

# Can the Timing of Labor Market Expansions be Explained by (the Transitional Dynamics of) the Burdett-Mortensen Model? \*

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## Abstract

We document three new facts about aggregate dynamics in US labor markets over the last 15 years, drawing in part from newly available datasets. These facts suggest a new view of how business cycles evolve and mature. We investigate whether this view is consistent with the transitional dynamics of the Burdett and Mortensen (1998) equilibrium search model, that we analyze in detail.

**Keywords:**

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# 1 Introduction

We document three new facts about aggregate dynamics in US labor markets over the last 15 years, drawing in part from newly available datasets. These facts suggest a new view of how business cycles evolve and mature. We investigate whether this view is consistent with the transitional dynamics of the Burdett and Mortensen (1998) equilibrium search model, that we analyze in detail.

**The facts.** First, annual BED firm-level data on job flows available since 1992 show that small firms (in terms of employment) concentrated most of their job creation in the early part of the 1990's expansion, and promptly expanded their employment after 2001. Conversely, large firms concentrated most of their 1990's job creation after 1996, and again failed to create jobs in the first part of the 2000's expansion. This pattern is observed across nine firm size classes and is exemplified in Figures 1 and 2 which plot employment shares and average labor force growth rates for four different classes.<sup>1</sup> The recoveries of the early 1990's and 2000's were "jobless" mainly at large firms, while the strong job creation of the late 1990's, in the mature phase of the expansion, was concentrated mainly in large firms. Second, monthly CPS data available since 1994 show that the employer-to-employer (EE) transition rate was flat well into the second half of the 1990's, picked up late in that expansion, and again declined in the 2001 recession and thereafter, only recently showing signs of recovery (Figure 3). Third, BLS public data on average real earnings show a flat profile in the first part of both the 1990's and 2000's expansions, a sharp increase in 1997-1999 and (possibly) since the Fall of 2006 (Figure 4). All in all, EE rates, wages and employment in large firms appeared to comove: sluggish early in the last two expansions and brisk in the late stages of the 1990's expansion (and possibly of the current one, since late 2006).

These facts suggest the following pattern. Early in an expansion, the large pool of unemployed workers sustains firms' monopsony power. Wages remain low, firms hire mostly from unemployment, relatively few workers quit from job to job. As the reservoir of unemployment dries out, more and more of the new hires arrive from other jobs. As poaching becomes the main source of hiring,

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<sup>1</sup>On all figures, vertical lines are placed at NBER business cycle dates.

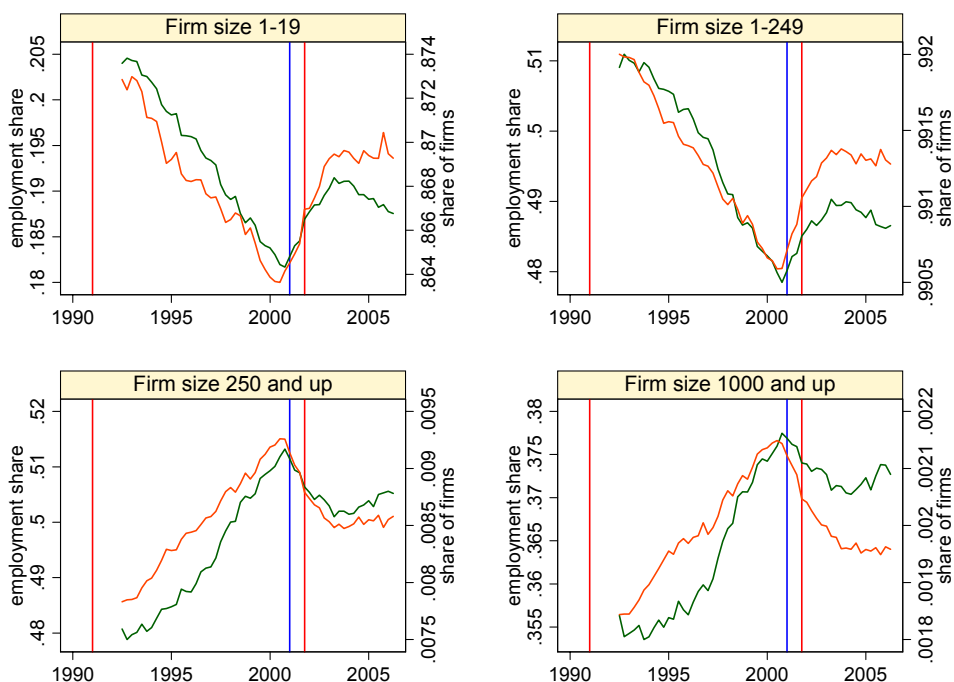


Fig. 1: Fraction of firms and employment shares — small *vs.* large firms

average wages and earnings rise and the EE rate picks up. If workers quit mostly from small, low-paying firms to large, high-paying firms, the growth in the employment of large firms will be fuelled by the stock of employment at small firms, which takes some time to replenish after a recession. Hence, employment at small firms rises faster and peaks earlier than at large firms. The erosion in firms' monopsony power reduces average mark-ups several years into an expansion, potentially creating favorable conditions for a new recession.

To understand whether this pattern can be consistent with equilibrium behavior, we study the transitional dynamics of the Burdett and Mortensen (1998) wage posting model with heterogeneous firms. We assume that the economy is hit by an unanticipated, positive aggregate productivity shock, and we study the convergence to the new steady state. Firms post and commit to wage paths that depend only on calendar time (or, equivalently, on the unemployment rate.) We focus

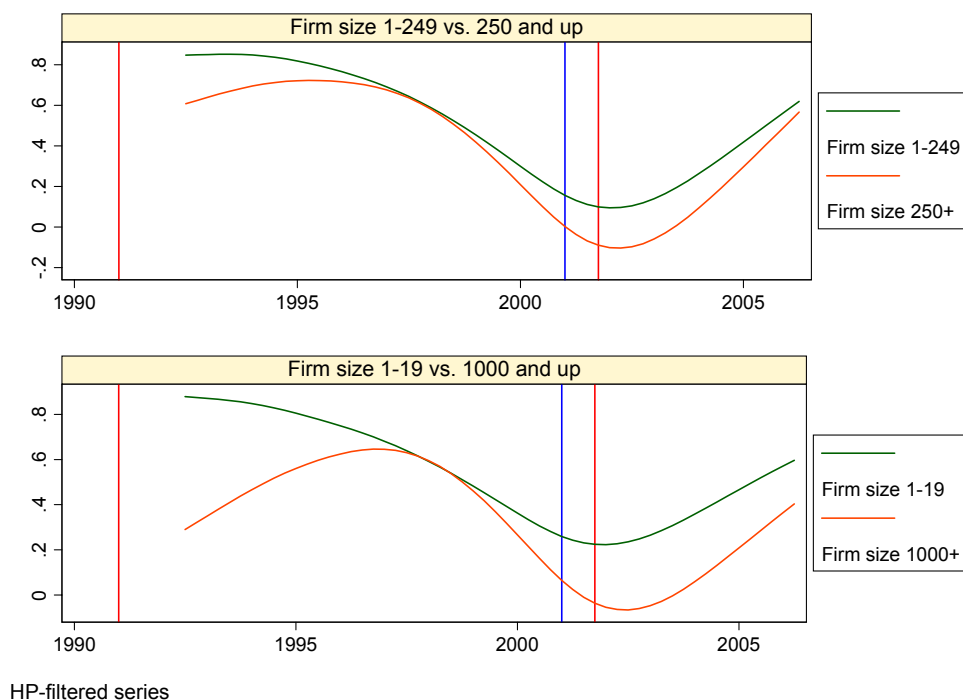


Fig. 2: Employment growth — small *vs.* large firms

on Rank-Preserving Equilibria (RPE) of the wage-posting game, where at each point in time more productive firms always offer higher wages, so the rank of each firm in the wage offer distribution is invariant. We analyze both the simpler case of myopic workers, who only care about the current wage offer when deciding whether to quit unemployment or the current job, and the more complex case of forward-looking workers, who make transitions based on the intertemporal value offered by each job. Firms are always forward-looking and maximize the present discounted value of their profits. We obtain a system of partial differential equations in time and firm productivity that completely characterize RPE dynamics and that can be solved numerically.

Section 2 presents the model and explains our solution strategy. Details and results of a simple calibration exercise are given in Section 3.

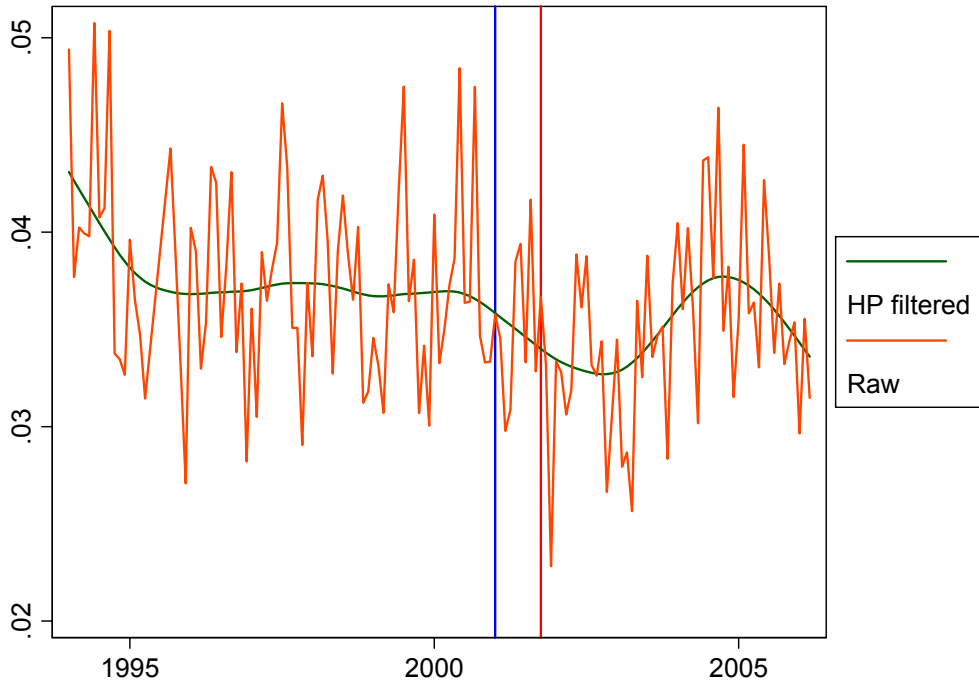


Fig. 3: Job-to-job quit rates

## 2 Transition Dynamics of the BM Model

### 2.1 Assumptions, Notation, and Problem Statement

**The economic environment.** Time is continuous. The labor market is populated by a unit-mass of workers who can be either employed or unemployed. It is affected by search frictions in that unemployed workers can only sample job offers sequentially at some finite Poisson rate  $\lambda_0 > 0$ . Employed workers are allowed to search on the job, and face a sampling rate of job offers of  $\lambda_1 > 0$ . Firm-worker matches are dissolved at rate  $\delta > 0$ . Upon match dissolution, the worker becomes unemployed.

All workers are ex-ante identical: they are infinitely lived, risk-neutral, equally capable at any job, and they attach a common lifetime value of  $U_t$  to being unemployed at date  $t$ .

Workers face a measure  $N$  of active firms operating constant-return technologies with heteroge-



Fig. 4: Wages and output per hour

neous productivity levels  $p \sim \Gamma(\cdot)$  among firms. For (quantitative) reasons that will become clear below, we assume that the sampling of firms by workers is not uniform in that a type- $p$  firm has a sampling weight of  $v(p) > 0$ . Sampling weights are normalized in such a way that their cumulated sum  $\Phi(p) = \int_{\underline{p}}^p v(x) d\Gamma(x)$  is a (sampling) cdf, i.e.  $\Phi(\bar{p}) = 1$ . The sampling density of a type- $p$  firm is therefore  $v(p) \gamma(p)$ . This naturally encompasses the conventional case of uniform sampling which has  $v(p) = 1$  for all  $p$ .

At some initial date  $t$ , each firm of a given type  $p$  commits to a wage profile  $\{w_s(p)\}_{s \in [t, +\infty)}$  over the infinite future. We generalize the Burdett and Mortensen (1998) restrictions placed on the set of feasible wage contracts to a non-steady-state environment by preventing firms to index wages on anything else than calendar time. We thus rule out, among other things, wage-tenure contracts (Stevens, 2004; Burdett and Coles, 2003), offer-matching or individual bargaining (Postel-

Vinay and Robin, 2002; Dey and Flinn, 2005; Cahuc, Postel-Vinay and Robin, 2006), or contracts conditioned on employment status (Carrillo-Tudela, 2007).

Any such profile  $\{w_s(p)\}_{s \in [t, +\infty)}$  offered by any type- $p$  firm yields a continuation value of  $V_t(p)$  to any worker employed at that firm at date  $t$ . The (time-varying) sampling distribution of job values is denoted as  $F_t(\cdot)$ , and its relationship to the sampling distribution of firm types  $\Phi(\cdot)$  will be discussed momentarily. Because from the workers' viewpoint jobs are identical in all dimensions but the wage profile, employed jobseekers quit into higher-valued jobs only.<sup>2</sup> This gradual self-selection of workers into better jobs implies that the distribution of job values in a cross-section of workers—which will be denoted as  $G_t(\cdot)$ —differs from the sampling distribution  $F_t(\cdot)$ .

**The control problem.** Firms post wage profiles over an infinite horizon that solve the following problem:

$$\Pi_t^*(p) = \max_{\{w_s(p)\}} \int_t^{+\infty} (p - w_s(p)) \ell_s(p) e^{-r(s-t)} ds \quad (1)$$

$$\text{subject to: } \dot{\ell}_s(p) = -(\delta + \lambda_1 \bar{F}_s(V_s(p))) \ell_s(p) + \frac{v(p)}{N} (\lambda_0 u_s + \lambda_1 (1 - u_s) G_s(V_s(p))) \quad (2)$$

$$\dot{V}_s(p) = (r + \delta + \lambda_1 \bar{F}_s(V_s(p))) V_s(p) - \lambda_1 \int_{V_s(p)}^{+\infty} x dF_s(x) - w_s(p) - \delta U_s \quad (3)$$

$$w_t(p) \geq \underline{w}, \quad (4)$$

where  $\ell_t(p)$  denotes a type- $p$  firm's workforce at date  $t$  (which, incidentally, implies that the *density* of firm types among workers at date  $t$  is given by  $N\ell_t(p)\gamma(p)/(1 - u_t)$ ),  $\underline{w}$  is the exogenous institutional minimum wage, and  $r$  is the discount rate, which is common to firms and workers.<sup>3</sup> When solving (1), the typical firm also is also constrained by its given initial size  $\ell_t(p)$ .

At the individual firm's level, the sampling and cross-sectional distributions of job values  $F_t(\cdot)$  and  $G_t(\cdot)$  are given macroeconomic quantities. So is the unemployment rate  $u_t$  which solves:

$$\dot{u}_t = \delta(1 - u_t) - \lambda_0 u_t, \quad u_0 \text{ given.} \quad (5)$$

<sup>2</sup>For simplicity, it is also assumed that any job offer posted in equilibrium is preferred to unemployment, i.e.  $\inf_p V_t(p) \leq U_t$  at all  $t$ . Specifically, we assume the existence of a constant and exogenous institutional minimum wage  $\underline{w}$  which is sufficiently high for unemployed workers accept even the least valuable job offer.

<sup>3</sup>Although we following standard practice we impose a common discount rate on firms and workers, this restriction is by no means essential. Indeed other cases, such as the case of myopic workers for example, are of potential interest (see below).

Problem (1) is therefore a standard (non-autonomous) optimal control problem, the Lagrangian of which is defined by:

$$\begin{aligned} \mathcal{L}_t(p) = & (p - w_t(p)) \ell_t(p) + m_t(p) (w_t(p) - \underline{w}) \\ & + \pi_t(p) \left\{ -(\delta + \lambda_1 \bar{F}_t(V_t(p))) \ell_t(p) + \frac{v(p)}{N} (\lambda_0 u_t + \lambda_1 (1 - u_t) G_t(V_t(p))) \right\} \\ & + \nu_t(p) \left\{ (\rho + \delta + \lambda_1 \bar{F}_t(V_t(p))) V_t(p) - \lambda_1 \int_{V_t(p)}^{+\infty} x dF_t(x) - w_t(p) - \delta U_t \right\}, \quad (6) \end{aligned}$$

where  $\nu_t(p)$  [ $\pi_t(p)$ ] is the costate associated with  $V_t(p)$  [ $\ell_t(p)$ ] and  $m_t(p) \geq 0$  is the Lagrange multiplier associated with the minimum wage constraint (4).

Optimality conditions are:

$$\nu_t(p) = -\ell_t(p) + m_t(p) \quad (7)$$

$$\begin{aligned} \dot{\nu}_t(p) = & r\nu_t(p) - (r + \delta + \lambda_1 \bar{F}_t(V_t(p))) \nu_t(p) \\ & - \lambda_1 f_t(V_t(p)) \ell_t(p) \pi_t(p) - \frac{\lambda_1 v(p)}{N} (1 - u_t) g_t(V_t(p)) \pi_t(p) \end{aligned} \quad (8)$$

$$\dot{\pi}_t(p) = (r + \delta + \lambda_1 \bar{F}_t(V_t(p))) \pi_t(p) - p + w_t(p) \quad (9)$$

$$m_t(p) \geq 0, \quad w_t(p) \geq \underline{w}, \quad m_t(p) (w_t(p) - \underline{w}) = 0 \quad (10)$$

$$\lim_{t \rightarrow +\infty} e^{-rt} \pi_t(p) = \lim_{t \rightarrow +\infty} e^{-rt} \nu_t(p) = 0. \quad (11)$$

Supplementing this latter set of conditions with the state equations (2), (3) and (5), we obtain a system of partial differential equations characterizing the solution to an individual firm's maximization problem for a given (profile of) sampling distribution(s)  $F_t(\cdot)$ . The main difficulty then lies in characterizing the *equilibrium*  $F_t(\cdot)$ , i.e. the profile of sampling distribution which is consistent with the above dynamic system simultaneously for the whole population of firms. In the following subsection we introduce an equilibrium restriction which helps getting round this difficulty.

## 2.2 Rank-Preserving Equilibria

**Definition.** We define a *Rank-Preserving Equilibrium* (RPE) as a dynamic equilibrium in which firms post values that are strictly increasing in  $p$  for all  $t$ . This has the consequence that workers rank firms according to productivity at all dates. Hence the following two properties hold true at



all dates under the RP assumption:

$$F_s(V_s(p)) \equiv \Phi(p),$$

$$(1 - u_s) G_s(V_s(p)) = N \int_{\underline{p}}^p \ell_s(x) d\Gamma(x).$$

(The latter identity reflects a count of how many workers are employed at firms of type  $p$  or less.)

**Employment and firm sizes in a RPE.** We now consider the stock of workers employed at a firm of type- $p$  or less,  $N \int_{\underline{p}}^p \ell_s(x) d\Gamma(x)$ . In a RPE (assuming one exists), those firms hire workers from unemployment and lose workers to their more productive competitors (firms of type higher than  $p$ ). The stock of workers under consideration thus evolves according to:<sup>4</sup>

$$\int_{\underline{p}}^p \dot{\ell}_t(x) d\Gamma(x) = \frac{\lambda_0 u_t}{N} \Phi(p) - [\delta + \lambda_1 \bar{\Phi}(p)] \int_{\underline{p}}^p \ell_t(x) d\Gamma(x). \quad (12)$$

This latter equation now solves as:

$$\int_{\underline{p}}^p \ell_t(x) d\Gamma(x) = e^{-[\delta + \lambda_1 \bar{\Phi}(p)]t} \left( \int_{\underline{p}}^p \ell_0(x) d\Gamma(x) + \frac{\lambda_0 \Phi(p)}{N} \int_0^t u_s e^{[\delta + \lambda_1 \bar{\Phi}(p)]s} ds \right) \quad (13)$$

Now differentiating with respect to  $p$ , one obtains a closed-form expression for the workforce of any type- $p$  firm:

$$\begin{aligned} \ell_t(p) = e^{-[\delta + \lambda_1 \bar{\Phi}(p)]t} & \left( \ell_0(p) + \lambda_1 t v(p) \int_{\underline{p}}^p \ell_0(x) d\Gamma(x) \right. \\ & \left. + \frac{\lambda_0 v(p)}{N} \int_0^t [1 + \lambda_1 \bar{\Phi}(p)(t-s)] u_s e^{[\delta + \lambda_1 \bar{\Phi}(p)]s} ds \right). \quad (14) \end{aligned}$$

The steady-state versions of (13) and (14) are:

$$\ell_\infty(p) = \frac{\delta(1 - u_\infty)(\delta + \lambda_1)}{N[\delta + \lambda_1 \bar{\Phi}(p)]^2} v(p) \quad \text{and} \quad N \int_{\underline{p}}^p \ell_\infty(x) d\Gamma(x) = \frac{\delta(1 - u_\infty)\Phi(p)}{\delta + \lambda_1 \bar{\Phi}(p)}, \quad (15)$$

where  $u_\infty = \frac{\delta}{\delta + \lambda_0}$  is the steady-state rate of unemployment.

This is the point at which the necessity for sampling weights appears. Note from equation (15) that the steady-state size ratio of the largest to the smallest firm in the market is  $\left(1 + \frac{\lambda_1}{\delta}\right)^2 \frac{v(\bar{p})}{v(\underline{p})}$ .

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<sup>4</sup>Not that the following law of motion can also be obtained by integration of (2) w.r.t.  $p$ . Details available on request.

With uniform sampling ( $v(p) \equiv 1$  throughout), this ratio would equal  $\left(1 + \frac{\lambda_1}{\delta}\right)^2$ , which is in the order of 25-30 given standard estimates of  $\lambda_1$  and  $\delta$ . Now of course the data counterpart of that size ratio is virtually infinite. More generally, it appears that the BM model requires a sampling distribution that is very heavily skewed toward high-productivity firms in order to replicate the observed distribution of firm sizes.

Before going any further into characterizing Rank-Preserving Equilibria, we should notice that the analysis of firm size and employment dynamics carried out in this paragraph would apply to any job ladder model in which a similar concept of RPE can be defined. Indeed nothing in the dynamics of  $\ell_t$  or  $u_t$  depends on the particulars of the wage setting mechanism, so long as this latter is such that employed jobseekers move from lower-ranking into higher-ranking jobs in the sense of a time-invariant ranking. Therefore, this model's predictions about everything relating to firm sizes are in fact much more general than the wage- (or value-) posting assumption retained in the BM model.

### 2.3 Existence of a RPE: Solving the Control Problem

**Interior solutions.** We now go back to the dynamical system characterizing the behavior of the typical individual firm, and analyze it in a RPE. The system in question is comprised of the set of optimality conditions (7) - (11) plus the set of state equations (2), (3) and (5). We first focus on intervals of time when the solution is interior, i.e. such that  $m_t(p) = 0$  and  $w_t(p) > \underline{w}$ . In this situation  $\nu_t(p) = -\ell_t(p)$ . Substitution of (7) into (8), and combination with (2) thus yields:

$$\frac{v(p)}{N} (\lambda_0 u_t + \lambda_1 (1 - u_t) G_t(V_t(p))) = \lambda_1 \pi_t(p) \left( f_t(V_t(p)) \ell_t(p) + \frac{v(p)}{N} (1 - u_t) g_t(V_t(p)) \right). \quad (16)$$

Next defining the shadow value to the *firm-worker match* (rather than to the firm) of the marginal unit of labor as  $\mu_t(p) = \pi_t(p) + V_t(p)$ , combination of (3) and (9) yields:

$$\dot{\mu}_t(p) = (r + \delta + \lambda_1 \bar{F}_t(V_t(p))) \mu_t(p) - \lambda_1 \int_{V_t(p)}^{+\infty} x dF_t(x) - \delta U_t - p, \quad (17)$$

which is supplemented by the transversality condition  $\lim_{t \rightarrow +\infty} e^{-rt} \mu_t(p) = 0$ . The RP assumption finally changes the system (16) - (17) into:

$$\left( \frac{\lambda_0 u_t}{N} + \lambda_1 \int_{\underline{p}}^p \ell_t(x) d\Gamma(x) \right) V_t'(p) = 2\lambda_1 \gamma(p) \ell_t(p) \pi_t(p) \quad (18)$$

$$\dot{\mu}_t(p) = (r + \delta + \lambda_1 \bar{\Phi}(p)) \mu_t(p) - \lambda_1 \int_p^{+\infty} V_t(x) d\Phi(x) - \delta U_t - p \quad (19)$$

$$\lim_{t \rightarrow +\infty} e^{-rt} \mu_t(p) = 0. \quad (20)$$

Differentiation of (19) w.r.t.  $p$  yields (primes denote differentiation w.r.t.  $p$  while dots denote time differentiation):

$$\dot{\mu}_t'(p) = (r + \delta + \lambda_1 \bar{\Phi}(p)) \mu_t'(p) + \lambda_1 \gamma(p) v(p) (V_t(p) - \mu_t(p)) - 1. \quad (21)$$

This, together with (18), gives the following system of two PDEs in  $(\mu_t'(p), \pi_t(p))$ :

$$\mu_t'(p) = (r + \delta + \lambda_1 \bar{\Phi}(p)) \mu_t'(p) - \lambda_1 \gamma(p) v(p) \pi_t(p) - 1 \quad (22)$$

$$\mu_t'(p) = \pi_t'(p) + \frac{2\lambda_1 \gamma(p) \ell_t(p)}{\frac{\lambda_0 u_t}{N} + \lambda_1 \int_{\underline{p}}^p \ell_t(x) d\Gamma(x)} \pi_t(p).$$

This can be solved numerically, subject to some initial and boundary conditions. ‘Initial’ conditions are given by the steady-state solution to (22), which is characterized as:

$$\mu_\infty'(p) = \frac{1 + \lambda_1 \gamma(p) v(p) \pi_\infty(p)}{r + \delta + \lambda_1 \bar{\Phi}(p)} \quad (23)$$

$$\pi_\infty(p) = \frac{(\delta + \lambda_1 \bar{\Phi}(p))^2}{r + \delta + \lambda_1 \bar{\Phi}(p)} \left( \int_{\underline{p}}^p \frac{dx}{(\delta + \lambda_1 \bar{\Phi}(x))^2} + \frac{\pi_\infty(\underline{p}) (r + \delta + \lambda_1)}{(\delta + \lambda_1)^2} \right).$$

Now turning to boundary conditions, standard arguments prove that the lowest-type firms have no reason to pay more than the minimum wage. While this implies that the minimum wage constraint (4) will bind at all dates for the lowest-type firm, it also implies that the following (time-invariant) boundary conditions are satisfied:

$$\pi_t(\underline{p}) \equiv \frac{p - w}{r + \delta + \lambda_1} \quad (24)$$

$$\mu_t'(\underline{p}) \equiv \frac{1 + \lambda_1 \gamma(\underline{p}) v(\underline{p}) \pi_t(\underline{p})}{r + \delta + \lambda_1},$$

where the second condition is obtained by combining the first one with the  $\mu'_t(p)$  equation in (22). These simple boundary conditions can be further simplified by imposing  $\underline{p} = \underline{w}$ , a kind of free-entry condition holding throughout the adjustment toward the new steady state, which implies  $\pi_t(\underline{p}) \equiv 0$ . The minimum productivity  $\underline{p}$  that can survive in the market is  $\underline{w}$ , as any firm with  $p > \underline{w}$  can make positive profits by offering  $\underline{w}$ , and possibly even more by offering a higher wage while no firm with  $p < \underline{w}$  can ever make any profits.

Once (22) is solved for  $(\mu'_t(p), \pi_t(p))$ , wages can be retrieved from (9) (written under the RP assumption):

$$w_t(p) = p - (r + \delta + \lambda_1 \bar{\Phi}(p)) \pi_t(p) + \dot{\pi}_t(p), \quad (25)$$

which has the following familiar steady-state solution:

$$w_\infty(p) = p - (\delta + \lambda_1 \bar{\Phi}(p))^2 \left( \int_{\underline{p}}^p \frac{dx}{(\delta + \lambda_1 \bar{\Phi}(x))^2} + \frac{\underline{p} - \underline{w}}{(\delta + \lambda_1)^2} \right). \quad (26)$$

**The minimum wage constraint.** The only firm for which the minimum wage constraint (4) is binding at the steady state characterized above is the lowest-type firm,  $\underline{p}$ . It may be the case, however, that the constraint temporarily binds for some higher-type firms over the transition to that steady state, in which case the economy no longer behaves according to (22) as  $m_t(p)$  becomes strictly positive for some  $p$  at some dates.

The Appendix describes an algorithm that constructs an equilibrium in which  $\underline{w}$  is indeed temporarily binding for some firms (at the lower end of the  $p$ -distribution) under the restriction that it can only bind over some initial period. In other words, any firm can choose to post the minimum wage for a while right after the occurrence of the productivity shock, but once it ceases to do so it is not allowed to return to the minimum wage. Simulations, however, will prove that the minimum wage is only offered by the lowest- $p$  firms in equilibrium.

### 3 Calibration Exercises

#### 3.1 Simulating an Expansion

In order to simulate the economy's response to a one-time, permanent and unanticipated aggregate productivity shock, we further specify the model as follows. We assume that any firm's productivity parameter  $p$  is the product of an aggregate productivity index  $y$  (common to all firms) and a firm-specific random effect  $\theta$ . We further assume that there is an exogenous number  $N_0$  of potential firms, each with a fixed value of  $\theta$  drawn from some exogenous underlying distribution  $\Gamma_0$ . Because for any potential firm productivity is  $p = y \times \theta$ , the only profitable firms in the presence of a wage floor  $\underline{w}$  are those with  $\theta \geq \underline{w}/y$ . The distribution of productivity levels among active firms will thus be given by:

$$\Gamma(p) = \frac{\Gamma_0(p/y) - \Gamma_0(\underline{w}/y)}{1 - \Gamma_0(\underline{w}/y)}, \quad (27)$$

and the number of active firms will be  $N = N_0 (1 - \Gamma_0(\underline{w}/y))$ .

We model a 'boom' as a permanent 2 percent increase in  $y$  (from  $y = 1$  to  $y = 1.02$ ). We further assume that this productivity increase causes the job finding rate  $\lambda_0$  to increase by 8 percent,<sup>5</sup> and the arrival rate of offers to employed jobseekers,  $\lambda_1$ , to increase by 1.6 percent. If the wage floor  $\underline{w}$  does not react, the shock causes entry of  $\Delta N = N_0 (\Gamma_0(\underline{w}) - \Gamma_0(\underline{w}/1.02))$  firms at the bottom of the productivity distribution, all starting off with a size of zero. The distribution of productivity across active firms jumps instantly following (27).

#### 3.2 Baseline calibration

**Productivity dispersion.** A sampling distribution of firm types is first calibrated following the Bontemps *et al.* (2000) procedure in such a way that the predicted steady-state wage distribution fits the business-sector wage distribution observed in the CPS. Specifically, equation (15) implies that the steady-state cross-section CDF of wages,  $G_w^*(\cdot)$  (say), is defined by

$$\Phi(p) = \frac{(\delta + \lambda_1) G_w^*(w(p))}{\delta + \lambda_1 G_w^*(w(p))} \Rightarrow \Phi'(p) = \frac{(\delta + \lambda_1) g_w^*(w(p)) w'(p)}{(\delta + \lambda_1 G_w^*(w(p)))^2}. \quad (28)$$

<sup>5</sup>This is based on an elasticity of labor market tightness with respect to productivity of 8 and an elasticity of the job finding rate w.r.t. labor market tightness of 0.5, both consensual numbers.

Differentiation of (26) then yields:

$$w'(p) = 2\lambda_1\Phi'(p) \frac{p - w(p)}{\delta + \lambda_1\overline{\Phi}(p)} \Rightarrow p(w) = w + \frac{\delta + \lambda_1 G_w^*(w)}{2\lambda_1 g_w^*(w)}. \quad (29)$$

A lognormal distribution is fitted to a sample of wages from the 2006 CPS and then used to construct a sample of firm types using the above relationship. The sampling distribution  $\Phi(\cdot)$  that rationalizes this sample in a steady state (and given values of  $\delta$  and  $\lambda_1$ ) is then retrieved using (28).

Firm size category	Cum. fraction of firms $[\Gamma(p)]$	Cum. emp. share $[G_w^*(w(p))]$
1-4	0.535	0.052
5-9	0.742	0.114
10-19	0.868	0.192
20-49	0.949	0.303
50-99	0.976	0.387
100-249	0.991	0.493
250-499	0.996	0.565
500-999	0.998	0.633
1000 and up	1.00	1.00

BED data, all years pooled.

Table 1: Firm sizes and employment shares

Once a sampling distribution has been obtained, the underlying distribution of firm types ( $\Gamma(p)$ ) and sampling weights ( $v(p)$ ) are calibrated based on the employment share-firm size relationship found in the BED data. Table 1 summarizes the information conveyed by the BED data about that relationship. The data in table 1 is found to be well fitted by the following parametric relationship:

$$\Gamma(p) = \left( \frac{1 - e^{-\alpha_1 G_w^*(w(p))}}{1 - e^{-\alpha_1}} \right)^{\alpha_2}, \quad (30)$$

with  $\alpha_1 = 8.0661$  and  $\alpha_2 = 0.5843$ . Sampling weights are finally retrieved as  $v(p) = \Phi'(p) / \gamma(p)$ .

Finally, we shift the support of the  $\Gamma(\cdot)$  distribution thus obtained so that its infimum is at  $p = 1$  and use it as our benchmark underlying distribution of firm types  $\Gamma_0(\cdot)$  (given the normalization  $y = 1$ ) as explained in the previous subsection.

**Other parameters.** Apart from productivity dispersion, our baseline parameterization is explicated in Table 2. The time unit is one month. The value of  $r$  reflects an annual discount rate of five

percent. The minimum wage is binding (in the sense that  $\underline{p} = \underline{w}$ ) since, being equal to 5, it exceeds the lower support of the distribution of potential firm productivity levels which was normalized at 1. Finally, the number chosen for  $N_0$  reflects an average firm size of 20.

Parameters (post-shock monthly values)						
$r$	$\delta$	$\lambda_0$	$\lambda_1$	$\underline{w}$	$y$	$N_0$
0.0043	0.025	0.40	0.12	5	1.02	0.0509

Table 2: Baseline parameterization

### 3.3 Simulation Results

**Unemployment, firm sizes and employment shares.** As can very easily be inferred from equation (5) The response of the unemployment rate to the positive shock hitting the economy is a simple monotonic adjustment toward the new (lower) steady-state value. The interesting feature of that adjustment is its speed: given our calibrated values of  $\delta$  and  $\lambda_0$ , 90% of the distance between the initial and the final steady state is covered in less than six months.

Figure 5 then shows how unemployment adjusts at single firms: it shows a plot of  $\ell_t(p)/\ell_0(p)$  for four different values of  $p$  corresponding to the 50th, 90th, 95th and 99.9th percentiles of the (post-shock) distribution of firm types,  $\Gamma(\cdot)$ .<sup>6</sup> Patterns of employment adjustment differ markedly across firm types—which translates into differences across firm size categories as low- $p$  firms are also smaller firms in the initial state of the labor market. One sees on Figure 5 that “large” firms tend to increase in size monotonically and gradually (the higher the firm in terms of  $p$ , the more gradual the adjustment). Conversely, “smaller” firms experience a short episode of rapid growth soon after the shock and then start shrinking back toward their final steady-state size, which they overshoot in the adjustment process. Firms at the 50th percentile of the  $\Gamma(\cdot)$  distribution (which places them at the 21st percentile of the sampling distribution  $\Phi(\cdot)$  and at the 4.5th percentile in terms of steady-state cumulated employment shares) even end up being smaller after the increase

<sup>6</sup>The normalization by  $1/\ell_0(p)$  is just there to rescale the paths and keep the picture legible. Moreover, on all Figures, circles on the axes indicate initial (steady-state) values of the various indicators plotted.

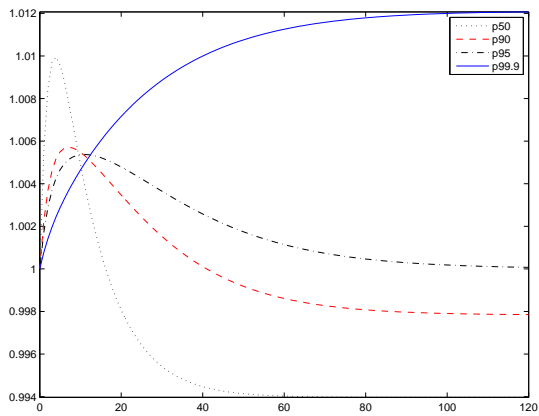


Fig. 5: Firm size dynamics

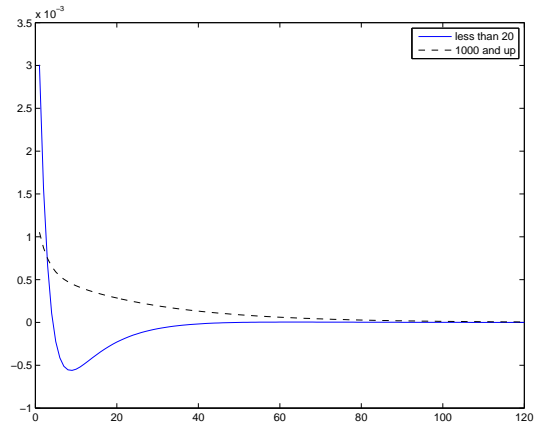


Fig. 6: Firm growth—small *vs.* large firms

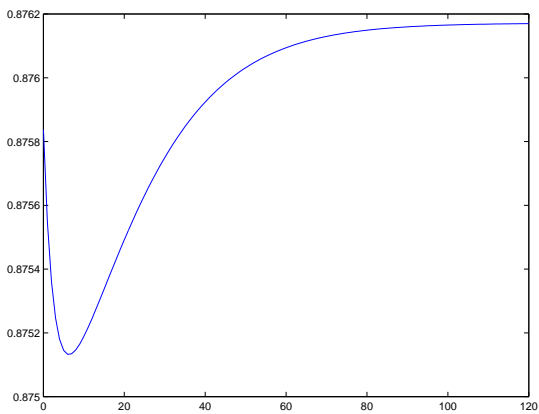


Fig. 7: Firm shares (firms < 20)

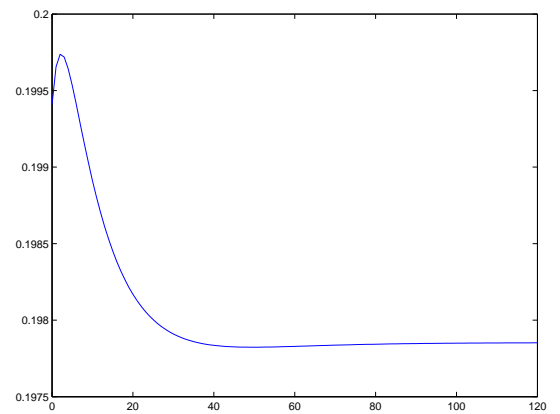


Fig. 8: Emp. shares (firms < 20)

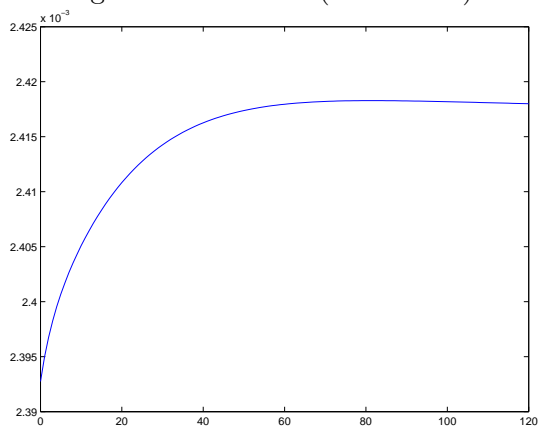


Fig. 9: Firm shares (firms  $\geq 1000$ )

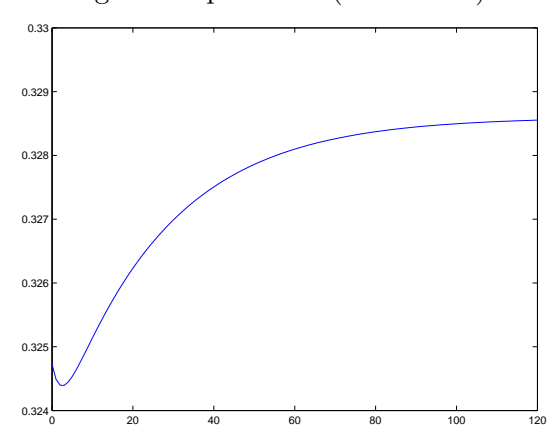


Fig. 10: Emp. shares (firms  $\geq 1000$ )



in productivity than in the initial steady state.

This pattern conforms with intuition: in the few months following the shock, most of the new hires are workers coming from unemployment which get disproportionately allocated to small (low- $p$ ) firms. After six months or so (given the magnitude of  $\lambda_0$ ), the unemployment pool dries out and poaching becomes the main channel of hiring. Poaching benefits larger, higher- $p$ , better-paying firms at the expense of smaller ones. It occurs later on in the expansion and is a much slower process than the initial siphoning of the unemployment pool as  $\lambda_1$  is about a third of  $\lambda_0$  in magnitude and the average offer acceptance rate of an employed jobseeker is less than one.<sup>7</sup>

For comparison with the descriptive evidence shown in the Introduction, the mechanism just described can be depicted in terms of employment shares and average growth rates by firm size category. This is done in Figures 6 to 10 which parallel Figures 1 and 2 from the Introduction.

**Job-to-job quits.** The response of the average job-to-job quit rate,  $\frac{\lambda_1 N}{1-u_t} \int_{\underline{p}}^{\bar{p}} \bar{\Phi}(x) \ell_t(x) d\Gamma(x)$ , is plotted on Figure 11. Apart from the initial jump caused by the assumed instant response of  $\lambda_1$  to the productivity shock, the average quit rate has an initial increasing phase which reflects the initial disproportionate inflow of new hires into small, low-productivity firms. These workers start getting poached away by larger firms relatively easily, while at the same time the unemployment pool quickly gets depleted and the excess inflow of workers into easy-to-poach positions slows down. As workers gradually get reallocated toward more productive, better-paying firms, poaching becomes more difficult (the acceptance rate of outside offers falls) and the quit rate falls.

**Wages and productivity.** Finally, Figures 12 and 13 plot the dynamic responses of mean wages and mean output per worker.

The path followed by mean output per worker results from a pure composition effect. After the initial upward jump caused by the sudden 2 percent increase in the productivity levels of all established firms, mean output per worker adjusts quasi-monotonically to its higher final steady-state value following the gradual reallocation of newly hired workers into more productive firms.

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<sup>7</sup>It actually equals  $N \int_{\underline{p}}^{\bar{p}} \bar{\Phi}(x) \ell_t(x) d\Gamma(x) / (1-u_t)$ . This becomes  $(1 + \delta/\lambda_1) \{1 - \delta \ln(1 + \lambda_1/\delta) / \lambda_1\}$  at a steady state, i.e. about 0.76 with our parameterization.

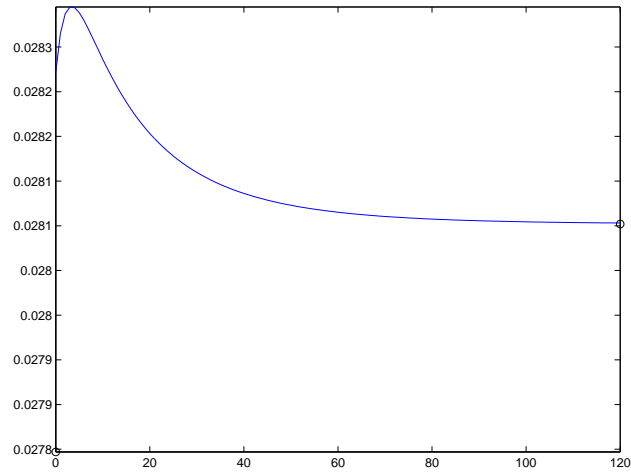


Fig. 11: Average job-to-job quit rate

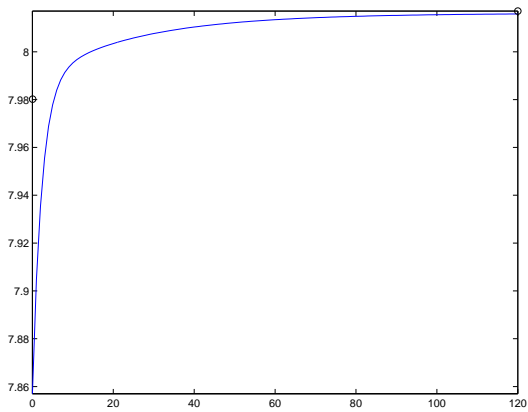


Fig. 12: Mean wage dynamics

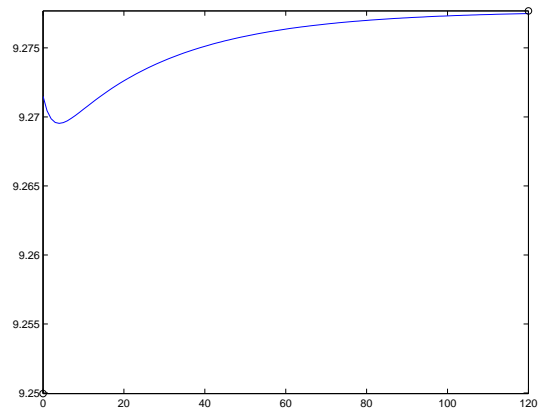


Fig. 13: Mean output per worker dynamics

The slight dip observed in the initial phase of that adjustment is due to the mass low-productivity firms suddenly becoming viable as a result of the positive aggregate shock on  $y$  and thus entering the market with an initial size of zero. These entrant firms drag average output per worker down in the early phase of the expansion as they hire some workers into low-productivity jobs.

The mechanisms generating the path followed by the mean wage are more intricate. First, the same composition effect as for mean output per worker operates for wages: there is an initial excess inflow of workers into low-paying firms and those workers gradually reallocate themselves into better-paying firms, thereby causing a sluggish positive response of the mean wage to the aggregate productivity shock. Note that, because of this composition effect, the aggregate mean wage would exhibit this sluggish adjustment pattern even if all firm-level wages would jump right onto their new steady-state values upon impact of the productivity shock.<sup>8</sup> Second, each firm-level wage follows a dynamic path of its own. The composition of these individual dynamic paths causes the initial downward jump in the mean wage. Intuitively, it is in the firms' interest to backload wage payments. In this version of the BM model, because firms are not allowed to index wages to individual tenure, they cannot backload at the individual level (as they would do in the wage-tenure models of Stevens, 2004 and Burdett and Coles, 2003). However they can index contracts to calendar time and benefit from future competition from higher-paying firms. Specifically, the prospect of receiving an offer from a better-paying firm later on makes up for the low wage that a single firm offers today. Of course the extent to which firms can piggyback on their (future) competitors in this way depends on the workers' horizon. In this particular instance it is assumed that workers are patient in that they have the same discount rate as employers. The model can easily be amended to allow for different discount rates for firms and workers, which can be shown to have a quantitative impact on the magnitude and sign of the initial jump in the mean wage as well as on the slope of the mean wage adjustment path.<sup>9</sup>

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<sup>8</sup>This is precisely the situation that would arise under the special assumption of infinitely impatient workers (worker with an infinite rate of future discount). The full details of that special case are available on request.

<sup>9</sup>Details available on request. See also the remark in footnote 8.

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## Appendix

### Numerical equilibrium determination.

The algorithm we use to numerically characterize the dynamic equilibrium is based on the restriction that, if the minimum wage constraint binds for some firms, it will do so at early stages of the expansion only. In other words, any firm can choose to post the minimum wage for a while right after the productivity shock, but once it ceases to do so it is not allowed to return to the minimum wage. Simulations will prove that an equilibrium with exactly this pattern exists.

In order to construct that equilibrium, we proceed through the following steps.

**Step 1.** Consider some productivity level  $p_0$  such that the functions  $\pi_t(p_0)$  and  $\mu'_t(p_0)$  are known. (In effect the algorithm is started at  $p_0 = \underline{p}$  for which those functions are known from (24).) Pick a step size  $h$ .

**Step 2.** Construct a candidate  $\pi_t(p_0 + h)$  using the second (static) differential equation in (22), such as:<sup>10</sup>

$$\tilde{\pi}_t(p_0 + h) = \pi_t(p_0) + h \times \left( \mu'_t(p_0) - \frac{2\lambda_1\gamma(p_0)\ell_t(p_0)}{\frac{\lambda_0 u_t}{N} + \lambda_1 \int_{\underline{p}}^{p_0} \ell_t(x) d\Gamma(x)} \pi_t(p_0) \right). \quad (31)$$

**Step 3.** Construct a candidate wage path for type- $(p_0 + h)$  firms from  $\tilde{\pi}_t(p_0 + h)$  and equation (9):

$$\tilde{w}_t(p_0 + h) = p_0 + h - (r + \delta + \lambda_1 \bar{\Gamma}(p_0 + h)) \tilde{\pi}_t(p_0 + h) + \dot{\tilde{\pi}}_t(p_0 + h). \quad (32)$$

**Step 4.** Construct  $w_t(p_0 + h)$  and  $\pi_t(p_0 + h)$  as follows:

- If  $\tilde{w}_t(p_0 + h) \geq \underline{w}$  at all dates, set  $w_t(p_0 + h) = \tilde{w}_t(p_0 + h)$  and  $\pi_t(p_0 + h) = \tilde{\pi}_t(p_0 + h)$  for all  $t$ .
- If  $\tilde{w}_t(p_0 + h) < \underline{w}$  for  $t \in [0, t^*]$ , set  $w_t(p_0 + h) = \tilde{w}_t(p_0 + h)$  and  $\pi_t(p_0 + h) = \tilde{\pi}_t(p_0 + h)$  for all  $t > t^*$  and set  $w_t(p_0 + h) = \underline{w}$  and:

$$\pi_t(p_0 + h) = \tilde{\pi}_{t^*}(p_0 + h) e^{-(r+\delta+\lambda_1\bar{\Gamma}(p_0+ h))(t^*-t)} + \frac{p_0 + h - \underline{w}}{r + \delta + \lambda_1 \bar{\Gamma}(p_0 + h)} \left( 1 - e^{-(r+\delta+\lambda_1\bar{\Gamma}(p_0+ h))(t^*-t)} \right)$$

for  $t \in [0, t^*]$ . (Note that  $t^*$  may depend on  $p_0$ .)

**Step 5.** Use  $w_t(p_0 + h)$  and  $\pi_t(p_0 + h)$  constructed at step 4 to solve for  $\mu'_t(p_0 + h)$  in the first equation of (22):

$$\mu'_t(p_0 + h) = \int_t^{+\infty} [1 + \lambda_1\gamma(p_0 + h)v(p_0 + h)\pi_t(p_0 + h)] e^{-[r+\delta+\lambda_1\bar{\Phi}(p_0+ h)](s-t)} ds.$$

**Step 6.** Start over at step 1 substituting  $p_0 + h$  for  $p_0$ .

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<sup>10</sup>The following uses a simple Euler approximation. In practice we use a 2-step Runge-Kutta approximation for numerical accuracy.