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A Competitive Theory of Equilibrium Mismatch

Javier A. Birchenall
University of California at Santa Barbara

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Abstract

This paper studies why unemployment and vacancies coexist in a model in which jobs are assigned to heterogeneous workers. Jobs are assigned under uncertain and sequential trade frictions. Thus, a 'worker-job' match takes place only in markets where trade occurs. This gives rise to a competitive market equilibrium with unemployed workers and vacant jobs as assignments to markets where trade is expected but not realized. The assignment can be decentralized through contingent wages that 'seem rigid.' The model is integrated with a typical real business cycle model and with models of equilibrium search unemployment. In response to neutral productivity shocks, the economy traces out a *Beveridge curve* –a negative co-movement between unemployment and vacancies.

[Preliminary and incomplete]

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Communications to: Javier A. Birchenall

Department of Economics, 2127 North Hall

University of California, Santa Barbara CA 93106

jabirche@econ.ucsb.edu

1 Introduction

This paper studies labor-market mismatch –why worker unemployment and job vacancies coexist– in a model in which jobs are assigned to heterogeneous workers.[1] The purpose of this paper is to provide a simple, yet general, theoretical framework for examining the business cycle fluctuations of wages, unemployment, and vacancies in a setting in which mismatch is microfounded and prices and quantities are simultaneously determined in a *competitive general equilibrium*.

The paper first provides an exposition of the conditions under which mismatch exists in models where information is perfect. Our analysis is in the spirit of recent work that examines the microeconomic foundations of mismatch (see, e.g., Lagos [31]; Shimer [51]). Mismatch in previous studies, however, takes place as a form of classical disequilibrium in which unemployment and vacancies are associated with excess supply of workers and excess supply of jobs.

The paper next proposes one possible microeconomic foundation for mismatch based on trading frictions given by uncertain and sequential trade, UST. Under UST considerations, trading is uncertain in some markets and uncertainty resolves sequentially. Job assignments take place before uncertainty is fully resolved although not necessarily before the beginning of the trading process (as typically considered in Arrow-Debreu markets). Because jobs are assigned before uncertainty is fully resolved, a 'worker-job' match only takes place in markets where trade occurs. This gives rise to a competitive market equi-

[1] The role of job assignments has a long tradition in economics but analyses in the labor market have focused mainly on understanding the determinants of the structure of wages and the distribution of earnings. (Sattinger [46] provides a survey of assignment models in the labor market.) Additional examples of assignment models include Becker [4], Koopmans and Beckmann [28], Shapley and Shubik [48], and Shimer [49]. A general description of non-atomic assignment models can be found in Grestky et al. [20].

librium with unemployed workers and vacant jobs as equilibrium assignments to markets where trade is expected but not realized. (The frictional model builds on work by Prescott [43], Butters [9], and Eden [14].[2])

The following example describes how mismatch arises under uncertain and sequential trade. Consider two groups of workers and uncertainty about worker arrival. Suppose workers differ by skill. The first group of workers is composed by unskilled workers who arrive almost certainly. Once jobs are assigned to these workers, trading in this market ends; assigned jobs cannot be taken away later on and recall is not an option. The arrival of a second group of skilled workers is uncertain. If the second group arrives, their arrival occurs after trading has ended in the first market.

Firms might not assign all jobs to the first group of workers because they speculate that more skilled workers will arrive later on. This speculation is essential to generate mismatch. If some jobs are assigned to skilled workers, but these workers fail to arrive, some jobs will be vacant. If all jobs are assigned to the first group, the second group of workers will be unemployed (if they arrive). Finally, if no jobs are assigned to the first group of workers and the second group of workers fails to arrive, the labor market will display mismatch; workers in the first group will be unemployed while the jobs assigned to the second group of workers will remain vacant.

As evident form the previous example, *competitive prices* play a fundamental role in mismatch because the opportunity cost of a job determines which workers should receive

[2] The UST model was first developed by Prescott [43] and Butters [9] to study trading uncertainty. In Prescott [43], sellers of motel rooms set prices before they know how many buyers will appear. (In Butters [9], sellers send price offers to potential costumers.) Cheaper rooms are sold first and therefore, in equilibrium, price and the probability of making a sale are traded off. When the realization of demand is low, capacity is low and vacancies arise. In the paper, we follow the competitive version of the UST model by Eden [14]. (See also Eden [16].)

jobs and the number of jobs to be assigned to each of the employable workers. In current views of mismatch, e.g., Lagos [31] and Shimer [51], quantity aspects of the labor market such as market tightness are central for business cycle fluctuations because the opportunity cost of a job is simply determined exogenously.

It is perhaps important to highlight that the trading frictions central to this paper can be understood as affecting the number of market participants but not any specific bilateral trade opportunity, which is the main consideration in search and matching models, e.g., Mortensen and Pissarides [41]. (Previous models on mismatch and some of the deficiencies discussed in the literature are the subject of a separate section, Section 2.) Because agents here interact with the market, wages are set competitively. Moreover, despite the fact that the assignment of jobs is irreversible and made before uncertainty is resolved, the plans in the UST model are time consistent and socially efficient. Plans can also be interpreted as contingencies in Arrow-Debreu markets so one may think of job assignments as posted vacancies and of market prices as advertised market wages. As a consequence of the informational structure of the economy, in addition to worker's productivity, competitive wages are characterized by the probability of trade. Hence, equally productive workers could have different wages if their arrival to the firm (or to the market, actually) is not equally certain.[3]

There are important advantages of relying on uncertain and sequential trade frictions beyond the ones previously mentioned. In addition to wage heterogeneity, the frictional

[3] An equilibrium distribution of prices is an efficient response to trading uncertainty because it reduces the expected costs of excess capacity (Prescott [43] and Butters [9]). Additional aspects of the distribution of prices have been considered by Carlton [12], Eden and Griliches [17], and Eden [14]. See also Weitzman [54] for a model of wage rigidity. Rotemberg and Summers [45] also employ Prescott's [43] model to study labor hoarding and labor productivity in a model in which prices are set before demand is known.

model generates wages that 'seem rigid' and allocations that 'seem immobile.' Wages will appear to be rigid because advertised wages will not change with the arrival of workers once trading has begun. By the sequential nature of the resolution of uncertainty, firms have no incentive to change the job assignments when information about arrival is partially revealed. As job assignments are unchanged, competitive assignments and wages will be unresponsive to market conditions or rigid during any given trading round. For example, the fact that a first group of workers has traded provides no additional information for the job assignment of subsequent batches of workers. The firm has already taken into account this arrival and the possible arrival of additional workers later on in the initial job assignment. Moreover, since only wages *actually paid* by the firm are observed, average wages for actual transactions would not only seem to be "sticky" but also "too high," as in Eden ([16], 214).

We enrich the static job assignment model in two main ways. First, we allow for worker mobility and equilibrium search, and second, we consider job creation and the response to aggregate productivity shocks. In particular, we examine an assignment version of Lucas and Prescott's [34] search unemployment model and introduce job vacancies and job mobility between locations (whereas in Lucas and Prescott [34] only worker mobility is allowed). Our dynamic assignment is the first model that considers a joint allocation of capital and workers in a search equilibrium with aggregate productivity shocks. Because the aggregate distribution of workers becomes a state variable, we describe the dynamic economy using sequences of aggregate distributions of the labor force and prove the existence of a sequential competitive equilibria as in anonymous sequential games with aggregate uncertainty (see, e.g., Bergin and Bernhardt [5]; [6]; Miao [36]). These general considerations yield a more complete and interesting view of the labor market in a general equilibrium system with heterogeneous agents.

The focus on how aggregate productivity shocks affect unemployment and vacancies in a competitive equilibrium is of particular interest because the labor market's response to productivity shocks is very intuitive in this paper.[4] A positive productivity shock reduces the cost of a job and increases current advertised wages and future expected wages. This makes worker search more expensive and thus reduces unemployment. The decline in the opportunity cost of a job also makes speculation less costly and this increases job assignments to all markets including those were trade will not take place. Then, job vacancies increase and the economy traces out a *Beveridge curve*.

The rest of the paper proceeds as follows. Section 2 describes previous theories of mismatch. We focus on recent micro-foundations to the aggregate matching function and on equilibrium unemployment. Section 3 studies frictionless unemployment and frictionless mismatch. Section 4 studies the UST frictions and mismatch in a static competitive equilibrium. Section 4 also discusses the properties of the competitive wages that decentralize the assignment. Section 4 then generalizes the assignment model to study equilibrium search and business cycles. Section 5 presents some quantitative explorations. Section 6 concludes this paper. The Appendix discusses the recursive representation of the dynamic equilibrium and the solution method for the simulations.

[4]This paper focuses on labor-market mismatch but informational frictions in the form of search and uncertain and sequential trade may be relevant for the determination of equilibrium assignments and prices in other markets including, but not limited to, the housing market, financial and asset markets, marriage markets, and other general transportation models.

2 Previous theories of mismatch

Matching models: The main approach to study frictional labor markets is that of Mortensen and Pissarides [40] and [41]; the MP model henceforth. In the MP model, mismatch exist because informational frictions prevent workers from meeting suitable employers. Frictions are introduced through a reduced-form "matching" function that depends on the number of unemployed workers and the number of available job vacancies. (Pissarides [42] presents a general treatment of this literature.)

The MP model has been criticized by Lagos [31] because a reduced-form matching function fails to provide an explicit account of how informational frictions affect the labor market. Quantitatively, Shimer [50] has also shown that a calibrated version of the MP model can account for only a small fraction of the business cycle volatility of unemployment and vacancies.[5]

Mismatch models: The paper is related to the frictionless models of Lagos [31] and Shimer [51], but the treatment proposed here differs in many ways from previous studies. Mismatch in Lagos [31] and Shimer [51] assume that workers and jobs are related in *fixed proportions* and that the short-side of the market is always served.[6] In both previous settings, prices are exogenous and mismatch takes place because there are locations with

[5] Several attempts have been proposed to reconcile the MP model with business cycle fluctuations based on endogenous labor supply decisions, alternative bargaining arrangements for wages or wage rigidity, alternative calibrations, or the introduction of additional productivity shocks. (Hall [23] presents a detailed survey not attempted here. See also Mortensen and Nagypál [39] and Hagedorn and Manovskii [22].)

[6] An example of labor-market mismatch that has received some attention in labor and urban economics is that of *spatial mismatch.* A spatial mismatch suggests that unemployed workers and firms are not uninformed but that they are in the "wrong location," Coulson et al. [13]; Brueckner and Martin [10]. Shimer [51] discusses additional examples of disequilibrium views of mismatch and proposes a random assignment related to Lagos [31]. Efficiency considerations in random assignments are discussed by Hylland and Zeckhauser [25]. Sattinger [47] and Stevens [52] discuss applications of queueing theory relevant for mismatch.

excess supply of workers and locations with excess supply of jobs (although in Shimer [51] the value of the excess supply of labor is zero).

Lagos [31] studied the matching of taxis and passengers. In Lagos [31], taxis are free to locate at any point in a linear city but the market assignment features vacant taxis and unserved passengers. Mismatch exists only when there is a small number of taxis. Our frictionless model generalizes Lagos [31] in a familiar labor market assignment setting. We also make explicit the conditions needed for mismatch. We show that a mismatch is possible even if the number of jobs is small. In Lagos [31], there is an excess supply of workers only when the aggregate number of jobs is small. But excess supply of workers could also arise if the opportunity cost of a job is sufficiently high. Since the aggregate number of jobs and their opportunity cost bear no relation in disequilibrium models, both factors can generate a frictionless mismatch.[7] This possibility relates the assignment model studied by Lagos [31] to the analysis of frictionless unemployment proposed by Akerlof [1] and emphasizes the role of the opportunity cost of a job in unemployment. Finally, we show that the alternative view proposed by Lagos [31] is based on a *knife-edge* condition and on an *inefficient* assignment of jobs.

Equilibrium unemployment: We consider several notions of equilibrium unemployment. Under UST, there are workers who stay in their current location and speculated on receiving a job but at the end of the trading round were not able to trade.[8] Following the

[7] One can hold the analysis of Lagos [31] as an example of disequilibrium because prices are exogenously fixed and not explained endogenously in the model. The assignment function in Lagos [31] is a *Nash equilibrium*. Because prices are a free parameter, variations in prices can also generate mismatch. This possibility was not explored by Lagos [31]. There are no efficiency considerations in Lagos [31] either.

[8] In a frictionless model, equilibrium unemployment involves the lack of a job but a zero market price. This is the case since market clearing needs the *value* of excess supply to be zero. In Shimer [51], for example, this notion of market clearing is obtained by assuming Bertrand competition between firms and an exogenous return to non-market activities. This form of unemployment was studied by Akerlof [1] and

literature, see, e.g., Jones and Riddell [26], this notion of unemployment can be seen as *waiting unemployment*. The notion of waiting complements the idea of rest unemployment in Alvarez and Shimer [3]; Jovanovic [27]; Gouge and King [19], but it differs from the 'rest' literature since workers here are stationed in a given location waiting for their trading signal but these workers strictly prefer the option of waiting to any exogenous non-market or search return.[9]

The second notion of frictional unemployment is *search unemployment*. Search takes place as in Lucas and Prescott [34] so unemployment represents workers who are in transit between different job locations.[10] In our search economy, because of job mobility, the aggregate distribution of the labor force is a state variable. Moreover, because locations are heterogeneous, equilibrium prices generally depend on the distribution of people across locations. To establish the existence of a sequential equilibrium we rely on results from anonymous sequential games studied by Bergin and Bernhardt [5] and [6], and from Miao's [36] competitive analysis. As recognized by the literature, see, e.g., Kubler and Schmedders [29], models with individual stochastic heterogeneity and aggregate shocks are an attractive alternative to representative agent models but there is yet no agreed view on how to simulate these models because the choice of the state vector has consequences for the recursive representation of the economy.

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[2]. See also Birchenall [8] and Lucas [33] for related analyses.

[9] A potential drawback in the notion of unemployment in Shimer [51] and the notion of rest unemployment in Alvarez and Shimer [3], Gouge and King [19], and Jovanovic [27] is that unemployed workers earn a non-market return in the form of 'leisure.' These workers are clearly non-employed but it is unclear if they are unemployed or outside of the labor force, Jones and Riddell [26]. Here, workers are waiting for a job to start in the near future and they can be counted as outside of the labor force until uncertainty resolves.

[10] Rocheteau and Wright ([44], 188-192) studied search in a monetary economy similar to Lucas and Prescott [34] and introduced a notion of a vacancy under the assumption that not all agents enter into a competitive market. The notion of a vacancy here, a notion based on uncertain though not restricted trade, is obviously very different from that in Rocheteau and Wright [44].

3 A frictionless model

Consider an economy populated by workers who differ by their type. We associate the measure space $(\mathbf{X}, \mathcal{A}, \lambda)$ to the types which can be seen as skills following Akerlof [1] or locations following Lucas and Prescott [34]. We assume that $\mathbf{X}$ is a compact set and that λ is non-atomic.

Agents are represented by the non-negative and measurable function $l(x)$ which gives the labor force in location x. The mass of agents in $A \in \mathcal{A}$ is $\mu(A) = \int_A l(x) d\lambda$. Since $l(x)$ is a nonnegative function in $\mathbf{X}$, the nonatomic measure μ on $\mathcal{A}$ is specified via the function l relative to λ. Alternatively, μ has a density l with respect to λ, see, e.g., Billingsley ([7], Section 16). By convention, the total mass of agents is normalized to one, $\mu(\mathbf{X}) = 1$. Information about skills and market conditions is perfect.

Jobs are assignments of capital. The joint output of an individual in location x and capital k is $\phi(x)f(k)$ with $\phi : \mathbf{X} \to \mathbb{R}$ continuous, nonnegative and increasing for almost every x in $\mathbf{X}$ with $\phi(0) \geq 0$, and $f : \boldsymbol{K} \to \mathbb{R}$ continuously differentiable, strictly increasing, and strictly concave with $\lim_{k\to\infty} f_k(k) = 0$. The product of $\phi(x)$ and $f(k)$ is assumed to be essentially bounded. The capital assigned to a subset $B \in \mathcal{A}$ of agents is: $\int_B k(x)l(x)d\lambda$, also equal to $\int_B k(x)d\mu$. Feasibility then requires: $\int_{\mathbf{X}} k(x)d\mu \leq K$.

Unemployment: Given K and μ, the value of the assignment is:

$$W(K, \mu) = \max\left\{\int_{\mathbf{X}} \phi(x)f(k(x))d\mu : \int_{\mathbf{X}} k(x)d\mu \leq K\right\}.$$

In the next Theorem we establish the basic existence result:

Theorem 1 (Existence and uniqueness) *For a given pair* (K, μ)*, there is a unique job assignment* $k : \mathbf{X} \to \mathbf{K}$ *which is optimal for* $W(K, \mu)$*.*

Because of strict concavity in $f(k)$, there is a unique solution to the job assignment problem. Moreover, since the feasibility constraint is an isoperimetric side condition, the assignment can be characterized by the Kuhn-Tucker Theorem:

Proposition 1 (Frictionless unemployment) *Let* r *be the opportunity cost of a job and assume* $f_k(0) = \boldsymbol{\rho} < \infty$*. Then, it is optimal to assign no jobs to locations with* $\phi(x)\boldsymbol{\rho} < r$*.*

Proof. The opportunity cost of a job is the Lagrange multiplier on the feasibility constraint. Optimality requires $[\phi(x) f_k(k(x)) - r]\, k(x) = 0$, for almost all x in $\boldsymbol{X}$. If $f_k(0) = \boldsymbol{\rho} < \infty$, it is optimal to assign no jobs to (almost) all locations x for which $\phi(x)\boldsymbol{\rho} - r < 0$ because those locations would misuse scarce jobs. ■

There are no special difficulties with the assignment of jobs described in Proposition 1.[11] In the assignment, r determines the *extensive margin* for employment or the set of locations with jobs, and the *intensive margin* or the amount of jobs to be assigned to each employable location. (A similar distinction is present in Lucas [33] for the overtime margin.) For example, it is optimal to assign no jobs to workers who are unable to cover the opportunity cost of a job as any assignment will be a diversion from profitable opportunities (Akerlof [1] and [2]).

[11] The setting in the model is a simplification of non-atomic assignment models, e.g., Grestky et al. [20]. In Grestky et al. [20], the solution is characterized in a space of nonnegative measures. Grestky et al. [20] show that $W(K, \mu)$ is continuous, increasing, bounded and differentiable in K. $W(K, \mu)$ is linearly homogeneous in μ (i.e., $W(K, \alpha\mu) = \alpha W(K/\alpha, \mu)$ for $\alpha > 0$) so the problem can be seen as a linear optimization on a convex domain. Overall, the analysis of Grestky et al. [20] encompasses a larger class of objects than the ones considered in this paper because capital is homogeneous here.

Under typical conditions, the feasibility constraint determines the opportunity cost of a job, $r(K, \mu)$. Equilibrium unemployment is then given by the measure of workers in *excess supply*:

$$U(K, \mu) = 1 - \int_{\phi(x)\boldsymbol{\rho} \geq r(K,\mu)} d\mu. \tag{1}$$

Unemployment represents a situation in which potential workers are unable to rent the capital needed so that they can be employed. Unemployment, however, is consistent with a Walrasian (or competitive) equilibrium since equilibrium prices ensure that the *value* of excess supply is zero. In a decentralized assignment, wages would satisfy: $w(x; r(K, \mu)) = \phi(x) f(k(x; r(K, \mu))) - r(K, \mu) k(x; r(K, \mu))$ for all x in $\mathbf{X}$. Thus, the labor market clears: workers of sufficiently low productivity are in excess supply but they will not earn positive wages if hired.

Mismatch: A Leontief technology generates unemployment because if $f(k) = \min[k, 1]$, $\boldsymbol{\rho} = 1$ at all values of capital (see, e.g., Akerlof [1]).[12] Under some special conditions considered by Lagos [31], a Leontief technology also gives rise to a frictionless mismatch. The Planner assignment is:

$$W(r, K, \mu) = \max \left\{ \int_{\mathbf{X}} \phi(x) \min[k(x), 1] \, d\mu : \int_{\mathbf{X}} k(x) d\mu \leq K \right\}. \tag{2}$$

Before describing the Planner solution, it is illustrative to consider a market assign-

[12] Lagos' [31] model describes the production of taxi-rides. In Lagos [31], $k(x)$ denotes the taxis per capita assigned to location x and $f(k(x)) = \min[k(x), 1]$ represents the probability that a passenger located in x will find a taxi; i.e., if $k(x) \geq 1$ the probability of finding a taxi is one and if $k(x) < 1$, the probability is $k(x)$. Mortensen [38], Nagypál [35], and Shimer [51] also rely on a Leontief technology to describe micro-level meeting technologies in which worker-job matchings are one-to-one and the short side is served.

ment. As in Lagos [31], assume that average productivity is equal across locations:

$$\left(\phi(x)\min\left[1, \frac{1}{k(x)}\right] - r\right) k(x) = 0. \tag{3}$$

Equation (3) also identifies two margins in the assignment of jobs. First, since $\boldsymbol{\rho} = 1$, workers with $\phi(x) < r$ should be unemployed (as in Proposition 1). Second, for employed workers, job assignments are such that $\phi(x)/k(x) = r$. The opportunity cost r, however, is exogenous and there is no market clearing:

Proposition 2 (Frictionless mismatch) *Assume workers and jobs are related in a fixed proportion, i.e., $f(k) = \min[k, 1]$. Then, given (r, K, μ) the competitive assignment in equation (3) exhibits excess supply of jobs in almost all locations with $\phi(x) > r$ simultaneously with excess supply of workers in almost all locations with $\phi(x) < r$. If $\phi(0) \geq r$, workers are in excess supply if $rK < \int_{\mathbf{X}} \phi(x) d\mu$.*

Proof. The excess supply of labor is as in equation (1). Since $k(x) = \phi(x)/r > 1$ for $\phi(x) > r$, the number of jobs assigned per worker is larger than one. This excess supply of jobs exists in almost all employable locations and regardless of the number of jobs available in the market. The total amount of jobs assigned is: $\int_{\phi(x)\geq r} k(x)d\mu = \frac{1}{r}\int_{\phi(x)\geq r} \phi(x)d\mu$. Notice that when $\phi(0) \geq r$, all workers are employable but the assignment would generate excess supply of workers if K is small. ■

Proposition 2 describes a market in which vacant jobs and unemployed workers coexist as excess supply of jobs and excess supply of workers. The assignment of jobs is described in Figure 1. To interpret the proposition and the figure, first recall that the mass $\mu\{x :$

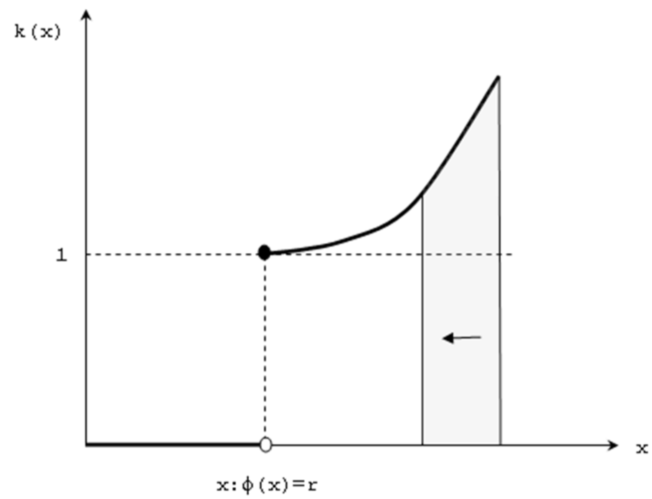

Figure 1: Frictionless mismatch. More attractive types are employed first and in excess supply of jobs (the shaded area) . The excess supply of workers depends on r and K. If $\phi(0) \geq r$, all workers should receive jobs (if feasible) but if K is small, the non-shaded areas will not receive jobs so they will also feature excess supply of workers (and hence mismatch).

$\phi(x) < r\}$ is in excess supply (as in Proposition 1). Notice also that jobs are assigned in excess supply. For types x with $\phi(x) > r$ there are more jobs than workers, $k(x) > 1$ (see Figure 1). Notice also that even if $\phi(0) \geq r$, there may still be mismatch if K small. This case is discussed in Lagos ([31], Proposition 2).

In summary, frictionless mismatch is possible because workers and jobs are in excess supply. This is so either because there are few jobs to be assigned or because the opportunity cost of a job is too high. One can argue that K and r should be related but since there is no equilibrium in the model, variations in K and in r are entirely independent.[13] A frictionless mismatch, however, is clearly *inefficient* and holds only as a *knife-edge*:

[13] Here there is an 'aggregate matching function' $\min\{K, r^{-1}\int_{\phi(x)\geq r}\phi(x)d\mu\}$ that specifies the total number of assignments. Assume $\phi(0) = r$. In this case, the matching function is $\min\{\boldsymbol{\phi}K, \mu(\mathbf{X})\}$, with $\boldsymbol{\phi} = \phi(0) \div \int_{\mathbf{X}} \phi(x)d\mu$, which is the exact form presented in Lagos ([31], Section V). Also, "market clearing" is exceptional. The number of jobs assigned equals the number of workers only at the marginally employed location; see equation (3) and Figure 1.

Corollary 1 (Efficiency) *The market assignment in equation (3) is socially inefficient.*

Corollary 2 (Knife-edge) *Let* $f(k) = \left[\alpha k^{(\boldsymbol{\sigma}-1)/\boldsymbol{\sigma}} + (1-\alpha)\right]^{\boldsymbol{\sigma}/(\boldsymbol{\sigma}-1)}$ *denote the production function of the job assignment. If* $\boldsymbol{\sigma} = 0$*, workers and jobs are in excess supply in a market assignments as shown in Proposition 2. If* $0 < \boldsymbol{\sigma} < 1$*, workers are in excess supply as shown in Proposition 1 but there is no excess supply of jobs, while if* $\boldsymbol{\sigma} \geq 1$ *there is no excess supply of jobs or workers.*

In a Planner assignment (2), types with $\phi(x) < r$ will receive no jobs, while types with $\phi(x) \geq r$ will receive at least one job per worker. This extensive margin is identical in both arrangements. The intensive margin in a Planner assignment is simply given by $k(x) = 1$ for employable types. Both differ because the market will not take into account the marginal reduction in worker-job finding rates imposed as more jobs are given to productive workers. It is possible to see that a competitive case generates 'congestion' because the market assignment equalizes *average* and not *marginal* productivity as needed for efficiency.[14] In fact, in a Leontief case, marginal conditions cannot be used to describe optimal allocations.

As Corollary 2 shows, in a CES technology with $0 < \boldsymbol{\sigma} < 1$, $\boldsymbol{\rho} = \alpha^{\boldsymbol{\sigma}/(\boldsymbol{\sigma}-1)}$ there will be some unemployment as in Proposition 1. However, if some substitution is allowed, the excess supply of jobs in the most attractive locations disappears and the model reaches a market equilibrium.

[14] The inefficiency in Lagos [31] is typical of directed search or common resource problems (e.g., in the 'tragedy of the commons'). In the directed search literature, an additional job offer lowers the probability of having a successful match in a given micro-market and this generates an externality, see Moen [37]. In Lagos' [31] taxicab economy congestion is easier to describe. The benefit from each location, $\phi(x)$, times the probability that each taxi finds a passenger in such location, $1/k(x)$, are equal for all taxis. As all taxis are indifferent across locations, a competitive assignment is not optimal.

4 A frictional model

This section describes an assignment of jobs in which mismatch arises optimally and as part of a competitive market equilibrium. There is no restriction on how workers and jobs are related and prices are endogenous. Once we describe the nature of the informational friction and the basic assignment model, we consider a general model and its implications for the business cycle properties of the labor market.

4.1 Uncertain and sequential trade

Trading frictions here are as in Prescott [43] and Butters [9]. In order to describe the informational friction, it is useful, as in Eden [14] and [16], to think of heterogeneous workers who arrive in batches. Workers arrive sequentially to the market. There is no uncertainty on the size of each batch of workers, $l(x)$, although such a possibility could be allowed.

Let N denote the number of workers who will arrive to the market. We assume that N is a random variable defined on $([0,1], \mathcal{L}, \Pi)$. Since $\mu(\mathbf{X}) = 1$, N will take values on the unit interval. If the realized value of N is $\tilde{N}$, trade will take place for workers with $\mu(\{x' \in \mathbf{X} : x' \geq x\}) \leq \tilde{N}$. If $\tilde{N} = 1$, all workers will arrive. If workers are distributed according to $\mu(\mathbf{X})$, the probability that workers whose type is at least x get to trade (or that trade in location x takes place) is:

$$q(x, \mu) = \Pr\{N \geq \mu(\{x' \in \mathbf{X} : x' \geq x\})\} = \int_{\mu(\{x' \in \mathbf{X} : x' \geq x\})}^{1} d\Pi.$$

An alternative view of the trading friction can be given in terms of the *markets* that open and not the number of *workers* who will arrive.[15] In this alternative view, N will take values in $(\mathbf{X}, \mathcal{A}, \Pi)$ and the probability of trade will be given by $q(x) = \Pr\{N \geq x\}$. In both cases, the trading frictions give a notion of uncertain or limited market participation. The distinction between whether markets or workers are prevented from trading is not important because there is no coordination in the economy. Uncertainty, notice, is sequential because $q(x, \mu)$ is decreasing in x (not in real time though).

4.2 The basic assignment

We first consider a static assignment. Since firms are risk neutral and job assignments are decided before N is known, the job assignment is given by:

$$W(K, \mu, q) = \max\left\{\int_{\mathbf{X}} \phi(x) f(k(x)) \Pr\{N \geq \mu(\{x' : x' \geq x\})\} d\mu : \int_{\mathbf{X}} k(x) d\mu \leq K\right\}. \quad (4)$$

As evident from the expression above, the setting of the problem is almost identical to the previous section so the existence results of Theorem 1 apply as well as the characterization of Proposition 1. For example, the first order condition for positive job assignments is: $\phi(x) f_k(k(x)) = r / \Pr\{N \geq \mu(\{x' \in \mathbf{X} : x' \geq x\})\}$.[16] Thus, the opportunity cost of a job r and the probability that an assignment for workers of type x takes place are traded-

[15] The trading frictions can be related to Calvo's [11] staggered price-setting model. In Calvo [11], firms receive a price-change signal that allows them to change their posted prices. In this paper, workers or markets will receive a stochastic and sequential signal –the green light– to trade. (The sequential resolution of uncertainty is also related to first-come first-serve sequential services typical of models of bank runs where the number of market participants is also an important variable.) We have assumed uncertainty about trading for low types but a lower limit of integration $\underset{\sim}{x}$ and a proper rescaling of the conditional probability of trade with $q(\underset{\sim}{x}, \mu) = 1$ will suggest that all workers with $x \leq \underset{\sim}{x}$ will trade with certainty.

[16] To highlight the difference of the UST model and standard treatments of uncertainty, assume a random fraction of all workers fails to arrive to the market. The fraction is denoted by $0 \leq F \leq 1$. Jobs must be assigned before F is known. The optimal assignment of jobs to workers with skill x satisfies: $\mathbb{E}\left[\phi(x) f_k\left(k(x)/F\right)\right] = r$. This paper can be generalized along this direction by having F play a role.

off. If trading is unlikely for x-type workers, a few jobs will be assigned for such a group because capacity will be misallocated if trade does not take place.

The properties of the assignment are also similar to the previous economy. Here, however, mismatch takes place as an equilibrium outcome:

Proposition 3 (Frictional mismatch) *Given* (K, μ, q)*, the solution to the static job assignment in equation (4) features mismatch. Unemployment and vacancies are:*

$$U(K, \mu, q) = 1 - \int_{\phi(x)q(x,\mu) \geq r(K,\mu,q)/\boldsymbol{\rho}} q(x, \mu) d\mu, \tag{5}$$

$$V(K, \mu, q) = \int_{\phi(x)q(x,\mu) \geq r(K,\mu,q)/\boldsymbol{\rho}} k(x)[1 - q(x, \mu)] d\mu, \tag{6}$$

and the opportunity cost of of a job $r(K, \mu, q)$ *is uniquely determined.*

Proof. First notice that there is a unique opportunity cost of capital r that ensures that all jobs are assigned ex-ante. In equilibrium, employment is the mass of workers with jobs in markets with trade, i.e., $E(K, \mu, q) = \int_{\phi(x)q(x,\mu) \geq r(K,\mu,q)/\boldsymbol{\rho}} q(x, \mu) d\mu$. Unemployed workers in equation (5) are of two kinds. First, some workers are in excess supply (as in Proposition 1), and second, some workers expected a job assignment and positive wages but failed to trade in the market. Following the literature, we consider this second notion of unemployment as *waiting unemployment*.

Vacancies are described next. As before, $k(x; r(K, \mu, q))$ denotes the ex-ante assignment of jobs. By the informational friction, some assigned jobs will be productive while some will be vacant. The expected number of worker-job matches is the number of assigned jobs to markets that open:

$$M(K,\mu,q)=\int_{\phi(x)q(x,\mu)\geq r(K,\mu,q)/\boldsymbol{\rho}} k(x;r(K,\mu,q))q(x,\mu)d\mu,$$

while job vacancies are the expected value of job assignments to markets that failed to open, $K-M(K,\mu,q)$ which is equation (6). ■

Under appropriate conditions, the assignment can be interpreted as a *market economy*:

Definition 1 (Competitive market equilibrium) *Given (K,μ,q), a competitive equilibrium for the job assignment (4) is a set of state-dependent market wages, a market price for capital, and an ex-ante assignment of jobs such that, taking prices and the probability of trade as given, job assignments maximize profits and markets clear.*

In terms of the *welfare implications*, the two welfare Theorems hold:

Theorem 2 (Welfare Theorems) *A competitive assignment of jobs in the frictional economy is Pareto optimal and the Pareto assignment can be implemented in a competitive market.*

Proof. The proof is standard as the relevant Pareto optimum is the one that maximizes the value of the job assignment. Finding the prices that decentralize the assignment is also a standard exercise. In a competitive economy, $r(K,\mu,q)$, the opportunity cost of a job (or the shadow price of capital) coincides with the market price. Wages are given by $w(x;r(K,\mu,q))=\phi(x)q(x,\mu)f(k(x;r(K,\mu,q)))-r(K,\mu,q)k(x;r(K,\mu,q))$. These wages decentralize the assignment. ■

The structure of unemployment, market wages, and the seemingly wage rigidity are discussed next. In the frictional model, unemployment would still take place at positive advertised wages or if $f_k(0) = \infty$. It is important to stress that this notion of unemployment is not based on excess supply. Some workers in equation (5) expected to receive a job assignment but were prevented from so by aggregate trading frictions. Unemployment is also socially efficient.

Wages are considered next. Wages in the frictional model are determined in perfectly competitive markets and hence flexible but they look unresponsive to market conditions or rigid because advertised wages will not change in response to the arrival of workers to the labor market. Assume trading has started and x markets have traded. The markets that have yet to open are markets for types with $x' \geq x$. By Bayes' rule, the expected productivity for markets $x' \geq x$, conditional on market x opening, is

$$\phi(x')f_k(k(x'))q(x', \mu|x) = \frac{\phi(x')f_k(k(x'))q(x', \mu)}{q(x, \mu)}. \tag{7}$$

The expected cost for a job assigned to workers with types $x' \geq x$, conditional on the trade for types x, is given by $r(K, \mu, q)/q(x, \mu)$. As it follows from equation (7), trade in x does not provide an incentive to reassign jobs to markets for types above x. As job assignments are unchanged, advertised wages will be unresponsive to the arrival of workers, see also Landsburg and Eden [32] and Eden [14].[17]

[17] As Eden [14] notes, it is important to distinguish between advertised and observed wages. 'Advertised' wages in equilibrium are given by $w(x, q(x, \mu); r(K, \mu, q))$ while observed wages only reflect a partial list of contracts. If only observed wages are considered to describe the labor market, the commodity space will clearly be misspecified.

4.3 The general assignment

This section generalizes the earlier analysis in directions that make the model applicable to quantitatively study the business cycle properties of the labor market. The main contribution of this sub-section is to allow for job creation, worker search, and to introduce neutral productivity shocks as drivers of business cycle fluctuations.

Shocks: Time is discrete and there are infinite number of periods. There is a single firm and a single representative consumer. At the beginning of period t, each location $x \in \mathbf{X}$ experiences an *idiosyncratic* productivity shock. Idiosyncratic shocks are defined on $(\mathbf{\Omega}, \mathcal{F})$ and given as a measurable function $\phi : \mathbf{X} \times \mathbf{\Omega} \to \mathbb{R}_+$. That is, for any $x \in \mathbf{X}$, the function $\phi(x, \cdot)$ is $(\mathcal{F}-)$ measurable and for any $\omega \in \mathbf{\Omega}$, the function $\phi(\cdot, \omega)$ is $(\mathcal{A}-)$ measurable. The transition function is $\Phi(A, x) = \Pr(\{\omega : \phi_{t+1}(x, \omega) \in A\})$ for all $A \in \mathcal{A}$. A location's productivity and its type are intimately related. The interpretation of idiosyncratic shocks is that if ω is drawn from $\mathbf{\Omega}$, location x's new 'type' is $\phi_{t+1}(x, \omega)$. The initial value $\phi_0(x)$ is a given and nonstochastic.

Let $\mathbf{z}_t$ denote the current *aggregate* productivity shock defined on $(\mathbf{Z}, \mathcal{Z})$ and let $\mathbf{z}^t = (\mathbf{z}_t, \mathbf{z}^{t-1})$ be the history of shocks defined in $\mathbf{Z}^t = \mathbf{Z} \times \mathbf{Z}^{t-1}$. We assume $\mathbf{Z}$ is countable and that $\mathbf{Z}$ and $\mathbf{\Omega}$ are compact metric spaces. The value of $\mathbf{z}_0$ is given. Aggregate shocks are a Markov process with transition function, $Q : \mathbf{Z} \times \mathcal{Z} \to [0, 1]$. The transition functions have the Feller property as defined by Stokey, Lucas, and Prescott ([53], Chapter 12).

State variables: Let $\mathcal{P}(\mathbf{X}, \mathcal{A})$ be the set of probability measures on $\mathbf{X}$ and define $(\mathbf{S}, \mathcal{S})$ by $(\mathbf{K} \times \mathbf{Z}, \mathcal{K} \times \mathcal{Z})$. The *state* in this economy is the distribution of the labor force across locations μ_t, the aggregate productivity shock $\mathbf{z}_t$, and the aggregate number of jobs K_t.[18] We write the state of the economy as a pair $(\mathbf{s}_t, \mu_t)$ that lies in $\mathbf{S}^t \times \mathcal{P}(\mathbf{X}, \mathcal{A})$. In order to properly treat the distribution of the labor force as a state variable, we assume that idiosyncratic shocks do not induce aggregate uncertainty. Because idiosyncratic shocks generate a transition in types, the distribution of the labor force in the future $\tilde{\mu}_{t+1}(\{\phi_{t+1}(x,\omega) \in A\})$ is a random measure, i.e.,

$$\tilde{\mu}_{t+1}(A) = \int_{\mathbf{\Omega}} \tilde{\mu}_{t+1}(\{\phi_{t+1}(x,\omega) \in A\})\Phi(x, d\omega).$$

Under the *no aggregate uncertainty* assumption put forward by Bergin and Bernhardt [5] and [6], the distribution of the labor force becomes deterministic $\mu_{t+1}(A) = \int_{\mathbf{X}} \Phi(A,x)\mu_{t+1}(dx)$ for all realizations $\omega \in \mathbf{\Omega}$ and all $A \in \mathcal{A}$. For this assumption to hold, the distribution of individual shocks across locations $\mathbf{X}$, for a given ω, is the same as the distribution over states $\mathbf{\Omega}$ for any given location. This interpretation is in the spirit of the Law of Large Numbers for the continuum, see , e.g., Bergin and Bernhardt [5] and Feldman and Gilles [18].[19]

Decisions about job creation are analogous to decisions about capital accumulation. There is an initial aggregate number of jobs K_0. Given a history of aggregate variables $\mathbf{s}^t$

[18] The state of the economy differs from the one in Lucas and Prescott [34] since the opportunity cost of a job, and hence wages, depends on the *distribution* of the labor force. In Lucas and Prescott [34], the state of the economy is the size of the workforce and the productivity shock of a *representative* micro-market. In Gouge and King [19], there are aggregate productivity shocks but no capital accumulation so the distribution of the labor force is not a state variable in their setting.

[19] Abusing notation, one can write $x_{t+1} = \phi_{t+1}(x_t, \omega)$. Then, $\tilde{\mu}_{t+1}(\{\phi_{t+1}(x_t,\omega) \in A\})$ can be seen as $\tilde{\mu}_{t+1}(\{x_{t+1} \in A\})$ which is a random measure. The no aggregate uncertainty condition assumes a stochastic process whose transition $\Phi(A, x_t)$ can be chosen independent of x_t. That is, it assumes that $\tilde{\mu}_{t+1}(\{x_{t+1} : \phi_{t+1}(x_t,\omega) \in D\}) = v(D)$ for all $\omega \in \mathbf{\Omega}$ and that $\Pr(\{\omega : \phi_{t+1}(x_t,\omega) \in D\}) = v(D)$ for all $x_t \in \mathbf{X}$. The no aggregate uncertainty condition we consider, as in Bergin and Bernhardt [5], is conditional on a given history of aggregate shocks.

and a distribution of the labor force $\mu_t(\mathbf{s}^{t-1}, \mu^{t-1})$, the job creation rate is $i_t(\mathbf{s}^t, \mu_t)$. Jobs are destroyed exogenously at a rate δ, i.e., $K_{t+1}(\mathbf{s}^t, \mu_t) = i_t(\mathbf{s}^t, \mu_t) + (1-\delta)K_t(\mathbf{s}^{t-1}, \mu_{t-1})$ for all possible histories of aggregate variables. Finally, the price of a unit of output delivered in period t is $p_t(\mathbf{s}^t, \mu_t)$.

In what follows we consider a scarce-land case. Production is $f(k, e)$. The assignment problem consists of allocating jobs and workers across locations. The number of jobs per employed worker in x is a function $k_t(x, \mathbf{s}^t, \mu_t)$ with $k_t : \mathbf{X} \times \mathbf{S}^t \times \mathcal{P}(\mathbf{X}, \mathcal{A}) \to \mathbf{K}$. Employment in location x is $e_t(x, \mathbf{s}^t, \mu_t)$ but employment is allowed to differ from the labor force available at the beginning of period t, $l_t(x, \mathbf{s}^{t-1}, \mu_{t-1})$. We assume that $f(k, e)$ is strictly decreasing and convex in e and that $\lim_{k\to\infty} f_k(k, e) = 0$ and $\lim_{e\to 0} f_e(k, e) = \infty$. As in the previous sub-section, $q_t(x, \mu_t)$ is the probability of trade in location x but N_t is not a state variable.

Worker search: The following discussion about worker search is inspired by Lucas and Prescott [34]. Workers are allowed to move between locations before trading begins and after observing $\mathbf{z}_t$ and $\phi_t(x, \omega)$. Movements are of two types. Workers who move *out* of location x are represented by $m_t^-(x, \mathbf{s}^t, \mu_t)$ while workers who move *in* are $m_t^+(x, \mathbf{s}^t, \mu_t)$. No location will have workers leaving and arriving in the same period. Search is frictional because workers who move in forego their employment opportunities for one period. Employment is $e_t(x, \mathbf{s}^t, \mu_t) = l_t(x, \mathbf{s}^{t-1}, \mu_{t-1}) - m_t^-(x, \mathbf{s}^t, \mu_t)$ while workers who arrive to any given location due so just before the beginning of the next round of trade, i.e.,

$$l_{t+1}(x, \mathbf{s}^t, \mu_t) = l_t(x, \mathbf{s}^{t-1}, \mu_{t-1}) + m_t^+(x, \mathbf{s}^t, \mu_t) - m_t^-(x, \mathbf{s}^t, \mu_t). \tag{8}$$

The general assignment problem is defined next:

Definition 2 (General assignment) *A worker-job assignment is a sequence of opportunity costs of search and opportunity costs of a job,* $\{\theta_t(\mathbf{s}^t,\mu_t), r_t(\mathbf{s}^t,\mu_t)\}$*, job creation decisions* $\{i_t(\mathbf{s}^t,\mu_t)\}$*, ex-ante worker search decisions* $\{m_t^+(x,\mathbf{s}^t,\mu_t), m_t^-(x,\mathbf{s}^t,\mu_t)\}$*, and ex-ante job assignments* $\{k_t(x,\mathbf{s}^t,\mu_t)\}$*, that solve:*

$$W_0(\mathbf{s}_0,\mu_0) = \max\left\{\mathbb{E}_t\sum_{t=0}^{\infty} p_t(\mathbf{s}^t,\mu_t)\left[\mathbf{z}_t Y_t(\omega,\mathbf{s}^t,\mu_t) - i_t(\mathbf{s}^t,\mu_t)\right]\right\}, \text{ with}$$

$$Y_t(\omega,\mathbf{s}^t,\mu_t) = \int_{\mathbf{X}} \phi_t(x,\omega) q_t(x,\mu_t) f(k_t(x,\mathbf{s}^t,\mu_t), e_t(x,\mathbf{s}^t,\mu_t)) e_t(x,\mathbf{s}^t,\mu_t) d\lambda,$$

subject to feasibility in job assignments: $\int_{\mathbf{X}} k_t(x,\mathbf{s}^t,\mu_t) e_t(x,\mathbf{s}^t,\mu_t) d\lambda \leq K_t(\mathbf{s}^{t-1},\mu_{t-1})$*, and* $K_{t+1}(\mathbf{s}^t,\mu_t) = i_t(\mathbf{s}^t,\mu_t) + (1-\delta)K_t(\mathbf{s}^{t-1},\mu_{t-1})$*, and feasibility in worker assignments: equation (8) and*

$$\int_{\mathbf{X}} m_\mu^+(x,\mathbf{s}^t,\mu_t) d\lambda = \int_{\mathbf{X}} m_\mu^-(x,\mathbf{s}^t,\mu_t) d\lambda, \tag{9}$$

for all $t \geq 0$ *and for all* $\mathbf{s}^t$ *in* $\mathbf{S}^t$.

The aim of the general problem is to assign workers and jobs. Feasibility in job assignments is as before and hence not discussed. The only new decision for the firm is $i_t(\mathbf{s}^t,\mu_t)$. Its first order condition is: $p_t(\mathbf{s}^t,\mu_t) = \mathbb{E}_t\left[p_{t+1}(\mathbf{s}^{t+1},\tilde{\mu}_{t+1})\left(1 + r_{t+1}(\mathbf{s}^{t+1},\tilde{\mu}_{t+1}) - \delta\right)\right]$, with intertemporal prices to be determined from the consumption side of the economy. Expectations are taken with respect to $\mathbf{s}_{t+1}$ using standard notation.[20]

[20] That is, let $P : (\mathbf{S},\mathcal{S}) \to [0,1]$ be the transition function for $\mathbf{s}_t = (K_t,\mathbf{z}_t)$. This transition function is standard, see Stokey, Lucas, and Prescott ([53], Theorem 9.13). Decisions about job creation depend on the decisions of the representative consumer, but since this problem is standard, decisions about job creation are continuous so the transition function for $\mathbf{s}_t$ has the Feller property, see Stokey, Lucas, and Prescott ([53], Theorem 9.14).

The main requirement for equilibrium is that the distribution $\tilde{\mu}_{t+1}$ has to be 'consistent' with the implied law of motion of the labor force. Because the aggregate distribution of the labor force is a state variable, we first condition on the *sequence* of aggregate distributions and then we study the consistency requirements of equilibria. (This approach is essentially the one in Bergin and Bernhardt [5] and Miao [36].) As we noted above, μ_t is a function from $\mathbf{S}^t \times \mathcal{P}(\mathbf{X}, \mathcal{A})$ to $\mathcal{P}(\mathbf{X}, \mathcal{A})$. Let $\mathcal{P}(\mathbf{X}, \mathcal{A})^{\mathbf{S}^t}$ denote the set of such functions. Let $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A}) = \times_{t=1}^{\infty} \mathcal{P}(\mathbf{X}, \mathcal{A})^{\mathbf{S}^t}$ and endowed it with the product (or pointwise convergence) topology. A sequence of distributions $\{\mu\}$ is an element of $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$.

The Euler equations for worker mobility together with market clearing fully characterize the worker assignment in this economy:

Proposition 4 (Search unemployment) *For a given $\{\mu\}$ and a history $\mathbf{s}^t \in \mathbf{S}^t$, there exists a continuous worker assignment $\{m_t^-(x, \mathbf{s}^t, \mu_t), m_t^+(x, \mathbf{s}^t, \mu_t)\}$ that solves $W_0(\mathbf{s}_0, \mu_0)$. The assignment of workers satisfies the following Euler equations:*

$$\left[\theta_t(\mathbf{s}^t, \mu_t) - w_t(x, \cdot) - \frac{\mathbb{E}_t\left[p_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1}) w_{t+1}(\phi_{t+1}(x, \omega), \cdot)\right]}{p_t(\mathbf{s}^t, \mu_t)}\right] m_t^-(x, \mathbf{s}^t, \mu_t) = 0, \quad (10)$$

$$\text{and} \quad \left[\frac{\mathbb{E}_t\left[p_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1}) w_{t+1}(\phi_{t+1}(x, \omega), \cdot)\right]}{p_t(\mathbf{s}^t, \mu_t)} - \theta_t(\mathbf{s}^t, \mu_t)\right] m_t^+(x, \mathbf{s}^t, \mu_t) = 0, \quad (11)$$

with wages given by: $w_t(x, e_t(x, \mathbf{s}^t, \mu_t), \mathbf{s}^t, \mu_t)$ and expectations defined as: $\mathbb{E}_t\left[w_{t+1}(\cdot)\right] = \int_{\mathbf{S}^{t+1}} \int_{\mathbf{\Omega}} w_{t+1}(\phi_{t+1}(x, \omega), e_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \mu_{t+1}), \mathbf{s}^{t+1}, \mu_{t+1}) \Phi(x, d\omega) P(\mathbf{s}^t, d\mathbf{s}_{t+1})$, for all $x \in \mathbf{X}$.

Proof. Notice that since feasibility in worker mobility involves an isoperimetric constraint the assignment can also be characterized by the Kuhn-Tucker Theorem. The multipliers on the isoperimetric constraints are the shadow prices. The opportunity cost of search $\theta_t(\mathbf{s}^t, \mu_t)$ is the Lagrange multiplier on equation (9). As before, wages are: $w_t(x, e_t(x, \mathbf{s}^t, \mu_t), \mathbf{s}^t, \mu_t) = \phi_t(x) q_t(x, \mu_t) \left[f(\cdot) - f_k(\cdot) k_t(x, \mathbf{s}^t, \mu_t) + f_e(\cdot) e_t(x, \mathbf{s}^t, \mu_t) \right]$.

Substitute the definition of employment and equation (8) into the objective function. Then, the previous job assignment becomes a standard optimal dynamic problem. To establish continuity in $m_t^-(x, \mathbf{s}^t, \mu_t)$ and $m_t^+(x, \mathbf{s}^t, \mu_t)$, first use the fact that the transition functions satisfy the Feller property and that continuity for $\mathbf{s}^t$ is preserved by integration, see, e.g., Bergin and Bernhardt ([6], Lemmas 1 and 2). Second, continuity with respect to μ_t follows from the continuity in f and q_t since μ_t affects wages only through the opportunity cost of a job and in q_t. ■

Proposition 4 describes the optimal mobility decisions in the worker assignment. The proposition can be understood as in Lucas and Prescott [34] because there are 3 cases to consider:

Case A: Some or all current workers in x leave, $m_t^-(x, \mathbf{s}^t, \mu_t) > 0$. For these workers, the expected benefit of staying, current and future advertised wages, is smaller than the opportunity cost of moving, $\theta_t(\mathbf{s}^t, \mu_t)$, see equation (10).

Case B: No additional worker arrives and no worker leaves location x. If current and future wages exceed the return to search in equation (10), $m_\mu^-(x, \mathbf{s}^t, \mu_t) = 0$. In this case, workers are not willing to lose their expected wage in order to search and $l_{t+1}(x, \mathbf{s}^t, \mu_t) = l_t(x, \mathbf{s}^{t-1}, \mu_{t-1})$.

Case C: Workers move into x if $\mathbb{E}_t \left[p_{t+1}(\cdot) w_{t+1}(\phi_{t+1}(x, \omega), \cdot) \right]$, the future expected value of the location, is larger than the opportunity cost of searching, $\theta_t(\mathbf{s}^t, \mu_t)$, see equation (11).

Finally notice that for each history $\mathbf{s}^t$, the assignment of jobs and workers needs to be consistent with the evolution of the labor force so $\theta_t(\mathbf{s}^t, \mu_t)$ must ensure that the workers who search for jobs are 'absorbed' by the future labor market. As in Lucas and Prescott [34] notice that the assignment of searchers is arbitrary because search is not coordinated or directed to locations with rents.

In order to close the model, assume the profits generated by the firm are transferred to a representative consumer. Consumption choices are given by $c_t(\mathbf{s}^t, \mu_t)$ with $u : \mathbb{R}_+ \to \mathbb{R}$ as an increasing, strictly concave, continuously differentiable and bounded utility function. The discount factor satisfies $0 < \beta < 1$. For all $\mathbf{s}^t \in \mathbf{S}^t$, the representative consumer makes intertemporal choices based on the following problem:

$$B_0(\mathbf{s}_0, \mu_0) = \max \left\{ \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t(\mathbf{s}^t, \mu_t)) \right] : \sum_{t=0}^{\infty} p_t(\mathbf{s}^t, \mu_t) c_t(\mathbf{s}^t, \mu_t) \leq W_0(\mathbf{s}_0, \mu_0) \right\}. \quad (12)$$

For a given $\{\mu\}$, the Euler equation associated with this problem is:

$$u_c(c_t(\mathbf{s}^t, \mu_t)) = \beta \int_{\mathbf{S}^{t+1}} u_c(c_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1})) \left(1 + r_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1}) - \delta\right) P(\mathbf{s}^t, d\mathbf{s}_{t+1}), \quad (13)$$

which is standard in capital theory, see, e.g., Stokey, Lucas, and Prescott [53].

Equilibrium: Next we examine the evolution of the aggregate distribution of the labor force and establish the existence of a sequential competitive equilibrium. At the end of period t, when searchers arrive, the mass of agents in $A \in \mathcal{A}$ is $\int_A l_{t+1}(x, \mathbf{s}^t, \mu_t) d\lambda$. The distribution of the labor force at the beginning of period $t+1$ is then:

$$\begin{aligned} \mu_{t+1}(A, \mathbf{s}^t, \mu_t) &= \int_{\mathbf{\Omega}} \int_A l_{t+1}(\phi_{t+1}(x,\omega), \mathbf{s}^t, \mu_t) \Phi(x, d\omega) d\lambda, \\ &= \int_{\mathbf{X}} l_{t+1}(x, \mathbf{s}^t, \mu_t) \Phi(A, x) d\lambda, \end{aligned} \tag{14}$$

where the second line follows by the no aggregate uncertainty condition.

All the relevant information needed to describe equilibria in this economy is contained in $\mu_{t+1}(A, \mathbf{s}^t, \mu_t)$. For example, equilibrium requires consistency between the previous economic decisions about job creation, savings, job assignments, and worker mobility with the sequence of distributions $\{\mu\}$ we have considered thus far. That is, any distribution μ_{t+1} on $\mathcal{P}(\mathbf{X}, \mathcal{A})$ whose marginal distribution agrees with the distribution μ_t is consistent with μ_t at time $t+1$. Since equation (14) defines a mapping $T : \mathcal{P}^\infty(\mathbf{X}, \mathcal{A}) \to \mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$, a topological fixed point $\mu^* = T(\mu^*)$ induces a *sequential competitive equilibrium*:

Theorem 3 (Sequential competitive equilibrium) *There exists a sequential competitive equilibrium for the general assignment.*

Proof. The proof follows the arguments in the proof of Miao ([36], Theorem 1) and applies Brouwer-Schauder-Tychonoff Fixed-Point Theorem to T. One needs to show that the domain of T is compact, that T maps $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$ into itself, and that T is continuous. A positive depreciation rate and Inada conditions imply that the number of jobs is uniformly bounded or that there is a maximum sustainable stock of jobs, $\bar{K}$. For every equilibrium sequence, $\bar{K} \geq \int_{\mathbf{X}} k_0(x, \mathbf{s}_0, \mu_0) e_0(x, \mathbf{s}_0, \mu_0) d\lambda$, and $\bar{K} \geq \int_{\mathbf{X}} k_t(x, \mathbf{s}^t, \mu_t) e_t(x, \mathbf{s}^t, \mu_t) d\lambda$ so job assignments are uniformly bounded. (The marginal utility of consumption $u_c(c)$ is assumed bounded so the transversality condition in consumption is satisfied.) The space of aggregate shocks $\mathbf{Z}$ is also bounded and compact by assumption. This makes $\mathbf{S}$ bounded and compact. Because $\mathbf{X}$ is assumed compact, $\mathcal{P}(\mathbf{X}, \mathcal{A})$ is also compact. Endowed with the product (or pointwise convergence) topology, the set $\mathcal{P}^{\infty}(\mathbf{X}, \mathcal{A})$ is then compact. By equations (14) and (9), $T(\mu)$ lies in $\mathcal{P}^{\infty}(\mathbf{X}, \mathcal{A})$. That is, the cost of search ensures that the mapping T returns a distribution function in $\mathcal{P}^{\infty}(\mathbf{X}, \mathcal{A})$. Finally, T is continuous because the mobility decisions and assignments are continuous, see Proposition 4. ■

Characterizing the general assignment is difficult because the distribution of the labor force is a state variable. It is difficult, for example, to establish uniqueness and this has important consequences for the specification of the state vector in a recursive representation, see, e.g., Kubler and Schmedders [29] and Miao [36]. In essence, because workers need to form expectations about future wages, any future equilibrium must be consistent with these expectations. If there are multiple equilibria, workers need additional information to determine which equilibrium was expected last period. The fact that more than one distribution of the labor force may be consistent with current conditions, is a typical generic problem of economies with frictions and heterogeneous agents, see, e.g., Kubler and Schmedders [29].

4.4 The Beveridge curve

This sub-section examines the predicted response of unemployment and vacancies to an aggregate productivity shock and shows that unemployment and vacancies negatively covary tracing out a *Beveridge curve*. Throughout this section we consider allocations along an equilibrium distribution of the labor force $\{\mu^{*}(\mathbf{s}^{\infty}; \mathbf{s}_0, \mu_0)\}$. The employment rate is:

$E_t(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} q_t(x, \mu_t^*) e_t(x, \mathbf{s}^t, \mu_t^*) d\lambda$, or the measure of workers who are not searching for a job and are able to receive jobs. The unemployment rate is:

$$U_t(\mathbf{s}^t, \mu_t^*) = 1 - \int_{\mathbf{X}} q_t(x, \mu_t^*) e_t(x, \mathbf{s}^t, \mu_t^*) d\lambda,$$

with *waiting unemployment* as the measure of workers who do not search but did not receive their assigned jobs, $U_t^{\text{waiting}}(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} [1 - q_t(x, \mu_t^*)] e_t(x, \mathbf{s}^t, \mu_t^*) d\lambda$.

It is important to stress that in contrast to search, where workers are not unemployed in any particular locale, waiting unemployment takes place *in* the labor market while workers wait for the trading signal. Also, unemployment in the general model is not based on excess supply as in the models above. Furthermore, as in equation (6), vacancies are job assignments to locations with employable workers but where trade was not possible:

$$V_t(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} [1 - q_t(x, \mu_t^*)] k_t(x, \mathbf{s}^t, \mu_t^*) e_t(x, \mathbf{s}^t, \mu_t^*) d\lambda. \tag{15}$$

The coexistence of vacancies and unemployment also gives rise to a Beveridge curve:

Proposition 5 (The Beveridge curve) *In the general assignment, in response to a positive productivity shock, vacancies increase and unemployment declines, i.e., the economy traces out a Beveridge curve.*

Proof. The proposition describes an intuitive labor market response to changes in aggregate productivity. First consider vacancies, equation (15). An exogenous increase in productivity influences vacancies by two margins. First, because the optimal job assignment for potentially employable workers satisfies $\mathbf{z}_t \phi_t(x) q_t(x, \mu_t^*) f_k(k_t(x, \cdot), e_t(x, \cdot)) =$

$r_t(\mathbf{s}^t, \mu_t^*)$, an increase in productivity is equivalent to a reduction in the opportunity cost of a job that makes speculation in $k_t(x, \mathbf{s}^t, \mu_t^*)$ less costly. By this channel, vacancies increase when productivity increases.

Second, vacancies also change because the set of potentially employable types changes. The response of employment $e_t(x, \mathbf{s}^t, \mu_t^*)$ depends on the changes in current and expected future advertised wages. Current wages increase because the job assignments increase and because productivity increases. A productivity shock also leads to a dynamic response typical of the real business cycle literature. A positive productivity shock increases expected future productivity and induces households to substitute consumption intertemporally. This dynamic effect induces job creation and raises expected advertised wages causing $e_t(x, \mathbf{s}^t, \mu_t^*)$ to increase. By this channel, vacancies also increase when productivity increases. The response in unemployment is the inverse of the response in the employment margin for vacancies. Because employment increases with $\mathbf{z}_t$, unemployment declines. ■

5 A quantitative exploration [TO BE COMPLETED]

This section provides a parametrization designed to explore the quantitative properties of the model. While the paper is mainly concerned with methodological aspects, it is useful to determine the quantitative predictions in terms of the Beveridge curve and the volatility of unemployment and vacancies.

The model is parametrized next. Assume Π is exponential and uncertainty about market trade. Then, $q(x) = \exp\{-\chi(1-\alpha)x\}$. Assume Cobb-Douglas production function with a constant unit of land in each location, $f(k, e)e = k^\alpha e^{1-b}$ with $0 < \alpha < 1$ and

$0 < b < 1-\alpha$. Assume also a log-utility function, $u(c) = \ln c$ in equation (12). Productivity shocks are given by $\ln \mathbf{z}_t = \rho \ln \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$ with $\sigma^2_{\boldsymbol{\varepsilon}}$ as the variance of $\boldsymbol{\varepsilon}_t$. Idiosyncratic shocks are also AR(1) with the same correlation but variance σ^2_{ϕ}. Wages under the previous parametrization are:

$$w_t(x, e_t(x, \mathbf{s}^t, \mu_t), \mathbf{s}^t, \mu_t) = \frac{1-\alpha-b}{\alpha^{\alpha/(\alpha-1)}} \exp\{-\chi x\} \left[\frac{\mathbf{z}_t \phi_t(x)}{r_t(\mathbf{s}^t, \mu_t)^{\alpha}}\right]^{1/1-\alpha} e_t(x, \mathbf{s}^t, \mu_t)^{-b/(1-\alpha)},$$

with $r_t(\mathbf{s}^t, \mu_t)$ as the equilibrium price from the static job assignment.

The following eight parameters need to be determined. The time discount factor β, the job destruction rate δ, the persistence of productivity shocks ρ, the variance of innovations $\sigma^2_{\boldsymbol{\varepsilon}}$ and σ^2_{ϕ}, the Cobb-Douglas parameters α and b, and the friction parameter χ. Additionally, we need to specify a model period. We consider each period to be a quarter and assume a job destruction rate of $\delta = 10$ percent. This measure is larger than conventional measures of quarterly capital depreciation in real business cycles models but we follow Shimer's [50] estimate of the quarterly exogenous separation rate. The discount factor is assumed to be $\beta = 1/1.045^{1/4}$. In the deterministic steady-state of the Euler equation (13), the opportunity cost of a job would then be 11 percent. For the shares in production we assume $\alpha = 0.35$ and $b = 0.05$.

The parameters that govern $\ln \mathbf{z}_t$ can be estimated directly from quarterly data on labor productivity. Because the production function is Cobb-Douglas, the difference between total factor productivity and labor productivity is not important as both are proportional to each other. As suggested by Shimer [50], we employ the seasonally adjusted quarterly real average output per person in the non-farm business sector constructed by the BLS.

We also employ the Hodrick-Prescott filter with a smoothing parameter of $10,000$. The estimates obtained are $\rho = 0.895$ and $\sigma_{\varepsilon} = 0.0083$. In typical real business cycle models (Kydland and Prescott [30]), the previous parameters are not very different. The estimates of ρ and σ_{ε} are also similar to those in Hagedorn and Manovskii [22]. The volatility of idiosyncratic shocks σ_{ϕ}^2 and χ are key determinants unemployment and vacancies. We calibrate both to obtain an average measure of the total unemployment rate and vacancy rate of 6 and 8 percent.

Table 1. Parameter values for the simulations.

Parameter	Baseline value	Description
β	0.99	Time discount factor
δ	0.10	Job destruction rate
ρ	0.895	Persistence of productivity shocks
σ_{ε}	0.0083	Volatility of aggregate shocks, $\mathbf{z}_t$
σ_{ϕ}	0.415	Volatility of idiosyncratic shocks, $\phi_t(x)$
α	0.35	Capital share in production
β	0.05	Land share in production
χ	0.001	Frictional parameter for market trading

Note: The parameters are discussed in the text. The solution algorithm is in the Appendix.

Because the distribution μ_t is a state variable, we need to reduce the dimensionality of this distribution. Several alternatives are possible, but here we discretize $\mathbf{X}$ and treat μ_t as a vector of dimension $1 \times L$. Then, the state of the economy lies in $\mathbf{K} \times \mathbf{Z} \times [0,1]^{L \times 1}$. Alternatively, one can use a limited number of moments of the distribution, or parameterize the distribution itself. Since the current stage is exploratory, and since there is no agreed method on how to quantitatively study frictional economies with heterogeneous agents, we leave alternative strategies for future work.

[TO BE COMPLETED]

6 Conclusion

The purpose of this paper was to examine an assignment of jobs in which unemployed workers and vacant jobs coexist in a competitive market equilibrium. We first showed that a frictionless model can generate equilibrium unemployment since low productive workers must be banned from production in order to maximize output (as in Akerlof [1]). We also showed that a frictionless model may feature mismatch as demonstrated by Lagos [31] although this is only possible as a market disequilibrium (Proposition 2). Mismatch in frictionless models is socially inefficient and based on the assumption that workers and jobs are related in a fixed proportion (Corollaries 1 and 2). Prices are also a free parameter and one can generate excess supplies by variations in the opportunity cost of a job and not only by the aggregate number of jobs as initially considered by Lagos [31].

The paper also proposed a frictional model which was the main focus of the article. Frictions are given by aggregate uncertainty about the number of market participants or about trading opportunities. Job assignments are made before uncertainty is resolved and hence unused factors arise optimally and in a competitive equilibrium (Proposition 3 and Theorem 2). Because assignments are made before uncertainty is resolved, some assigned jobs will remain vacant at the end of the trading period. Similarly, some workers will not able to trade and so they will be unemployed. It is important to stress that all forms of equilibrium unemployment are perfectly voluntary and socially efficient. Moreover, because of the trading frictions, state-contingent wages seem unresponsive to market conditions.

We enriched the job assignment to allow for worker mobility as in the equilibrium search model of Lucas and Prescott [34]. The generalization also introduced aggregate and idiosyncratic shocks. Because jobs are also mobile, the economy cannot be studied using a representative location as the distribution of the labor force across locations is a state variable. To surmount the difficulties that arise in this case we considered a sequential equilibrium and proved its existence as in anonymous sequential games with aggregate uncertainty (see, e.g., Bergin and Bernhardt [5]; [6]; Miao [36]). The generalizations we studied are important because they induce the correct business cycle association between unemployment and vacancies (Proposition 5). While the quantitative analysis is exploratory, the model generates a Beveridge curve and may potentially lead to large fluctuations in unemployment and vacancies.

7 Appendix

This Appendix presents additional results regarding the recursive representation of the general assignment, and the parameterized expectation method employed in the quantitative section.

7.1 Recursive equilibrium

The discussions of the general assignment have thus far only considered a sequential representation of the problem. As show by Miao [36], and Bergin and Bernhardt [6], one can establish an equivalence with a recursive equilibrium if the state-space is rich enough:

Proposition 6 (Recursive representation I) *Given $\{\mu\}$, for each t, there exists a value function $W_t(\mathbf{s}^t, \{\mu\})$ that gives the value of the assignment and satisfies:*

$$W_t(\mathbf{s}^t, \{\mu\}) = \max\left\{\mathbf{z}_t Y_t(\omega, \mathbf{s}^t, \mu_t) - i_t(\mathbf{s}^t, \mu_t) + \mathbb{E}_t \beta_t W_{t+1}(\mathbf{s}^{t+1}, \{\mu\})\right\},$$

with aggregate output $Y_t(\omega, \mathbf{s}^t, \mu_t)$ defined as in the text and with the discount factor determined from the consumption side of the economy. That is, $\beta_t = p_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1})/p_t(\mathbf{s}^t, \mu_t)$

Proof. Because we have conditioned on $\{\mu\}$, the proof is standard for dynamic programs, see, e.g., Stokey, Lucas, and Prescott ([53], Chapter 4). ■

The worker assignment can also be described using a recursive representation:

Proposition 7 (Recursive representation II) *Given $\{\mu\}$, for each t, there exists a value function $v_t(x, \mathbf{s}^t, \{\mu\})$ that gives the value of employment in location x and satisfies:*

$$v_t(x, \mathbf{s}^t, \{\mu\}) = \max \left\{\theta_t(\mathbf{s}^t, \mu_t), w_t(x, \mathbf{s}^t, \mu_t) + \min \left[\theta_t(\mathbf{s}^t, \mu_t), \mathbb{E}_t \beta_t v_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \{\mu\})\right]\right\},$$

with wages $w_t(x, \mathbf{s}^t, \mu_t)$ defined as in Proposition 4, and with jobs assigned based on a static assignment.

Proof. Let $g_t(x, \mathbf{s}^t, \mu_t) = \max_{k_t(x,\mathbf{s}^t,\mu_t)} \{q_t(\cdot)\mathbf{z}_t\phi_t(x)f(k_t(x, \cdot), e_t(x, \cdot)) - r_t(\cdot)k_t(x, \cdot))\} e_t(x, \cdot)$, represent the value of a locations once jobs are assigned. Consider an n-period truncation. In period n, the value of any given locations from the worker's perspective is $v_n(x, \mathbf{s}^n, \{\mu\}) = w_n(x, \mathbf{s}^n, \mu_n)$ because $m^+(x, \mathbf{s}^n, \mu_n) = m^-(x, \mathbf{s}^n, \mu_n) = 0$. For period $n-1$, the worker assignment solves:

$$\max_{m^-,m^+} \left\{g_{n-1}(x, \mathbf{s}^{n-1}, \mu_{n-1}) + \mathbb{E}_{n-1}\left[\beta_{n-1} g_n(\phi_{t+1}(x, \omega), \mathbf{s}^n, \{\mu\})\right]\right\},$$

subject to the feasibility conditions of the general assignment. Using the n-period truncation and Proposition 4, if some (or all) workers leave location x, then $v_{n-1}(x, \mathbf{s}^{n-1}, \{\mu\}) = \theta_{n-1}(\mathbf{s}^{n-1}, \mu_{n-1})$. Also, $v_{n-1}(x, \mathbf{s}^{n-1}, \{\mu\}) = w_{n-1}(x, \mathbf{s}^{n-1}, \mu_{n-1}) + \mathbb{E}_{n-1}\left[\beta_{n-1} v_n(\phi_{t+1}(x, \omega), \mathbf{s}^n, \{\mu\})\right]$ if there are no movements. If there are worker arrivals, $\mathbb{E}_{n-1}\left[\beta_{n-1} v_n(\phi_{t+1}(x, \omega), \mathbf{s}^n, \mu_n)\right] = \theta_{n-1}(\mathbf{s}^{n-1}, \mu_{n-1})$. Combining all previous cases yields the functional equation $v_{n-1}(x, \mathbf{s}^{n-1}, \{\mu\})$ above. Induction will work for periods $0 \le t \le n$. If $\lim_{n\to\infty} \mathbb{E}_{n-1}\left[\beta_{n-1} v_n(x, \mathbf{s}^j, \{\mu\})\right]$ is defined, the sequential problem can be represented by the value function, see, e.g., Stokey, Lucas, and Prescott ([53], Theorem 4.2). ■

Because equilibrium prices generally depend on the distribution of people across locations, it is natural to include the aggregate distribution μ_t as a state variable. But the current distribution of the labor force μ_t may not be enough to describe the state of the economy and for that reason we conditioned on the whole sequence $\{\mu\}$. For instance, when there are search frictions, workers need to form expectations about future prices and the future equilibrium decisions must be consistent with these expectations. As discussed in Miao [36] and Kubler and Schmedders [29], if the competitive equilibrium is globally unique for all possible initial values, the current distribution of the labor force would be a sufficient statistic for the state of economy. Uniqueness, however, cannot be checked from primitives.

7.2 Solution method

To solve the model we use a parameterized expectations algorithm, see, e.g., den Haan and Marcet [21]. First, we generate a sequence of aggregate and idiosyncratic productivity shocks $\{(\mathbf{z}_t, \phi_t(x) : t = 0, 1, ..., T \text{ and } x \in \tilde{\mathbf{X}}\}$. These realizations are held fixed in all subsequent steps and all decisions can be seen as functions of these sequences of realizations.

The general assignment satisfies a series of Euler equations for worker mobility and consumption. Let $\mathbb{E}_t\left[y_{t+1}(\mathbf{s}_{t+1}, \mu_{t+1})\right]$ be the conditional expectation of an Euler equation for variable y_{t+1}. The algorithm approximates the conditional expectations in the Euler equations by a first-order exponentiated polynomial,

$$\Theta(\mathbf{s}_t, \mu_t) = \exp\left\{\sum\nolimits_x \boldsymbol{\gamma}'_{\mu,x,y}\mu_t(x) + \boldsymbol{\gamma}_{z,y} \ln \mathbf{z}_t + \boldsymbol{\gamma}_{k,y} \ln K_t\right\},$$

with $\boldsymbol{\gamma}$ as a vector of coefficients to be found. (The choice of function is polynomial since such a function can approximate any general function well.) We initialize $\boldsymbol{\gamma}$ by drawing a random sequence of worker assignments and using a non-linear least squares fit of each Euler equation:

$$\hat{\boldsymbol{\gamma}} \in \arg\min T^{-1} \sum\nolimits_t \left\| \mathbb{E}_t \left[\vec{y}_{t+1}(\mathbf{s}_{t+1}, \mu_{t+1}) \right] - \vec{\Theta}(\mathbf{s}_t, \mu_t) \right\|^2 .$$

Given the parameterized Euler equations, we use bisection methods to solve the assignment of workers and jobs and find the market clearing conditions for any period t. Once equilibrium prices have been determined, we update the expectations by re-estimating the parameters $\boldsymbol{\gamma}$ until convergence is achieved.

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