) needs careful treatment. To obtain a proper impulse response, we should identify the orthogonal structural shock ε_t from the reduced form shock u_t. In general, this relationship is written as

$$u_t = LR\varepsilon_t$$

where L is the Choleski decomposition of Σ, that is, a lower triangular matrix, and R is an orthogonal matrix, $RR' = I$. The Choleski decomposition is unique so that we can pin down L matrix, but choosing R is controversial, namely identification problem in VARs. Del Negro and Schorfheide (2004) suggests to construct R as follows. Let $A_0(\theta)$ be the initial impact of ε_t on $\mathbf{y}_t$ in the DSGE model. In our state space representation, (17) and (18), this initial impact can be decomposed by QR factorization (which is unique)

$$A_0(\theta) = \Theta^{(s)}(\theta)\Phi^{\varepsilon}(\theta) = L^*(\theta)R^*(\theta)$$

where $L^*(\theta)$ is lower triangular and $R^*(\theta)$ is orthogonal. Now we can identify the structural shock by letting

$$u_t = LR^*(\theta)\varepsilon_t$$

This rotation matrix $R^*(\theta)$ is chosen so that the DSGE's and the DSGE-VAR's impulse responses approximately coincide in case of no misspecification.

3.3 Data

Most of data were obtained through KOSIS (Korean statistical information service)[6] maintained by Korea National Statistical Office and ECOS (Economic statistics system)[7] maintained by the Bank of Korea. Seasonally adjusted real GDP is divided by population 15 years and older and its growth rate is calculated as 100 times the first difference in logs. The interest rate is the overnight call rate. The core inflation rate is calculated from core CPI as 400 times the first difference in logs. The nominal hourly wage is obtained by dividing total wage by total hours worked and its inflation is again calculated as 400 times the first difference in logs. The nominal exchange rate depreciation is calculated as 100 times the first difference in logs of the effective exchange rate published by the Bank of International Settlement (BIS).[8] The international oil price relative to domestic price level is obtained by dividing WTI crude oil spot price by CPI and being normalized after taking logs. Finally, the crude oil import is obtained from Korea Petroleum Association[9] and then seasonally adjusted by X12 method available from EViews. Per capita term is obtained by dividing it by population 15 years and older, and then quarter-to-quarter growth rate is calculated as 100 times the first difference in logs. Data are available for 1993:Q2–2008:Q4.

4 Empirical Results

We begin this section by explaining the specification of prior distributions of structural parameters of the DSGE model. In the following discussion on the "optimal" DSGE prior weight, we also consider two variants of our baseline DSGE model. One lacks oil in consumption basket and the other excludes oil from inputs of production. We discuss how a fit changes as we move away from our baseline model. We also look into impulse response functions from our DSGE models and compare them with those from DSGE-VARs. Finally, we investigate the behavior of oil price pass-through as the international crude oil prices surges in mid-2000s.

In what follows, we use DYNARE for estimation of both DSGE models and DSGE-VARs. For each specifications, we generate 125,000 draws from posterior distributions and the first 25,000 draws are discarded for convergence of Markov-chain.

4.1 Prior Distribution

Prior distribution in Bayesian analysis plays an important role in the estimation of DSGE models. By specifying them, we express our own view on plausible parameter values. Actually this process re-weight the information contained in the data that are used in actual estimation. That

[6] http://www.kosis.kr
[7] http://ecos.bok.or.kr
[8] http://www.bis.org/statistics/eer
[9] http://www.petroleum.or.kr

is, we can incorporate extra information that is possibly missing in estimation samples and is developed in the related literature.

To begin with, we calibrate several parameter values that are not identified in our representation. First, the substitution elasticity across differentiated labor ν_L that governs wage markup is set to 9 as in Medina and Soto (2005). The price markup parameter ν_H is not present in our linearized model. Noting that our model abstracts from capital and government spending, we set the steady state consumption-output ratio, C/Y, as 0.98, which stems from the average ratio of the sum of consumption, investment, and government spending to GDP in our sample. The steady state export share, X/Y, is 0.4 according to our sample. From these two ratios, we can derive other big ratios using steady state relationships reported in Appendix.

Table 1 lists the marginal prior distributions for the structural parameters of the DSGE model. In general, the prior distributions used in this study are quite diffuse. As usual, the rule of thumb in choosing the distribution family for each parameter is the shape of the support. Parameters that have limits on both end, usually confined between 0 and 1, follow the beta distribution. For those with positive unbound support we specify the gamma distribution, but standard deviations of shock processes follow inverse gamma distributions. Unbounded parameters are specified as normal distributions. The oil share in production α has mean 0.3. The model abstracts from capital input and we assign the capital share as oil share. With standard deviation 0.1, 90% coverage is [0.15,0.48]. Inverse of intertemporal substitution elasticity of labor τ has mean 1 and standard deviation 0.75 whose 90% coverage is [0.15,2.46]. Without preference shock as in our model, this parameter is often estimated quite small and even negative with aggregate data. Due to lack of information on the habit persistence parameter h, it is centered at 0.5 and standard deviation 0.2 to have [0.17,0.83] as 90% coverage. Elasticity between Home and Foreign goods in core consumption ϕ_Z has relatively low mean 0.3 and it is roughly around the calibrated value in the Bank of Korea model (BOKDSGE) by Kang and Park (2007). Its counter-part in foreign consumption is set to 1. The elasticity between oil and core consumption is also low as 0.33 since there is almost no substitute for oil in the Korean economy, especially when it comes to fuel for transportation. The elasticity between oil and labor input of production is not obvious and it is set to 0.5. For more discussion of the estimates of the elasticity of energy or oil with other inputs, see Backus and Crucini (2000). Calvo rigidity parameters for price θ_H and wage θ_L are equally set to have mean 0.7. This value implies that prices and wages are reset every 3 quarters on average. Standard deviations for θ_H and θ_L are 0.1 and 0.15, respectively. Hence, 90% coverage imply that prices are reset between 2.1 and 6.8 quarters and wages 1.7 and 11.9 quarters. Price (ξ_H) and wage (ξ_L) indexation to past inflation are all centered 0.5 and have common standard deviations 0.2. Monetary policy parameters ψ_π and ψ_y is set to have means from Taylor's (1993) values, 1.5 and 0.5, and 90% coverage, [1.19,1.84] and [0.17,0.97], respectively. We further specify weights on Foreign goods in core consumption ω_F and on oil in consumption ω_o. They are centered at 0.35 and 0.1, respectively. Persistence of shocks, $(\rho_A, \rho_o, \rho_{o^*}, \rho_{R^*}, \rho_{\pi^*}, \rho_{C^*})$ have the same specification, mean 0.75 and standard deviation 0.15.

4.2 Model Selection and DSGE Prior Weight

The main purpose of DSGE-VARs is to evaluate the (mis-)specification of DSGE models under consideration. To begin with, however, we investigate a direct estimation of structural parameters of our baseline model. Bayesian estimations of linearized DSGE models trace back to DeJong, Ingram, and Whiteman (2000), Landon-Lane (1998), and Schorfheide (2000), and they use Markov-chain Monte Carlo (MCMC) algorithm for posterior simulator while Kalman filter provides likelihood computations. As noted previously, a unified framework for model selection within Bayesian framework, the posterior odds ratio, makes this approach quite popular. Here we consider two restrictions on the baseline model described in Section 2. In our baseline economy, the entire volume of oil in domestic use is imported from foreign country and a fraction of oil imports is directly consumed among households. The first restricted model tackles this point and assumes that oil is not included in consumption basket (No Oil Consumption). On the contrary, oil is not used for production in the second restricted model (No Oil in Production). In this case, the production function depends only on labor input with decreasing returns to scale since the baseline model abstracts from capital inputs in production. The first row of Table 2 and right end points of Figure 1 reports the log marginal likelihood of data (the log marginal data density, $p(Y)$) of three models under consideration. The No Oil Consumption model attains the highest marginal data density, followed by the baseline and No Oil in Production models. That is, No Oil Consumption model best describes the data if these models are assigned the same prior probabilities. Actually, the marginal data density penalizes larger models, and hence to use it for model selection works just like to use information criteria, especially Schwarz (Bayesian) information criterion. This connection is well illustrated in Kass and Rafterty (1995).

Now we turn our attention to DSGE prior weight, that is, DSGE-VARs. Due to short sample periods we restrict the lags in VARs to 2. Note that λ should take values in $(m(p+1)/T, \infty)$ for DSGE-VARs to be estimable. DSGE models have VAR representations with the truncation at a particular lag order. This approximate VAR representation distinct DSGE-VARs from DSGE models even with infinite weight on DSGE priors, DSGE-VAR(∞). This discrepancy is obviously seen from differences between the first and the second rows in Table 2. Steep slopes in the last segments of lines in Figure 1 also signify relatively short lag orders.

For each of three specifications, we try various values for λ, $[0.4, 0.5, 0.75, 1, 1.25, 1.5, 2, \infty]$, and depict results in Table 2 and Figure 1. As usual, Figure 1 shows inverted U-shape curves of log marginal likelihood functions in λ. All three cases attain highest values when $\lambda = 0.5$. In normalized weight, $\lambda/(1+\lambda)$, DSGE models have roughly one third to achieve best fits in our model specification. While the baseline and the No Oil Consumption models behave similarly in terms of marginal likelihoods, No Oil in Production model shows a little bit different pattern. It attains the highest marginal likelihood at "optimal" DSGE prior weight but the lowest for the state space representation of the model. This result is quite confusing. However, we can argue that oil is actually important as an input of production, rather than as direct consumption. The changes in the marginal likelihood are much bigger when we close down productive motive of oil imports.

4.3 Posterior Estimates

Table 3 reports posterior estimates from the DSGE model and DSGE-VAR($\hat{\lambda}$). We first focus on estimates from the baseline model. In DSGE estimation, most of parameters show information gain through likelihood, that is, prior and posterior distributions are different. A couple of parameters, α and ψ_{pi}, have roughly the same prior and posterior means. However, 90% coverage shrinks as they move to posterior distributions, which implies that likelihoods bring on some extra information. The elasticity between oil and labor input of production ϕ_y attains very low posterior mean. This implies that oil and labor are not substitutable in production and hints a huge difference in marginal likelihoods between the baseline and No Oil in Production models. The model displays relatively high degrees of price θ_H and wage θ_L rigidities, 0.959 and 0.847, with 24.4 and 6.5 quarters of duration, respectively. The price rigidity is too high, however, its estimate reduces down to 12 quarters with DSGE-VAR($\hat{\lambda}$). The estimated slope of Phillips curve, $\beta/(1+\beta\xi_H)$, is 0.59, both for DSGE and DSGE-VAR($\hat{\lambda}$), and it is quite close to the Bank of Korea's calibration, 0.58. The weight on oil in consumption basket ω_o is estimated as 0.131. Persistence parameters are estimated high except one. The posterior mean of the persistence for foreign inflation shock ρ_{π^*} is 0.147. This estimate is even lower for DSGE-VAR($\hat{\lambda}$).

As pointed out in Del Negro and Schorfheide (2004), information about structural parameters of the DSGE model is gathered more slowly as the DSGE prior weight loosens, that is, becomes smaller. When λ is moving away from infinity priors on VAR parameters becomes less tighter. Therefore, we can expect that the posterior of DSGE-VAR($\hat{\lambda}$) is closer to the prior than the posterior distribution of the DSGE model. For many parameters it is verifiable, especially for substitution elasticity between oil and labor input of production ϕ_y and some of persistence parameters.

We also list posterior estimates for No Oil in Production model in the right panel of Table 3. We can achieve this specification by letting the oil share in production α be zero. In this case, however, the substitution elasticity between oil and labor input of production ϕ_y is not identified, so we fix it to be 0.5. Most of posterior distributions of structural parameters are different from those of the baseline model. The intertemporal substitution elasticity of labor τ becomes more reasonable. It even becomes comparable in DSGE-VAR($\hat{\lambda}$) with estimates from micro labor literatures. The habit persistence h and the wage rigidity θ_L decrease substantially. After we close down the oil use in production, the substitution elasticity between oil and core consumption ϕ_C and weight on oil consumption compared to core consumption both decreases. Households may want to reduce the exposure to oil shocks by consuming less oil in their consumption basket once it is not easy to substitute oil with other consumption material. Since oil is not used in production the technology shock persistence is estimated higher. Now let us look at parameters on the pass-through of oil price $\zeta_{o,t}$. While the persistence ρ_o decreases slightly both for DSGE and DSGE-VAR($\hat{\lambda}$), the standard deviation of the shock becomes bigger.

4.4 Impulse Response Functions

As seen previously, DSGE-VAR($\hat{\lambda}$) attains higher marginal likelihood than other two extremes: DSGEs and VARs. Basically, the DSGE-VAR($\hat{\lambda}$) is a Bayesian VAR (BVAR) with optimally weighted prior from the DSGE model. Hence, we can use it as the benchmark in evaluating the performance of the DSGE model. As is often the case with indirect inferences (e.g., Christiano, Eichenbaum, and Evans, 2005), the performance of a DSGE model is checked by comparing impulse response functions, one from a VAR and another from the DSGE model.

Figure 2(a) depicts impulse responses with respect to a monetary policy shock in the baseline economy. The posterior mean responses of the DSGE (solid line) and DSGE-VAR($\hat{\lambda}$) (dotted line) are given with 90% coverage band (gray area) for DSGE-VAR($\hat{\lambda}$). Responses of real international price of oil are omitted because this observable is purely exogenous and it responds only to its own shock $\widehat{pr}_{o^*,t}$ in the DSGE model. We can see that responses from the DSGE model trace out those of DSGE-VAR($\hat{\lambda}$). Most of responses from the baseline DSGE model show hump-shaped and prolonged effects, but these effects are quantitatively small compared to those from the DSGE-VAR($\hat{\lambda}$). This quantitative discrepancies are originated from relatively low value of $\hat{\lambda}$, that is, 0.5. Some initial responses have wrong signs, such as wage inflation and exchange rate depreciation. Figure 2(b) shows responses to a technology shock in the baseline economy. Again, response from the DSGE model mimics well those from DSGE-VAR($\hat{\lambda}$), but they are quantitatively weak. Likewise, initial responses of wage inflation and oil import have wrong sign. Hence, this exercise shows that the baseline DSGE model is at odds with the data in terms of the wage inflation.

Figure 3 reports the entire set of impulse responses of the baseline economy. In particular, the interest rate and oil import responses to an international oil price shock are located upper bound of 90% coverage. For comparison we also depict impulse response functions for No Oil in Production model in Figure 4.

4.5 Pass-Through of Oil Price Shocks

The baseline model for our analysis is constructed so that the exchange rate pass-through for all but oil is perfect. However, there is a discrepancy between international oil price and domestic oil price as in (13) and deviations from the law of one price is modeled as a stochastic process whose log-deviation is AR(1):

$$\hat{\zeta}_{o,t} = \rho_o \hat{\zeta}_{o,t-1} + \epsilon_{o,t}$$

Hence, we can see that $\hat{\zeta}_{o,t}$ takes value 0 if the pass-through is complete, and moves away from zero otherwise. From Table 3 it is obvious that the persistence of $\hat{\zeta}_{o,t}$ is quite high across specifications. Since $\hat{\zeta}_{o,t}$ makes one of underlying state variables of the state space representation, we can obtain the smoothed series via Kalman filter once structural parameter values are fixed. Figure 5 shows these smoothed pass-through variables from all of three specifications at their own posterior mean from DSGE-VAR($\hat{\lambda}$): Baseline (solid line), No Oil Consumption (dash-dotted line), and No Oil in Production (dashed line). Actual observations of log real international price of oil (dotted line) are also drawn for reference.

The international oil price is stable until 2003 and takes off around 2004. The smoothed pass-through in the baseline model is moving around 0.5 until 2004 and drops to around 0 afterwards. To explain changes in the pass-through, we consider the government's reaction to an oil price shock. First we note that one of the main tax revenue of Korean government is the gasoline tax. Roughly 58% of the gasoline price paid by Korean customer are counted as the government revenue. Hence, the government could have lowered the gasoline tax to alleviate burdens of households and this fiscal policy could have affected the pass-through, even though the behavior of the government is not explicitly modeled in our baseline economy. This story sounds more compelling when we investigate smoothed pass-through from other specifications. The pass-through from No Oil Consumption economy does not change in its qualitative nature over the oil shock period. Noting that oil is not directly consumed in this economy, the implied change in the gasoline tax would have no effect on the pass-through. In No Oil in Production economy, however, the size of drop in the smoothed pass-through is simply huge, which reassures our argument.

5 Conclusion

In this paper we present the model economy to study the importance of oil price shocks to each part of the economy. The model economy uses oil imports either as direct consumption or an input of production. Within Bayesian estimation framework including DSGE-VARs, the empirical analysis is performed based on the Korean aggregate data. Our findings are as follows. First, we argue that the production component of oil usage is more important than the consumption component in fitting the data. Second, impulse response comparisons between the DSGE model and the DSGE-VAR($\hat{\lambda}$) show that the baseline model behaves well qualitatively, but quantitative results are not satisfactory. Third, from investigation of the smoothed underlying states we can conclude that the pass-through of oil prices in Korean economy moves towards a perfect one as oil prices continue to rise.

Appendix: Equilibrium Conditions and Log-linearization

Households

Due to complete market, consumption across households is the same. That is, $C_t(j) = C_t$ for all j. It is also easy to check that households will choose the same level of real money balances M_t/P_t, core consumption Z_t, oil consumption $O_{c,t}$, Home goods consumption $C_{H,t}$, Foreign goods consumption $C_{F,t}$, and foreign bond holdings B_t^*. First detrend real variables and write in lower-case letters:

$$c_t = \frac{C_t}{\gamma^t}, \quad o_{C,t} = \frac{O_{C,t}}{\gamma^t}, \quad z_t = \frac{Z_t}{\gamma^t}, \quad c_{H,t} = \frac{C_{H,t}}{\gamma^t}, \quad c_{F,t} = \frac{C_{F,t}}{\gamma^t}$$

And define relative prices: $\widehat{pr}_{s,t} = \hat{p}_{s,t} - \hat{p}_t$. Then we can derive

$$\hat{c}_{H,t} = (\phi_z - \phi_c)\,\omega_F \hat{s}_t - \Big((1-\omega_F)\phi_c + \omega_F\phi_z\Big)\,\widehat{pr}_{H,t} + \hat{c}_t \tag{27}$$

$$\hat{c}_{F,t} = -\Big((1-\omega_F)\phi_z + \omega_F\phi_c\Big)\,\hat{s}_t + (\phi_z - \phi_c)\,(1-\omega_F)\widehat{pr}_{H,t} + \hat{c}_t \tag{28}$$

$$\hat{o}_{C,t} = -\phi_c \widehat{pr}_{o,t} + \hat{c}_t \tag{29}$$

Consumption Euler equation After detrending, the consumption Euler equation is written as

$$\mathbf{E}_t\left[\beta R_t \pi_{t+1}^{-1}\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\gamma\right)^{-1}\right] = 1$$

where

$$\mathcal{C}_t = c_t - hc_{t-1}$$

At steady state:

$$\bar{R} = \frac{\gamma\bar{\pi}}{\beta}, \qquad \overline{\mathcal{C}} = (1-h)\bar{c}$$

Hence we log-linearize it as

$$\hat{R}_t = \mathbf{E}_t\left[\hat{\pi}_{t+1} + \hat{\mathcal{C}}_{t+1} - \hat{\mathcal{C}}_t\right] \tag{30}$$

$$\hat{\mathcal{C}}_t = \frac{1}{1-h}\hat{c}_t - \frac{h}{1-h}\hat{c}_{t-1} \tag{31}$$

Bond Euler equation Now the bond Euler equation is written as

$$\mathbf{E}_t\left[\beta R_t^* \frac{e_{t+1}}{e_t}\pi_{t+1}^{-1}\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\gamma\right)^{-1}\Theta\left(\frac{e_t B_t^*}{P_{X,t}X_t}\right)\right] = 1 \tag{32}$$

The steady state is

$$\bar{R}^* = \frac{\gamma\bar{\pi}}{\beta\bar{\Theta}} \tag{33}$$

Let $\Theta_t = \Theta(e_t B_t^* / P_{X,t} X_t)$. Then from the property of $\Theta(\cdot)$, we have $\hat{\Theta}_t = \kappa \hat{b}_t^*$ where

$$b_t^* = \frac{e_t B_t^*}{P_{X,t} X_t}$$

With plug-in of (30) the log-linearization of (32) leads us to

$$R_t = R_t^* + \mathbf{E}_t \left[\Delta e_{t+1}\right] + \kappa \hat{b}_t^* \tag{34}$$

Labor supply Detrended real wages are

$$w_t = \frac{W_t}{\gamma^t P_t}, \qquad \widetilde{w}_t = \frac{\widetilde{W}_t}{\gamma^t P_t}$$

From wage setting behavior we can write

$$w_t^{1-\nu_L} = (1-\theta_L)\widetilde{w}_t^{1-\nu_L} + \theta_L \left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{\xi_L(1-\nu_L)} \left(\frac{\pi_t}{\bar{\pi}}\right)^{-(1-\nu_L)} w_{t-1}^{1-\nu_L} \tag{35}$$

Plug labor demand into the first order condition of the wage reset:

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \left(\left(1 - \frac{1}{\nu_L}\right) \frac{\widetilde{W}_t}{\gamma^t P_t} \frac{P_t \Gamma_{W,t}^k}{\gamma^k P_{t+k}} \mathcal{C}_{t+k}^{-1} - \chi_H \widetilde{H}_{t+k}^{\tau}\right) \widetilde{H}_{t+k}\right] = 0$$

where

$$\widetilde{H}_{t+k} = \left(\frac{P_t}{P_{t+k}} \frac{\Gamma_{W,t}^k}{\gamma^k}\right)^{-\nu_L} \widetilde{w}_t^{-\nu_L} w_{t+k}^{\nu_L} H_{t+k} \tag{36}$$

and

$$\Gamma_{W,t}^k = (\gamma\bar{\pi})^k \left(\frac{\pi_t}{\bar{\pi}} \frac{\pi_{t+1}}{\bar{\pi}} \cdots \frac{\pi_{t+k-1}}{\bar{\pi}}\right)^{\xi_L} \tag{37}$$

Rewrite to get

$$\left(1 - \frac{1}{\nu_L}\right) \mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \widetilde{w}_t \frac{P_t \Gamma_{W,t}^k}{\gamma^k P_{t+k}} \widetilde{H}_{t+k} \mathcal{C}_{t+k}^{-1}\right] = \mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \chi_H \widetilde{H}_{t+k}^{1+\tau}\right] \tag{38}$$

Plug (36) and (37) in

$$\begin{aligned} &\left(1 - \frac{1}{\nu_L}\right) \mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \widetilde{w}_t^{1-\nu_L} w_{t+k}^{\nu_L} \left(\frac{P_t \Gamma_{W,t}^k}{\gamma^k P_{t+k}}\right)^{1-\nu_L} H_{t+k} \mathcal{C}_{t+k}^{-1}\right] \\ &= \mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \chi_H \widetilde{w}_t^{-\nu_L(1+\tau)} w_{t+k}^{\nu_L(1+\tau)} \left(\frac{P_t \Gamma_{W,t}^k}{\gamma^k P_{t+k}}\right)^{-\nu_L(1+\tau)} H_{t+k}^{1+\tau}\right] \end{aligned} \tag{39}$$

Note that for $k \geq 1$

$$\Xi_{L,t,t+k} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\xi_L} \left(\frac{\pi_{t+1}}{\bar{\pi}} \frac{\pi_{t+2}}{\bar{\pi}} \cdots \frac{\pi_{t+k-1}}{\bar{\pi}}\right)^{\xi_L - 1} \left(\frac{\pi_{t+k}}{\bar{\pi}}\right)^{-1}$$

and $\Xi_{L,t,t} = 1$. Define the marginal rate of substitution between consumption for leisure with flexible wage, that is, all households are allowed to set their wages every period ($\theta_L \to 0$):

$$MRS_t = -\frac{U_{H,t}}{U_{C,t}} = \gamma^k \chi_H \mathcal{C}_t H_t^{\tau}$$

With detrending

$$mrs_t = \frac{MRS_t}{\gamma^t} = \chi_H \mathcal{C}_t H_t^{\tau}$$

Then, we have

$$\left(1 - \frac{1}{\nu_L}\right) \mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \widetilde{w}_t^{1-\nu_L} w_{t+k}^{\nu_L} \Xi_{L,t,t+k}^{1-\nu_L} H_{t+k} \mathcal{C}_{t+k}^{-1}\right]$$
$$= \mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_L)^k \widetilde{w}_t^{-\nu_L(1+\tau)} w_{t+k}^{\nu_L(1+\tau)} \Xi_{L,t,t+k}^{-\nu_L(1+\tau)} H_{t+k} \mathcal{C}_{t+k}^{-1} mrs_{t+k}\right]$$

Steady state:

$$\overline{\widetilde{w}} = \bar{w}$$
$$\left(1 - \frac{1}{\nu_L}\right) \bar{w} = \overline{mrs} = \bar{\mathcal{C}} \bar{H}^{\tau} = (1-h)\bar{c}\bar{H}^{1-\tau}$$

Loglinearizaton:

$$\sum_{k=0}^{\infty} (\beta\theta_L)^k \mathbf{E}_t \left[(1+\tau\nu_L)\widehat{\widetilde{w}}_t - \tau\nu_L \hat{w}_{t+k} - \widehat{mrs}_{t+k} + (1+\tau\nu_L)\widehat{\Xi}_{L,t,t+k}\right] = 0$$

where for $k \geq 1$

$$\widehat{\Xi}_{L,t,t+k} = \xi_L \hat{\pi}_t + (\xi_L - 1)\left(\sum_{r=1}^{k-1} \hat{\pi}_{t+r}\right) - \hat{\pi}_{t+k}$$

and from labor market clearing

$$\widehat{mrs}_t = \tau \hat{n}_t + \hat{\mathcal{C}}_t \tag{40}$$

Hence, we have

$$\widehat{\widetilde{w}}_t = (1-\beta\theta_L)\left(\frac{\tau\nu_L}{1+\tau\nu_L}\hat{w}_t + \frac{1}{1+\tau\nu_L}\widehat{mrs}_t\right) - \beta\theta_L\left(\xi_L\hat{\pi}_t - \mathbf{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \beta\theta_L \mathbf{E}_t\left[\widehat{\widetilde{w}}_{t+1}\right] \tag{41}$$

Now from wage aggregation (35)

$$\tilde{w}_t = (1-\theta_L)\widehat{\widetilde{w}}_t + \theta_L\left(\xi_L\hat{\pi}_{t-1} - \hat{\pi}_t + \hat{w}_{t-1}\right)$$

Rearrage to get

$$\widehat{\widetilde{w}}_t - \hat{w}_t = \frac{\theta_L}{1-\theta_L}\left(\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \xi_L\hat{\pi}_{t-1}\right) \tag{42}$$

From (41) and (42), we have

$$\frac{1+\beta\theta_L^2 + \tau\nu_L\theta_L(1+\beta)}{1+\tau\nu_L}\hat{w}_t - \theta_L\hat{w}_{t-1} - \beta\theta_L\mathbf{E}_t\left[\hat{w}_{t+1}\right] = \frac{(1-\beta\theta_L)(1-\theta_L)}{1+\tau\nu_L}\widehat{mrs}_t$$
$$- \left(\theta_L + \beta\theta_L\xi_L\right)\hat{\pi}_t + \theta_L\xi_L\hat{\pi}_{t-1} + \beta\theta_L\mathbf{E}_t\left[\hat{\pi}_{t+1}\right] \tag{43}$$

Home goods producers

Phillips Curve Detrend first:

$$y_{H,t} = \frac{Y_{H,t}}{\gamma^t}, \qquad o_{H,t} = \frac{O_{H,t}}{\gamma^t}$$

From (10), we can derive the aggregate relationship between labor and oil used in production. Hence, the log-linearization is

$$\hat{o}_{H,t} - \hat{n}_t = \phi_y \left(\hat{w}_t - \widehat{pr}_{o,t} \right) \tag{44}$$

From (9), (10), and (11), we can write

$$Y_{H,t}(i) \quad = \quad \frac{\gamma^t}{1-\alpha} \zeta_{A,t}^{1-\phi_y} w_t^{\phi_y} mc_t^{-\phi_y} N_t(i)$$

where mc_t denote the real marginal cost. With aggregation, we can write

$$G_t y_{H,t} = \frac{1}{1-\alpha} \zeta_{A,t}^{1-\phi_y} w_t^{\phi_y} mc_t^{-\phi_y} N_t \tag{45}$$

where

$$G_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu_H} di$$

It can be shown that $\hat{g}_t = 0$. Therefore, the log-linearization of (45) is

$$\hat{y}_{H,t} = (1-\phi_y)\hat{\zeta}_{A,t} + \phi_y \left(\hat{w}_t - \widehat{mc}_t \right) + \hat{n}_t \tag{46}$$

Given that the log-linearization of (11) is

$$\widehat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \widehat{pr}_{o,t} - \hat{\zeta}_{A,t} \tag{47}$$

From (44), (46), and (47), we have

$$\hat{y}_t = (1-\alpha)\hat{n}_t + \alpha \hat{o}_{H,t} + \hat{\zeta}_{A,t} \tag{48}$$

Noting that

$$\begin{aligned} \widetilde{Y}_{H,t+k} &= \left(\frac{\widetilde{P}_{H,t} \Gamma_{H,t}^k}{P_{H,t+k}} \right)^{-\nu_H} Y_{H,t+k} \\ \Lambda_{t,t+k} &= \beta^k \frac{P_t}{\gamma^k P_{t+k}} \frac{\mathcal{C}_t}{\mathcal{C}_{t+k}} \end{aligned}$$

The first order condition is now written as

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_H)^k \mathcal{C}_{t+k}^{-1} \frac{\widetilde{Y}_{H,t+k}}{\gamma^{t+k}} \left(\frac{\widetilde{P}_{H,t} \Gamma_{H,t}^k}{P_{H,t+k}} \frac{P_{H,t+k}}{P_{t+k}} - \frac{\nu_H}{\nu_H - 1} mc_{t+k} \right) \right] = 0$$

Now look at

$$\frac{\widetilde{P}_{H,t}}{P_{H,t+k}} \Gamma_{H,t}^k \quad = \quad \left(\frac{\widetilde{P}_{H,t}}{P_t} \right) \left(\frac{P_{H,t}}{P_t} \right)^{-1} \frac{P_{H,t}}{P_{H,t+k}} \Gamma_{H,t}^k \quad = \quad \left(\frac{\widetilde{P}_{H,t}}{P_t} \right) \left(\frac{P_{H,t}}{P_t} \right)^{-1} \Xi_{H,t,t+k}$$

where

$$\Xi_{H,t,t+k} = \left(\frac{\pi_{H,t}}{\bar{\pi}}\right)^{\xi_H}\left(\frac{\pi_{H,t+1}}{\bar{\pi}}\cdots\frac{\pi_{H,t+k-1}}{\bar{\pi}}\right)^{\xi_H-1}\left(\frac{\pi_{H,t+k}}{\bar{\pi}}\right)^{-1}$$

At steady state

$$\overline{mc} = 1 - \frac{1}{\nu_H}$$

The log-linearization is

$$\begin{aligned}\widehat{\widetilde{pr}}_{H,t} - \widehat{pr}_{H,t} &= (1-\beta\theta_H)\sum_{k=0}^{\infty}(\beta\theta_H)^k\,\mathbf{E}_t\left[\widehat{mc}_{t+k} - \widehat{pr}_{H,t+k} - \widehat{\Xi}_{H,t,t+k}\right]\\ &= (1-\beta\theta_H)\left(\widehat{mc}_t - \widehat{pr}_{H,t}\right) - \beta\theta_H\left(\xi_H\hat{\pi}_{H,t} - \mathbf{E}_t\left[\hat{\pi}_{H,t+1}\right]\right)\\ &\quad +\beta\theta_H\mathbf{E}_t\left[\widehat{\widetilde{pr}}_{H,t+1} - \widehat{pr}_{H,t+1}\right]\end{aligned}$$

Note that the aggregate price for Home goods is

$$P_{H,t}^{1-\nu_H} = (1-\theta_H)\widetilde{P}_{H,t}^{1-\nu_H} + \theta_H\left(\frac{\pi_{H,t-1}}{\bar{\pi}}\right)^{\xi_H(1-\nu_H)}\left(\bar{\pi}P_{H,t-1}\right)^{1-\nu_H}$$

Divide by $P_{H,t}^{1-\nu_H}$ and loglinearize it to get

$$\widehat{\widetilde{pr}}_{H,t} - \widehat{pr}_{H,t} = \frac{\theta_H}{1-\theta_H}\left(\hat{\pi}_{H,t} - \xi_H\hat{\pi}_{H,t-1}\right)$$

Then we have

$$\hat{\pi}_{H,t} = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H(1+\beta\xi_H)}\left(\widehat{mc}_t - \widehat{pr}_{H,t}\right) + \frac{\beta}{1+\beta\xi_H}\mathbf{E}_t\left[\hat{\pi}_{H,t+1}\right] + \frac{\xi_H}{1+\beta\xi_H}\hat{\pi}_{H,t-1} \tag{49}$$

Relative Prices We get the relationship between Home goods and general inflation from the following identity:

$$\frac{P_{H,t}}{P_t}\frac{P_t}{P_{t-1}}\frac{P_{t-1}}{P_{H,t-1}}\frac{P_{H,t-1}}{P_{H,t}} = 1$$

which leads to

$$\widehat{pr}_{H,t} = \widehat{pr}_{H,t-1} + \hat{\pi}_{H,t} - \hat{\pi}_t \tag{50}$$

From imperfect pass-through of oil prices (13)

$$\widehat{pr}_{o,t} = \hat{s}_t + \widehat{pr}^*_{o,t} + \hat{\zeta}_{o,t} \tag{51}$$

From the definition of real exchange rate (12)

$$\hat{s}_t = \hat{s}_{t-1} + \Delta\hat{e}_t + \hat{\pi}^*_t - \hat{\pi}_t \tag{52}$$

From the definition of CPI (5), the core consumption price level (6), and the law of one price

$$0 = \omega_o\widehat{pr}_{o,t} + (1-\omega_o)(1-\omega_F)\widehat{pr}_{H,t} + (1-\omega_o)\omega_F\hat{s}_t \tag{53}$$

The core CPI inflation is obtained from (5):

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\phi_C} = \omega_o \left(\frac{P_{o,t}}{P_t}\frac{P_t}{P_{t-1}}\right)^{1-\phi_C} + (1-\omega_o)\left(\frac{P_{Z,t}}{P_{Z,t-1}}\frac{P_{Z,t-1}}{P_{t-1}}\right)^{1-\phi_C}$$

which leads us to

$$\hat{\pi}_t = \omega_o \left(\widehat{pr}_{o,t} + \hat{\pi}_t\right) + (1-\omega_o)\left(\hat{p}_{Z,t} - \widehat{pr}_{Z,t-1}\right) \tag{54}$$

Again from (5) we also have

$$1 = \omega_o \left(\frac{P_{o,t}}{P_t}\right)^{1-\phi_C} + (1-\omega_o)\left(\frac{P_{Z,t}}{P_t}\right)^{1-\phi_C}$$

with (54) we have

$$\hat{\pi}_{Z,t} = \hat{\pi}_t - \frac{\omega_o}{1-\omega_o}\left(\widehat{pr}_{o,t} - \widehat{pr}_{o,t-1}\right) \tag{55}$$

Aggregate Equilibrium The log-linearization of Home goods market clearing (15) is

$$\begin{aligned}\hat{y}_{H,t} &= \frac{C_H}{Y_H}\hat{c}_{H,t} + \left(1-\frac{C_H}{Y_H}\right)\hat{c}^*_{H,t} \\ &= \frac{C_H}{Y_H}\hat{c}_{H,t} + \left(1-\frac{C_H}{Y_H}\right)\left(\hat{c}^*_t - \phi^*\left(\widehat{pr}_{H,t} - \hat{s}_t\right)\right) \end{aligned} \tag{56}$$

We can write the nominal GDP as

$$P_{Y,t}Y_t = P_tC_t + e_tP^*_{F,t}AC^*_t + P_{X,t}X_t - P_{M,t}M_t$$

where X_t and M_t denote export and import, respectively. The log-lineariztion is

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{X}{Y}\hat{x}_t - \frac{M}{Y}\hat{m}_t \tag{57}$$

Noting that Noting that $P_{X,t}X_t = P_{H,t}C^*_{H,t}$, we can write exports and its real price:

$$\begin{aligned}\hat{x}_t &= \hat{c}^*_t - \phi^*\left(\widehat{pr}_{H,t} - \hat{s}_t\right) \\ \widehat{pr}_{X,t} &= \widehat{pr}_{H,t}\end{aligned} \tag{58}$$

Noting that $P_{M,t}M_t = s_tP_tC_{F,t} + e_tP^*_{o,t}O_t$ and $O_t = O_{C,t} + O_{H,t}$, we can decompose into imports and its real price:

$$\hat{m}_t = \frac{C_F}{M}\hat{c}_{F,t} + \left(1-\frac{C_F}{M}\right)\hat{o}_t \tag{59}$$

$$\widehat{pr}_{M,t} = \frac{C_F}{M}\hat{s}_t + \frac{O}{M}\widehat{pr}^*_{o,t}$$

$$\hat{o}_t = \frac{O_C}{O}\hat{o}_{C,t} + \left(1-\frac{O_C}{O}\right)\hat{o}_{H,t} \tag{60}$$

Bond Accumulation Outstanding balance of the foreign bond evolves with surpluses from trades:

$$\frac{b_t^*}{R_t^*\Theta(b_t^*)} = \frac{e_t}{e_{t-1}} b_{t-1}^* \frac{P_{X,t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} \frac{P_t}{P_{X,t}} \frac{X_{t-1}}{\gamma^{t-1}} \frac{\gamma^t}{X_t} \frac{1}{\gamma} + 1 - \frac{P_{M,t}}{P_t} \frac{P_t}{P_{X,t}} \frac{M_t}{\gamma^t} \frac{\gamma^t}{X_t}$$

With (33), we have the steady state relationship:

$$(1-\beta)\frac{\bar{b}^*}{\gamma\bar{\pi}} = \frac{M}{X} - 1$$

Now the log-linearization gives

$$\begin{aligned} \beta(1-\kappa)\hat{b}_t^* \quad = \quad & \beta\hat{R}_t^* + \hat{b}_{t-1}^* + \hat{x}_{t-1} + \widehat{pr}_{X,t-1} + \Delta\hat{e}_t - \hat{\pi}_t \\ & - \left(1 - \frac{M}{X}\frac{\bar{\pi}\gamma}{\bar{b}^*}\right)\left(\hat{x}_t + \widehat{pr}_{X,t}\right) - \frac{M}{X}\frac{\bar{\pi}\gamma}{\bar{b}^*}\left(\hat{m}_t + \widehat{pr}_{M,t}\right) \end{aligned} \tag{61}$$

Monetary Policy Monetary policy (14) is interest rate targeting

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\psi_\pi\hat{\pi}_t + (1-\rho_R)\psi_y\left(\hat{y}_t - \hat{y}_{t-1}\right) + \epsilon_{R,t} \tag{62}$$

Steady State We need the following steady state ratio to write the log-linearized model:

$$\frac{C}{Y}, \quad \frac{X}{Y}, \quad \frac{C_H}{Y_H}, \quad \frac{M}{Y}, \quad \frac{C_F}{M}, \quad \frac{O}{M}, \quad \frac{O_C}{O}$$

We have the following steady state relationships:

$$\begin{gathered} \frac{O_c}{C} = \omega_o, \quad \frac{Z}{C} = 1-\omega_o, \quad \frac{C_H}{Z} = 1-\omega_F, \quad \frac{C_F}{Z} = \omega_F, \quad \frac{O_H}{Y_H} = \alpha \\ 1 = \frac{C}{Y} + \frac{X}{Y} - \frac{M}{Y}, \quad \frac{O}{Y} = \frac{O_C}{Y} + \frac{O_H}{Y}, \quad \frac{M}{Y} = \frac{O}{Y} + \frac{C_F}{Y}, \quad \frac{Y_H}{Y} = \frac{C_H}{Y} + \frac{X}{Y} \end{gathered}$$

Therefore, we can derive

$$\begin{gathered} \frac{Y_H}{Y} = \frac{C_H}{Z}\frac{Z}{C}\frac{C}{Y} + \frac{X}{Y} = (1-\omega_F)(1-\omega_o)\frac{C}{Y} + \frac{X}{Y} \\ \frac{C_H}{Y_H} = (1-\omega_F)(1-\omega_o)\frac{C}{Y}\left(\frac{Y_H}{Y}\right)^{-1} \\ \frac{M}{Y} = \frac{C}{Y} + \frac{X}{Y} - 1 \\ \frac{O}{Y} = \frac{O_c}{C}\frac{C}{Y} + \frac{O_H}{Y_H}\frac{Y_H}{Y} = \omega_o\frac{C}{Y} + \alpha\frac{Y_H}{Y} \\ \frac{C_F}{M} = 1 - \frac{O}{Y}\left(\frac{M}{Y}\right)^{-1} \\ \frac{O}{M} = 1 - \frac{C_F}{M} \\ \frac{O_C}{O} = \frac{O_C}{C}\frac{C}{Y}\left(\frac{O}{Y}\right)^{-1} = \omega_o\frac{C}{Y}\left(\frac{O}{Y}\right)^{-1} \end{gathered}$$

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Table 1: Prior Distribution

Name	Domain	Density	Mean	S.D.	Description
α	$[0,1)$	Beta	0.300	0.100	Oil share in production
τ	$\mathbf{R}^+$	Gamma	1.000	0.750	(inverse) EIS of labor
h	$[0,1)$	Beta	0.500	0.200	Habit persistence
κ	$\mathbf{R}^+$	Gamma	0.010	0.005	Elasticity: risk premium
ϕ_Z	$\mathbf{R}^+$	Gamma	0.300	0.400	Elasticity: H/F goods consumption
ϕ^*	$\mathbf{R}^+$	Gamma	1.000	0.400	Elasticity: H/F goods in foreign consumption
ϕ_C	$\mathbf{R}^+$	Gamma	0.330	0.150	Elasticity: Oil and core consumption
ϕ_y	$\mathbf{R}^+$	Gamma	0.500	0.300	Elasticity: Oil and labor input of production
θ_H	$[0,1)$	Beta	0.700	0.100	Calvo on price
θ_L	$[0,1)$	Beta	0.700	0.150	Calvo on wage
ξ_H	$[0,1)$	Beta	0.500	0.200	Price indexation
ξ_L	$[0,1)$	Beta	0.500	0.200	Wage indexation
ψ_π	$\mathbf{R}^+$	Gamma	1.500	0.200	Responsiveness on inflation
ψ_y	$\mathbf{R}^+$	Gamma	0.500	0.250	Responsiveness on output
ρ_R	$[0,1)$	Beta	0.750	0.100	Persistence: interest rate
$\gamma^{(Q)}$	$\mathbf{R}$	Normal	0.750	0.300	Growth rate
$r^{(A)}$	$\mathbf{R}^+$	Gamma	0.500	0.200	Steady state real interest rate
$\pi^{(A)}$	$\mathbf{R}^+$	Gamma	3.000	2.000	Target inflation rate
ω_F	$[0,1)$	Beta	0.350	0.100	Weight on foreign good consumption
ω_o	$[0,1)$	Beta	0.100	0.050	Weight on oil consumption
ρ_A	$[0,1)$	Beta	0.700	0.150	Persistence: technology
ρ_o	$[0,1)$	Beta	0.700	0.150	Persistence: oil price pass-through
ρ_{o^*}	$[0,1)$	Beta	0.700	0.150	Persistence: foreign oil price
ρ_{R^*}	$[0,1)$	Beta	0.700	0.150	Persistence: foreign interest rate
ρ_{π^*}	$[0,1)$	Beta	0.700	0.150	Persistence: foreign inflation
ρ_{C^*}	$[0,1)$	Beta	0.700	0.150	Persistence: foreign consumption
σ_R	$\mathbf{R}^+$	InvGamma	0.010	2	StDev: monetary policy
σ_A	$\mathbf{R}^+$	InvGamma	0.150	2	StDev: technology
σ_o	$\mathbf{R}^+$	InvGamma	0.150	2	StDev: oil-price pass-through
σ_{o^*}	$\mathbf{R}^+$	InvGamma	0.150	2	StDev: foreign oil price
σ_{R^*}	$\mathbf{R}^+$	InvGamma	0.050	2	StDev: foreign interest rate
σ_{π^*}	$\mathbf{R}^+$	InvGamma	0.050	2	StDev: foreign inflation
σ_{C^*}	$\mathbf{R}^+$	InvGamma	0.050	2	StDev: foreign consumption

Notes: For the inverse-gamma distribution, values in S.D. column denote degrees of freedom.

Table 2: The Fit of the Small Open Economy DSGE Model

Specification	λ	Baseline	No oil in consumption $\omega_o = 0$	No oil in production $\alpha = 0$
DSGE		-1202.84	-1189.35	-1226.28
DSGE-VAR	∞	-1154.89	-1136.24	-1211.31
	2	-1096.80	-1072.75	-1133.97
	1.5	-1086.68	-1072.64	-1120.70
	1.25	-1064.45	-1059.55	-1108.56
	1	-1054.63	-1043.78	-1049.00
	0.75	-1032.35	-1037.72	-1039.46
	0.5	-1030.69	-1029.09	-1011.40
	0.4	-1041.03	-1042.56	-1012.80

Figure 1: Log Marginal Likelihood

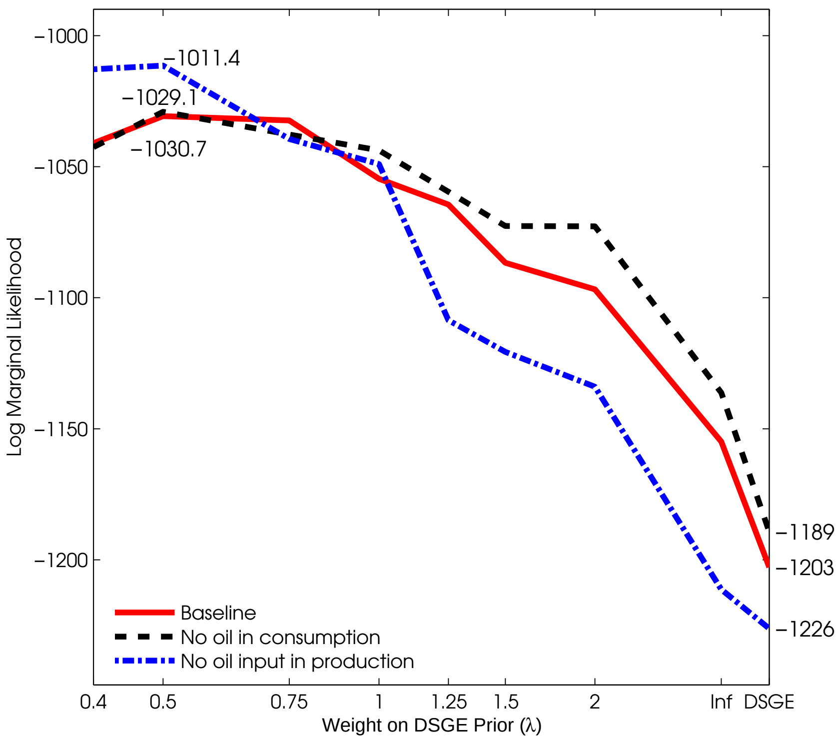
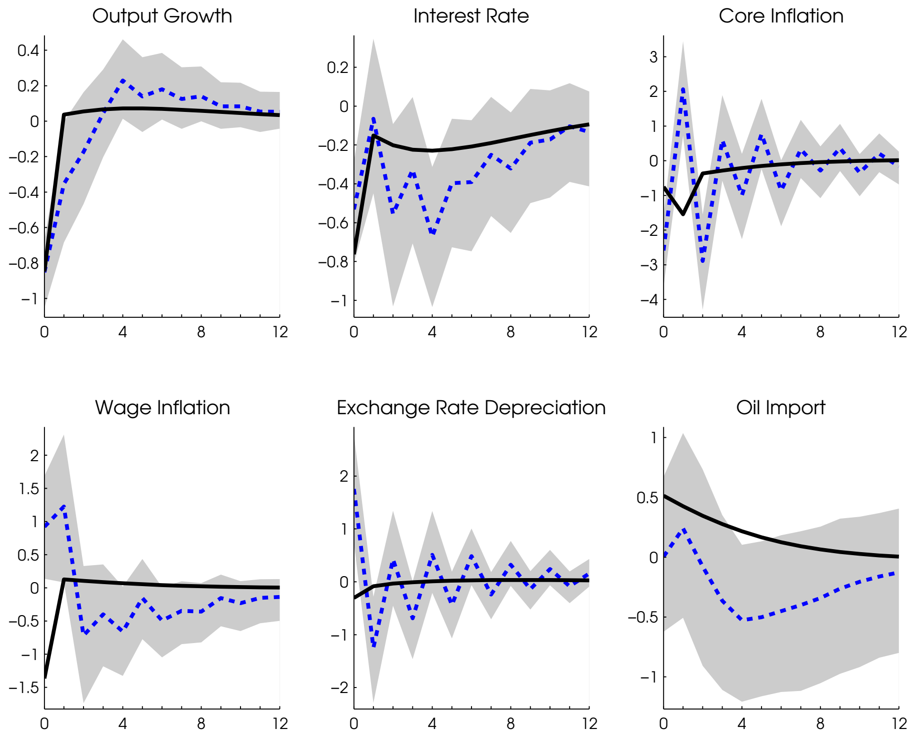


Table 3: Posterior Estimates: Baseline Model

	Baseline				No Oil in Production			
	DSGE		DSGE-VAR($\hat{\lambda}$)		DSGE		DSGE-VAR($\hat{\lambda}$)	
	Mean	90% Interval	Mean	90% Interval	Mean	90% Interval	Mean	90% Interval
α	0.297	[0.268,0.322]	0.184	[0.131,0.231]				
τ	0.152	[0.024,0.280]	0.488	[0.091,0.846]	0.438	[0.264,0.601]	1.674	[0.908,2.269]
h	0.684	[0.642,0.726]	0.832	[0.719,0.952]	0.161	[0.074,0.236]	0.233	[0.058,0.395]
κ	0.002	[0.001,0.003]	0.003	[0.001,0.006]	0.004	[0.002,0.006]	0.002	[0.000,0.003]
ϕ_Z	0.421	[0.319,0.496]	1.209	[0.822,1.674]	1.290	[1.025,1.539]	0.161	[0.000,0.381]
ϕ^*	1.260	[1.167,1.353]	1.112	[0.780,1.465]	1.753	[1.636,1.877]	0.989	[0.811,1.175]
ϕ_C	0.393	[0.368,0.416]	0.410	[0.320,0.507]	0.027	[0.006,0.049]	0.183	[0.113,0.256]
ϕ_y	0.049	[0.012,0.087]	0.150	[0.028,0.259]				
θ_H	0.959	[0.944,0.973]	0.917	[0.881,0.953]	0.988	[0.984,0.992]	0.916	[0.883,0.955]
θ_L	0.847	[0.819,0.872]	0.714	[0.599,0.803]	0.708	[0.644,0.797]	0.420	[0.322,0.529]
ξ_H	0.686	[0.640,0.731]	0.690	[0.555,0.824]	0.553	[0.472,0.636]	0.847	[0.751,0.941]
ξ_L	0.939	[0.907,0.980]	0.852	[0.735,0.964]	0.949	[0.910,0.990]	0.543	[0.359,0.696]
ψ_π	1.500	[1.446,1.547]	1.387	[1.223,1.540]	1.539	[1.440,1.615]	1.369	[1.222,1.545]
ψ_y	0.364	[0.255,0.483]	0.467	[0.201,0.722]	0.586	[0.525,0.651]	0.289	[0.139,0.439]
ρ_R	0.856	[0.832,0.882]	0.844	[0.804,0.880]	0.799	[0.757,0.844]	0.716	[0.660,0.778]
$\gamma^{(Q)}$	0.473	[0.376,0.565]	0.492	[0.295,0.695]	0.155	[0.035,0.341]	0.594	[0.383,0.775]
$r^{(A)}$	0.628	[0.537,0.713]	0.551	[0.404,0.699]	0.604	[0.452,0.699]	0.339	[0.204,0.491]
$\pi^{(A)}$	6.303	[5.519,7.434]	2.524	[1.010,3.882]	3.985	[2.757,5.446]	2.699	[1.670,3.780]
ω_F	0.210	[0.188,0.236]	0.116	[0.073,0.157]	0.193	[0.167,0.218]	0.136	[0.102,0.170]
ω_o	0.131	[0.125,0.137]	0.105	[0.072,0.133]	0.071	[0.058,0.082]	0.011	[0.003,0.019]
ρ_A	0.822	[0.783,0.857]	0.711	[0.592,0.828]	0.997	[0.994,0.999]	0.640	[0.504,0.773]
ρ_o	0.962	[0.948,0.976]	0.928	[0.865,0.987]	0.960	[0.944,0.978]	0.822	[0.680,0.969]
ρ_{o^*}	0.963	[0.943,0.983]	0.968	[0.934,0.998]	0.974	[0.949,0.999]	0.888	[0.806,0.977]
ρ_{R^*}	0.644	[0.595,0.689]	0.431	[0.310,0.537]	0.495	[0.387,0.581]	0.299	[0.136,0.424]
ρ_{π^*}	0.147	[0.127,0.170]	0.137	[0.065,0.211]	0.091	[0.043,0.146]	0.143	[0.060,0.226]
ρ_{C^*}	0.995	[0.991,0.998]	0.956	[0.919,0.992]	0.967	[0.954,0.978]	0.946	[0.902,0.986]
σ_R	0.008	[0.007,0.010]	0.003	[0.002,0.003]	0.010	[0.008,0.012]	0.003	[0.002,0.003]
σ_A	0.117	[0.099,0.134]	0.048	[0.034,0.061]	0.352	[0.223,0.467]	0.045	[0.029,0.059]
σ_o	0.289	[0.245,0.333]	0.077	[0.046,0.106]	0.480	[0.349,0.604]	0.179	[0.096,0.256]
σ_{o^*}	0.179	[0.151,0.206]	0.079	[0.055,0.101]	0.178	[0.152,0.204]	0.111	[0.080,0.143]
σ_{R^*}	0.014	[0.011,0.017]	0.010	[0.007,0.012]	0.016	[0.012,0.020]	0.010	[0.008,0.013]
σ_{π^*}	0.060	[0.050,0.068]	0.023	[0.015,0.030]	0.060	[0.051,0.069]	0.023	[0.017,0.030]
σ_{C^*}	0.081	[0.064,0.097]	0.044	[0.029,0.059]	0.151	[0.121,0.179]	0.030	[0.021,0.039]

Figure 2: Baseline Model Impulse Response Functions

(a) Monetary Shocks



(b) Techonology Shocks

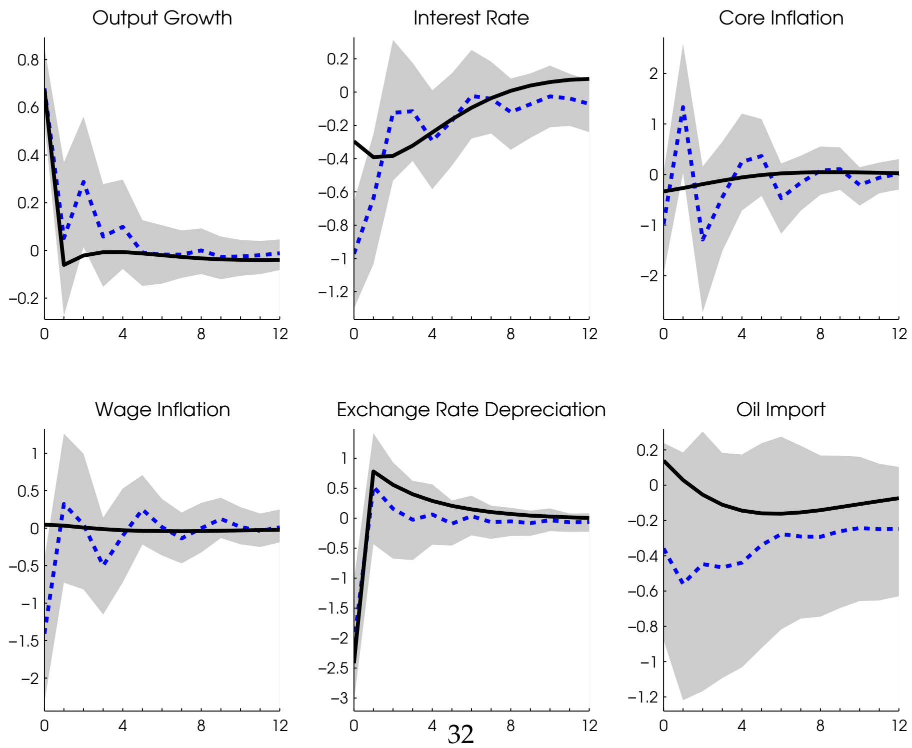


Figure 3: Impulse Response Functions: Baseline Model

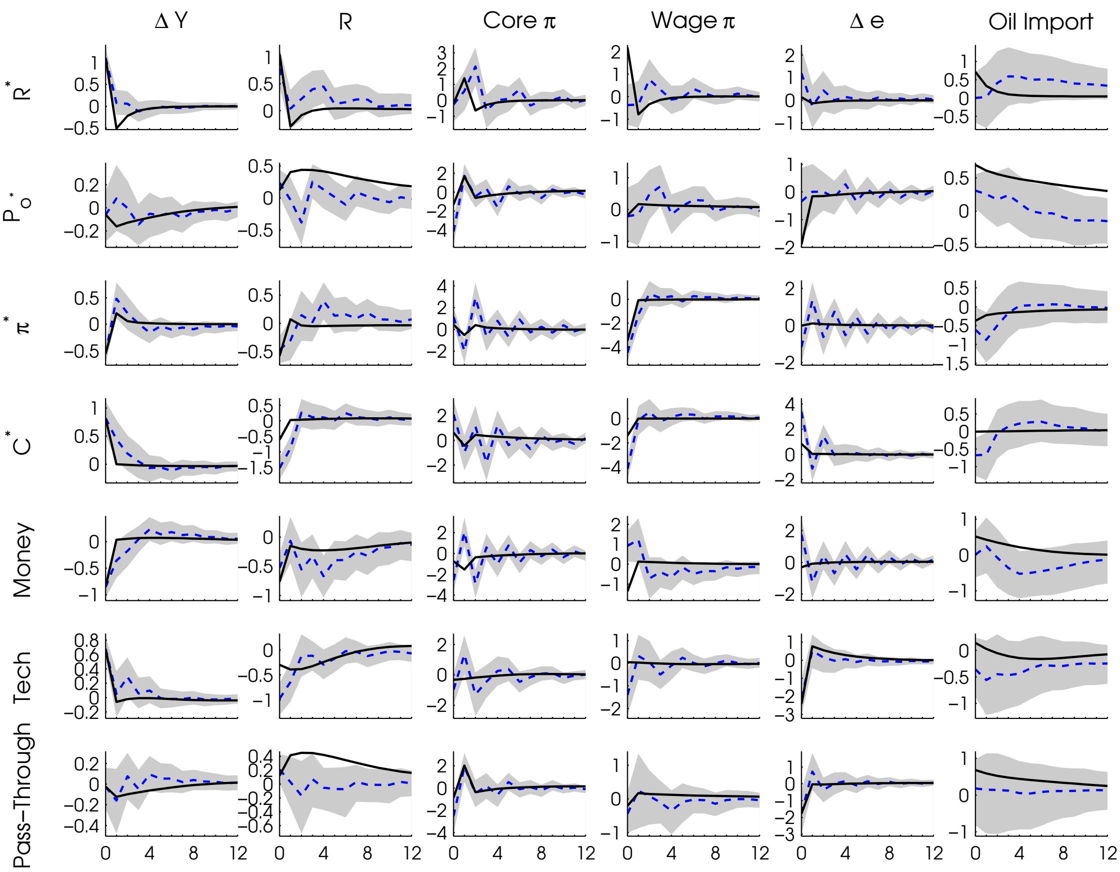


Figure 4: Impulse Response Functions: No Oil in Production Model

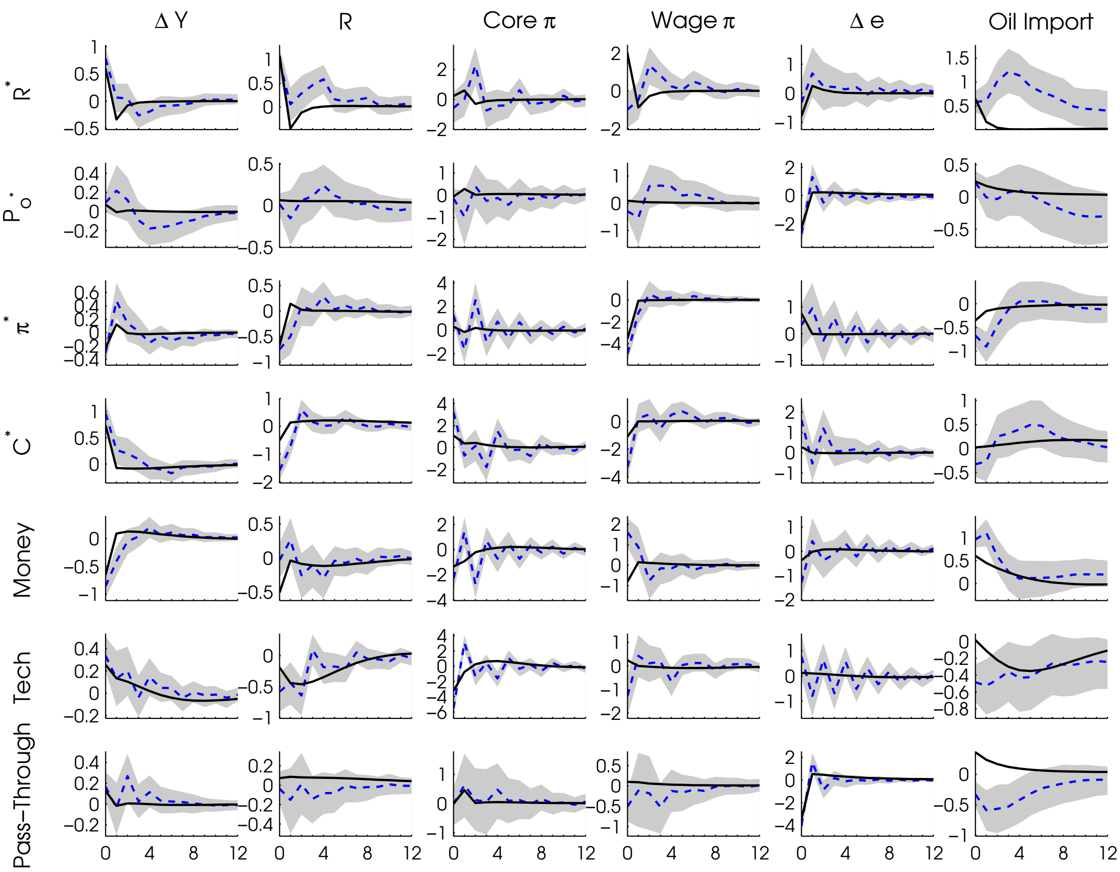


Figure 5: Pass-through of International Oil Price