

Financial Policy with Fully Rational Firms*

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ABSTRACT. This paper studies a model of firm financing where the firm cares about maximizing cash flows and smoothing dividend payments. Every period, firms decide how much to invest, how many stocks to issue and how many dividends to pay. In contrast to the existing literature, we assume that the firm is fully rational, in the sense that it recognizes the relationship between future dividends and stock prices. Under this assumption, the problem of the firm is of similar nature to a problem of optimal fiscal policy, since it chooses today's dividend and it makes promises to the market stockholders about future dividends that induce them to hold the stock. Financial policy may therefore be time inconsistent. First, we characterize several special cases where time consistency arises. Second, we study the full commitment (and time inconsistent) solution numerically in a setup with capital accumulation and incomplete financial markets. The fully rational policy is also compared to the case where the firm is naive, in the sense that it does not take into account the relationship between future dividends and prices. Our results suggest that growing firms that are fully rational will pay lower dividends at the beginning and promise higher dividends in the future to inflate the stock price. This allows them to raise cheaper external funds and grow faster.

1. INTRODUCTION

This paper studies a general equilibrium model of corporate finance where firms (managers), care about maximizing cash flows and smoothing dividend payments. Every period, firms decide how much to invest, how many stocks to issue and how much to pay out as dividends. We assume that markets are incomplete and that market investors and managers have conflicting objectives, arising from different degrees of risk aversion. In contrast to the existing literature, we also assume that firms are fully rational and recognize the relationship between future dividends and stock prices, in the sense that they can anticipate the reaction of the market investors to their choice of financial policy.

Our last assumption has several important implications. First, the problem of the firm is of a similar nature to a problem of optimal fiscal policy, since it chooses today's dividend and it makes promises to the market stockholders about future dividends that induce them to hold the stock. Given this, financial policy is likely to be time inconsistent. Second, the problem is not recursive and standard dynamic programming is not applicable. One of our contributions is therefore to adapt the techniques used in the optimal taxation literature to formulate and solve the problem recursively.

First, we characterize analytically several special cases where the financial policy is time consistent and others where it is not. We then study the full commitment (and time inconsistent) solution numerically in a setup with and without capital accumulation. The

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equilibrium allocations are compared to the case where the firm does not take into account the relationship between future dividends and prices.

Our paper is related to several strands in the literature. First, it relates to the literature on firm dynamics that started from Hopenhayn's (1992) entry and exit model and that analyzes financial policy in a dynamic infinite horizon setting. Some recent examples include Cooley and Quadrini (2001), Covas and Den-Haan (2007) and Quadrini and Jermann (2005) among many others. In this literature, firms maximize their market value subject to financing frictions. In contrast, we assume conflicting objectives for the market investors and the managers and we abstract from issues such as firm heterogeneity and the size and age distribution of firms, which are central to those papers. Instead, we focus and explicitly model the dividend, stock and stock price interactions by allowing managers to anticipate the effects of dividend policy on the firm's stock price.

Second, our paper is also related to a large strand of corporate finance literature. Since we study the full commitment equilibrium and the possibility of time inconsistent financial policy, we relate more closely to the seminal work of Modigliani and Miller (1961) and to the signalling literature, such as Bhattacharya (1979) and Miller and Rock (1985). In contrast to this literature, our paper assumes full information. Given this, it provides an alternative reason for the presence of time inconsistency. In addition, it rationalizes the fact that firms want to use dividends rather than repurchases under full information. The reason is that firms can use dividend payments to influence stock prices and obtain cheaper external funding.

A potentially extension that would be interesting involves studying the case where the firm takes into account the effects of dividend policy on both stock prices and households' consumption. We leave this for future research.¹

The paper is organized as follows. The model and its recursive formulation under full commitment is presented in Section 2. Section discusses time inconsistency and presents several examples where financial policy turns out to be time consistent. In Section 4 we analyze a three period example and Section 5 studies the infinite horizon economy. Finally, Section 6 summarizes and concludes.

2. THE MODEL

Time is discrete and indexed by $t = 0, 1, 2, \dots$ and the only source of uncertainty in the economy is an exogenous shock θ . The economy is populated by a continuum of identical investors and a firm. The cash flow of the representative firm is denoted by n . We study two different cases. In the first case (exchange economy), the cash flow is equal to the exogenous shock, $n_t \equiv \theta_t$. In the second case (production economy), the cash flow is also a function of the aggregate capital stock k , which is determined endogenously. In this last environment, $n_t \equiv n(\theta_t, k_{t-1}, k_t)$.

Every period t , the firm can issue new stocks that are traded at price p_t . Further, it distributes dividends d_t to the stockholders at the beginning of the period. If we let s_t be the quantity of stocks outstanding at t , with $s_{-1} = 1$, the budget constraint of the firm is equal to:²

$$d_t s_{t-1} \leq p_t (s_t - s_{t-1}) + n_t \tag{1}$$

The stocks issued by the firm are bought by household-investors. Households can also

¹Note that this case would be the closest one to the Ramsey optimal taxation literature. There is an important difference, however, since the Ramsey concept is typically associated with a benevolent government, while the firm is not benevolent in our framework.

²In the present setting, there is indeterminacy with respect to the choice of stocks and bonds and we focus therefore on stock issuance.

trade in bonds that are assumed to be in zero net supply. They solve the following problem:³

$$E_0 \sum_{t=0}^{\infty} \delta^t u(c_t) \text{ st.}$$

$$c_t + p_t (s_{h,t} - s_{h,t-1}) + p_t^b b_{h,t} \leq d_t s_{h,t-1} + b_{h,t-1} \quad (2)$$

In the previous equation, s_h and b_h denote the holdings of stocks and bonds of the households. The price of the safe bond is equal to:

$$p_t^b = \delta E_t \frac{u'(c_{t+1})}{u'(c_t)} \quad (3)$$

Optimality also implies that the stock price depends on the stream of dividends according to the following equation:

$$p_t = \delta E_t \frac{u'(c_{t+1})}{u'(c_t)} [p_{t+1} + d_{t+1}] = E_t \sum_{j=1}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} \quad (4)$$

where we have used the fact that $\lim_{j \rightarrow \infty} E_t \delta^j \frac{u'(c_{t+j})}{u'(c_t)} p_{t+j} = 0$.

In what follows, we call this relationship the price-dividend mapping. Note that this mapping reflects that the stock market is perfectly competitive. As stated in the introduction, we assume that the firm recognizes it and that it takes it into account when deciding on its financial policy. Authors who take stock prices as given ignore the interplay between dividends and stock prices.

We start by defining an exchange equilibrium where the cash flow and financial policy are taken as given. Next, we establish several results that will help us characterize how the cash flow and financial policy are determined optimally in a production equilibrium.

Definition 1: An exchange equilibrium with respect to a given cash flow $\{n_t\}_{t=0}^{\infty}$ and the firm's financial policy $\{d_t, s_t\}_{t=0}^{\infty}$ is given by a vector of allocations for the households $x_h \equiv \{c_t, s_{h,t}, b_{h,t}\}_{t=0}^{\infty}$ and by a vector of prices $\{p_t^b, p_t\}_{t=0}^{\infty}$ such that, (i) given the prices and financial choices of the firm, the vector x_h solves the problem of the households and (ii) markets clear. This implies that $c_t = n_t$, $s_{h,t} = s_t$ and $b_{h,t} = 0$ for all t .

The following Proposition establishes two results regarding the firms's financial policy that will be useful later on. To do this, we define the variables D_t and B_t as follows:

$$D_t \equiv E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} d_{t+j}$$

$$B_t \equiv E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} n_{t+j}$$

Note that D_t represents the present value of dividends and B_t represents the present value of cash flows.

Proposition 1: (i) For any sequence of capital and dividends, $\{d_t, k_t\}_{t=0}^{\infty}$, the period by period budget constraint of the firm in (1) is satisfied at all t if and only if the following constraints are satisfied:

$$s_{-1} E_0 \sum_{j=0}^{\infty} \delta^j \frac{u(c_j)}{u(c_0)} d_j = E_0 \sum_{j=0}^{\infty} \delta^j \frac{u(c_j)}{u(c_0)} n_j \quad (5)$$

$$\frac{B_t}{D_t} \text{ is measurable with respect to information up to } t-1 \text{ for all } t > 0. \quad (6)$$

³We implicitly assume that investors are subject to the natural borrowing limit. In addition stock issuance is subject to a no Ponzi condition.

(ii) Given a cash flow process $\{n_t\}_{t=0}^\infty$, there are many feasible financial choices $\{d_t, s_t\}_{t=0}^\infty$ that constitute an equilibrium. Further, under these different financial choices, the real allocations and firm value are unchanged.

Proof of Proposition 1: (i) To prove the Proposition, we first use the period-by-period constraints in (1) and the price Euler equation from the consumers' problem in (4), together with the No-Ponzi scheme assumption, to derive the following intertemporal budget constraint as follows:

$$\begin{aligned}
d_t s_{t-1} + p_t s_{t-1} &= \\
p_t s_t + n_t &= \\
\delta E_t \frac{u(c_{t+1})}{u(c_t)} [(p_{t+1} s_t + d_{t+1} s_t) + n_t] &= \\
\delta E_t \frac{u(c_{t+1})}{u(c_t)} \left[\delta E_{t+1} \frac{u(c_{t+2})}{u(c_{t+1})} (p_{t+2} s_{t+1} + n_{t+1}) + n_t \right] &= \dots \\
E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} n_{t+j} &\Rightarrow \\
s_{t-1} E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} d_{t+j} &= E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} n_{t+j} \tag{7}
\end{aligned}$$

Since this holds for all $t \geq 0$, the equation evaluated at $t = 0$ implies that (5) is satisfied. In addition, using the definitions of B_t and D_t , this condition implies $\frac{B_t}{D_t} = s_{t-1}$ so that (6) is satisfied. To prove the converse, we show that given (5), (6) and (4), we can construct a sequence of stock holdings such that (1) is satisfied. First, define S_t as follows:

$$S_t \equiv \frac{B_t}{D_t}$$

so that S_t is measurable with respect to information up to $t - 1$. Then

$$\begin{aligned}
D_t S_t &= n_t + E_t \sum_{j=1}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} n_{t+j} \\
&= n_t + \delta E_t \left[E_{t+1} \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} n_{t+j+1} \right] \\
&= n_t + \delta E_t \left[n_{t+1} + E_{t+1} \sum_{j=1}^{\infty} \delta^j \frac{u(c_{t+j})}{u(c_t)} n_{t+j+1} \right] \\
&= n_t + \delta E_t [D_{t+1} S_{t+1}]
\end{aligned}$$

But S_{t+1} is measurable with respect to information up to t , so that

$$D_t S_t = n_t + \delta S_{t+1} E_t [D_{t+1}]$$

Finally, noticing that $D_t = p_t + d_t$, we get the required period-by-period budget constraint as long as we choose $s_{t-1} = S_t = \frac{B_t}{D_t}$.

(ii) The second part of the proposition directly follows from the first. In particular, assume that the cash flow of the firm is given by $\{\tilde{n}_t\}_{t=0}^\infty$ and consider the equilibrium consumption process $\{\tilde{c}_t\}_{t=0}^\infty = \{\tilde{n}_t\}_{t=0}^\infty$. Consider any choice of stocks $\{\tilde{s}_t\}_{t=0}^\infty$ and let

$\{\tilde{s}_t^h\}_{t=0}^\infty = \{\tilde{s}_t\}_{t=0}^\infty$. Consistency with the budget constraint of the firm implies that the associated price has to satisfy the following equation:

$$\tilde{p}_t \tilde{s}_t = E_t \left(\sum_{j=1}^{\infty} \delta^j \frac{u'(\tilde{c}_{t+j})}{u'(\tilde{c}_t)} \tilde{n}_{t+j} \right)$$

where we have used the optimality condition of the households in (4) and the budget constraint of the firm in (1). Knowing the process for stocks and the associated price, we can then use equation (7) to derive the divided process $\{\tilde{d}\}$ that is consistent with the budget constraint of the firm, which satisfies:

$$\tilde{d}_t \tilde{s}_{t-1} + \tilde{p}_t \tilde{s}_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} \tilde{n}_{t+j}$$

Since the allocations $\{\tilde{c}_t\}_{t=0}^\infty = \{\tilde{n}_t\}_{t=0}^\infty$ and $\{\tilde{s}_t^h\}_{t=0}^\infty = \{\tilde{s}_t\}_{t=0}^\infty$ clear the markets and satisfy the budget constraint of the households, we have found an equilibrium with respect to the cash flow and financial policies $\{\tilde{n}_t\}_{t=0}^\infty$ and $\{\tilde{d}_t, \tilde{s}_t\}_{t=0}^\infty$. It is easy to see that we can find many other equilibria by changing $\{\tilde{s}_t\}_{t=0}^\infty$. In addition, since neither consumption \tilde{c}_t nor the value of the firm $\tilde{d}_t \tilde{s}_{t-1} + \tilde{p}_t \tilde{s}_{t-1}$ depend on the firm's financial choice, these variables are the same across the different financial policies. ■

The previous Proposition has several important implications. First, in contrast to a framework where markets are complete, the period by period budget constraint of the firm is not equivalent to the period zero consolidated budget constraint in (5). Under incomplete markets, the following constraint also needs to be satisfied:

$$E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} s_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} n_{t+j} \quad (8)$$

In other words, while many dividend sequences satisfy (5), not all of them are feasible, since they have to adjust so that equation (7) is satisfied. To see that this is the case, assume for example that households are risk neutral and consider the constant stream of dividends $d_t = d = (1 - \delta) E_0 \sum_{t=0}^{\infty} \delta^t n_t$, which clearly satisfy equation (5) under risk neutrality. The associated stock price is equal to $p = d \frac{\delta}{1-\delta}$. In such a case, the budget constraint of the firm will only be satisfied if

$$s_{t-1} = \frac{E_t \sum_{j=0}^{\infty} \delta^j n_{t+j}}{d + p}$$

which cannot be true, since the right hand side depends on information up to period t . This result will be useful later on.

Second, the proposition establishes a Modigliani Miller result, in the sense that many financial choices are feasible given a certain cash flow process. Note that this implies that financial policy will be irrelevant in economies where the cash flow is exogenous or investment is decided independently of the firms financial policy. Given this, we will mostly focus on production economies in what follows.

2.1. The Problem of the Firm. This section discusses the problem of the firm in a production economy under two different firm objectives that have different implications regarding the relevance of financial policy. Such a model allows for the study of how stock issuance can be used to finance firm investment and how the latter is related to the firm's dividend policy.

We assume that the firm owns and accumulates the capital stock that it uses for production each period. The cash flow of the firm is therefore given by:

$$n_t = \theta_t f(k_{t-1}) + (1 - \eta) k_{t-1} - k_t \quad (9)$$

where η is the depreciation rate of capital. The (cum-dividend) value of the firm V_0 at time $t = 0$ is equal to:

$$V_0 \equiv (p_0 + d_0) s_{-1} \quad (10)$$

In the literature, value maximization is the usual objective for the firm. Using the price-dividend mapping that arises from the optimization problem of the investors, the value of the firm V_0 can also be written as:

$$V_0 = s_{-1} E_0 \sum_{t=0}^{\infty} \delta^t \frac{u'(c_t)}{u'(c_0)} d_t = E_0 \sum_{t=0}^{\infty} \delta^t \frac{u'(c_t)}{u'(c_0)} n_t \quad (11)$$

where the last equality uses the budget constraint of the firm.

In addition, we consider a second objective where the firm maximizes the value of dividend payments according to an increasing and concave utility function v :

$$W_0 \equiv E_0 \sum_{t=0}^{\infty} \delta^t v(d_t) \quad (12)$$

As we will show later, this second objective, which we label as a risk averse firm in what follows, can be interpreted as a case where a manager owns a fixed fraction of the firm.

Finally, we consider several cases regarding what the firm internalizes when it makes the financing decisions. In the benchmark economy, we assume that firms take into account the effects of dividend policy on prices and we denote this equilibrium as *fully rational*.

It is important to note that we have implicitly taken this into account by rewriting the value maximization objective as in (11), since this formulation assumes that managers understand the relationship between prices and dividends and that he takes it into account. To make the point clearer, a manager that does not realize this relationship would treat p_0 as outside of his control and decide that the optimum is to pay everything out as dividends today and close down the firm. Note that, using the objective in (11) but taking prices as given in the budget constraint would be inconsistent. As to the second objective, while it does not require any use of the price-dividend mapping, note that this mapping still affects the optimal choices of the firm through the effect of the stock price on the firm's budget constraint.

In a later section, the fully rational equilibrium will be compared to the case where the firm is *naive*, in the sense that it does not take into account the effect of dividends on prices.⁴ In both cases, we maintain the assumption of full commitment.

2.2. The Fully Rational Firm. The problem of a fully rational firm is given by:

$$\max_{\{d,s,k\}} V_0 \text{ or } W_0 \text{ st.} \quad (13)$$

$$d_t s_{t-1} + k_t - (1 - \eta) k_{t-1} = p_t (s_t - s_{t-1}) + \theta_t f(k_{t-1}) \quad (14)$$

$$p_t = \delta E_t \left(\frac{u'(c_{t+1})}{u'(c_t)} (p_{t+1} + d_{t+1}) \right) \equiv E_t \left(\sum_{j=1}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} \right)$$

⁴From the above discussion, this only makes sense in the case of the second firm objective.

Equation (14) reflects that, in addition to stock issuance (external funds), the income of the firm is given by production (earnings or internal funds), which depends on past capital, today's productivity shock and the production function f . In other words, the firm can use internal and external funds to face the investment expense $k_t - (1 - \eta)k_{t-1}$ and pay dividends. Note that a naive firm will not take into account the last constraint, since it does not internalize the effects of dividends on stock prices.

Definition 2: A production equilibrium with fully rational firms is a vector of allocations for the households $x_h \equiv \{c_t, s_{h,t}, b_{h,t}\}_{t=0}^{\infty}$, a vector of allocations for the firms $x_f \equiv \{k_t, d_t, s_t\}_{t=0}^{\infty}$ and a vector of prices $\{p_t^b, p_t\}_{t=0}^{\infty}$ such that, (i) given the prices and x_f , the vector x_h solves the problem of the households, (ii) given the price-dividend mapping, the vector x_f solves the problem of the firm and (iii) markets clear. This implies that $c_t = n_t$, $s_{h,t} = s_t$ and $b_{h,t} = 0$ for all t .

The equilibrium under the different firm objectives is characterized in what follows. Consider first the *fully rational* equilibrium with risk averse firms. In this case, the Lagrangian of the problem is equal to:

$$L = E_0 \sum_{t=0}^{\infty} \delta^t [v(d_t) + \gamma_t u'(c_t) (\theta_t f(k_{t-1}) + (1 - \eta)k_{t-1} - k_t - d_t s_{t-1})] \\ + E_0 \sum_{t=0}^{\infty} \delta^t \gamma_t \left[(s_t - s_{t-1}) E_t \sum_{j=1}^{\infty} \delta^j u'(c_{t+j}) d_{t+j} \right]$$

where γ_t is the multiplier associated with the period t budget constraint⁵. The presence of expectations of future variables in the expression above implies that the problem is not recursive. Nevertheless, we can use the recursive contracts approach of Marcet and Marimon (1999) to write the Lagrangian recursively by introducing a new state variable as follows:

$$L = E_0 \sum_{t=0}^{\infty} \delta^t [v(d_t) + \mu_{t-1} u'(c_t) d_t + \gamma_t u'(c_t) (\theta_t f(k_{t-1}) + (1 - \eta)k_{t-1} - k_t - d_t s_{t-1})]$$

where the co-state variable μ_t follows the law of motion:

$$\mu_t = \mu_{t-1} + \gamma_t (s_t - s_{t-1}) \text{ with } \mu_{-1} = 0. \quad (15)$$

In the present setting, the multiplier μ_t captures the promises that have been made in the past about the dividend in period t , d_t . Since there are no past promises to be kept at the beginning of time, we have that $\mu_{-1} = 0$. On the other hand, at $t = 1$, there is an inherited promise from period 0, $\mu_0 = \gamma_0 (s_0 - s_{-1})$, which arises from the fact that p_0 is a function of the expectation of future dividend payments. Similarly, as we consider dividends further away in the future (d_2, d_3 etc.), these are linked with promises made in past periods. As reflected by its law of motion, the co-state μ_{t-1} adds up all of these past promises and summarizes them in a single number. More intuition can be obtained by considering the first order conditions arising from this problem. These are given by:

$$v'(d_t) = \gamma_t s_{t-1} u'(c_t) - u'(c_t) \mu_{t-1} \quad (16)$$

$$\gamma_t u'(c_t) p_t = E_t [\gamma_{t+1} u'(c_{t+1}) (d_{t+1} + p_{t+1})] \quad (17)$$

$$\gamma_t u'(c_t) = \delta E_t [\gamma_{t+1} u'(c_{t+1}) (\theta_{t+1} f'(k_t) + 1 - \eta)] \quad (18)$$

⁵For convenience, we have multiplied the budget constraints by $u'(c_t)$, essentially renormalizing the multipliers γ_t .

The last two equations represent the stock Euler equation (17) and the capital Euler equation (18) respectively, which are fairly standard. We therefore focus on the condition describing the optimal dividend choice (16). As we see, a marginal increase in d_t yields a direct utility benefit of $v'(d_t)$ but it has a cost in terms of lost resources at t that is equal to $\gamma_t s_{t-1} u'(c_t)$. A naive firm that does not realize the relationship between its stock price and its dividend policy would only have to consider these two effects.

On the other hand, a fully rational firm also has to take into account the fact that the dividend choice at time t will affect stock prices in all previous periods. In particular, a marginal increase in d_t also implies increases in the stock prices of all previous periods and this in turn affects the resources available in all these periods. If the firm has been issuing stocks, this price effect is positive, since it implies more funds raised for the same level of stock issuance. Conversely, if the firm has been repurchasing stocks in the past, a price increase has a negative effect on resources. Thus, the multiplier μ_{t-1} summarizes the effect of a marginal change in d_t on all previous periods' resources, and it can be positive or negative depending on the history of stock issuance and repurchase.

The above first order conditions, together with the budget constraint and price equations below, characterize the equilibrium:

$$s_{t-1} E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} n_{t+j} \quad (19)$$

$$p_t = E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} (d_{t+1} + p_{t+1}) \right] \quad (20)$$

Looking at the previous system of equations, several important remarks are worth noting. *First*, if we wanted to analyze an exchange economy, where the cash flow is exogenous, it is easy to see that the system of equations that would characterize the equilibrium allocations would be the same as above except that condition (18) would not be present. *Second*, with the additional assumption that θ is Markov, the optimal solution of the previous problem satisfies:

$$\begin{bmatrix} d_t \\ s_t \\ k_t \\ \gamma_t \end{bmatrix} = F(\theta_t, k_{t-1}, s_{t-1}, \mu_{t-1})$$

for a time-invariant function F , and $\mu_{-1} = 0$. Thus, despite having a maximization problem where the Bellman equation does not hold, by adding the co-state variable μ , we can make the solution recursive. This is due to the fact that, even though the whole past history is needed to make decisions at any point in time t , the recursive contracts formulation allows us to summarize all the relevant information in just one variable, μ_{t-1} . *Third*, since standard dynamic programming is not applicable, the solution is not the fixed point of a contraction mapping and convergence is therefore not guaranteed.

Fourth, such a law of motion delivers in general a time-inconsistent solution, although we will show below that there are some cases where the solution is actually time consistent. Here, it is important to note that time inconsistency may arise due to the fact that firms take into account the effects of dividend policy on stock prices. To see this, consider the naive firm. In this case, the equilibrium is characterized by equations (19), (20) and by the following first order conditions:

$$v'(d_t) = \gamma_t u'(c_t) s_{t-1} \quad (21)$$

$$\gamma_t u'(c_t) p_t = E_t [\gamma_{t+1} u'(c_{t+1}) (d_{t+1} + p_{t+1})] \quad (22)$$

$$\gamma_t u'(c_t) = \delta E_t [\gamma_{t+1} u'(c_{t+1}) (\theta_{t+1} f'(k_t) + 1 - \eta)] \quad (23)$$

It is easy to see that the law of motion of the system above can be written as a time invariant function in the natural state variables $(\theta_t, k_{t-1}, s_{t-1})$. In this case, the Bellman equation applies and the solution is time consistent.

Last, the equilibrium system of equations reflects that financial policy is fully determined when the firm is risk averse. In particular, the investment and financial decisions are linked in the present setup. To provide a more intuitive explanation of why this is the case, we now show that our firm objective implies that firms care about both maximizing cash flows and smoothing dividend payments. This can be done by plugging (5) into the objective function of the firm. The problem can then be rewritten as:

$$\max_{\{d,s,k\}} E_0 \sum_{t=0}^{\infty} \delta^t \left[v(d_t s_m) - \gamma_0 \frac{u'(c_t)}{u'(c_0)} d_t + \gamma_0 \frac{u'(c_t)}{u'(c_0)} n_t \right] \text{ st.} \quad (24)$$

(6)

where γ_0 is the Lagrange multiplier of (5). Expressed in this way it is clear that the manager would like to maximize the expected, discounted *weighted* sum of two elements. The first element is $v(d_t s_m) - \gamma_0 \frac{u'(c_t)}{u'(c_0)} d_t$ and it depends only on dividends, whereas the second element is the cash flow weighted by $\gamma_0 \frac{u'(c_t)}{u'(c_0)}$. This illustrates that investment and financing decisions are linked with risk averse firms. In contrast, suppose that firms would maximize their market value according to (11). Note that this would correspond to the case where managers just care about the last part of the previous objective. In this case, it is easy to show that they would set capital so that it satisfies the following equation:

$$1 = \delta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} (\theta_{t+1} f'(k_t) + 1 - \eta) \right] \quad (25)$$

In what follows, we refer to this value of capital as the value maximizing level of capital. In addition, it is also easy to see using the results of Proposition 1 that many financial policies would be consistent with that level of investment. In fact, since the level of investment is independent of the financial policy of the firm, financial policy is indeterminate. Given this, we focus on risk averse firms in what follows.

In this case, the presence of the part $v(d_t s_m) - \gamma_0 \frac{u'(c_t)}{u'(c_0)} d_t$ in the objective can be interpreted as the manager caring about minimizing the variability of dividends for a given cash flow. If the manager cared only about this part of the objective function, he would not choose capital efficiently (as under value maximization), since he would use it to smooth dividends. In fact, the optimal behavior of the manager has to balance the optimality of the capital choice that maximizes the cash flows and is best for the consumers with the desire to smooth dividends.

2.3. A Modified Setting. In this section, we show that a risk averse firm can be interpreted as one where managers hold a (fixed) number of stocks s_m . To see this, we now write a slightly different version of the model where the manager's stock holding is explicit. As before, we assume that there are two kinds of agents, managers and investors. Investors are all alike and they can invest in stocks of the firm that is run by the manager. The problem of the investors is given by:

$$\max_{\{c_t, s_{h,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u_h(c_t) \text{ st.}$$

$$c_t + p_t(s_{h,t} - s_{h,t-1}) = d_t s_{h,t-1}$$

All investors have the same utility and initial stock holdings. The usual first order condition for an interior solution for stocks is given by:

$$p_t = \delta E_t \frac{u'(c_{t+j})}{u'(c_t)} [p_{t+j} + d_{t+j}] = E_t \left(\sum_{j=1}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} \right)$$

The manager runs the firm and decides on the investment and financial policy. In particular, he decides how much to invest, how much stock to issue each period and how much to pay out as dividends. The income of the manager is tied to the dividend paid by the firm. This could be because the manager owns a fixed number of stocks or because the salary of the manager is proportional to the dividends. Both cases give rise to the same model outcome. The manager has no other means of saving or dissaving and his consumption is given by:

$$c_{m,t} = s_m d_t$$

where s_m can be interpreted as the fixed number of shares the manager owns or as the constant that determines his salary. The problem of the manager is therefore given by:

$$\max_{\{d,s,k\}} E_0 \sum_{t=0}^{\infty} \delta^t u_m(d_t) \text{ st.}$$

$$d_t s_{t-1} = p_t (s_t - s_{t-1}) + n_t$$

where $u_m(d_t) \equiv v(d_t s_m)$ and $s_t = s_{h,t} + s_m$ is the total number of stocks.

It is important to note that the way this model is presented, it assumes that the manager's stocks are fixed and the investors' stocks can change. Clearly, this implies that the total number of stocks is also variable. On the other hand, this setup can also be interpreted as one where managers and investors can both change their *proportion* of the firm owned, since the proportion of the manager in the firm changes with a change in the number of stocks. Since this modified version explicitly models the presence of two different agents, we will stick to this version in what follows.

3. TIME CONSISTENCY

It would seem clear that, in general, a firm can improve its stance by credibly promising a certain path for dividends. What follows is an intuitive statement of how it can do so. When the firm needs to raise external funds, it can do so by increasing the amount of stocks outstanding. Given this, the firm might want to drive today's price up by paying a low dividend today and promising at the same time a stream of high future dividends. When tomorrow comes and investors have already bought the firm's stock, the manager has an incentive to deviate if he is not fully committed. This is due to the fact that adjusting the dividends downwards will not affect the stock price, which depends only on future dividends. In other words, if the institutions allow him to do so, it seems that it is better for the manager to renege on past promises. Using this argument, it also seems that a firm that is issuing stocks (e.g. a growing firm) and that realizes the relationship between dividends and stock prices will tend to tilt the dividend profile in favour of future dividends.

As already noted before, the fact that time inconsistency may arise in the present setup is reflected formally in the recursive formulation of the last section. As we see, the same (time-invariant) policy function F has to be used for all periods with μ_{t-1} as an argument. Further, μ_{t-1} is determined endogenously every period from past actions and it captures promises that have been made about today's dividends. The fact that there are no past commitments in the first period is reflected in $\mu_{-1} = 0$. Further, that a manager is tempted to re-optimize is reflected in the fact that, if he was allowed to do so in period t' without

being restricted to honor past commitments, he would want to follow a policy that implies re-setting $\mu_{t-1} = 0$ and following the optimal policy F from then onwards. If the manager is fully committed to following the announced policy, however, he will plug in the actual μ_{t-1} in the policy function.

The previous two paragraphs are standard descriptions of how time-inconsistency can arise in models when a constraint such as (4) involves values of future choice variables. It is, however, not true that any model where future decision variables influence today's constraint displays time inconsistency. In what follows, we discuss some cases where the full commitment solution is time consistent, implying that the plans the manager makes for future dividends and stock issuances will indeed be fulfilled in the future, even if the manager is offered the opportunity to re-optimize. The following proposition shows that time consistency arises in exchange economies, where the cash flow is exogenous, or in production economies when the value maximizing level of capital is implemented.

Proposition 2.

(i) Consider the full commitment solution $x_0^* \equiv \{c_t^*, d_t^*, s_t^*, p_t^*\}_{t=0}^\infty$ in an exchange economy. Furthermore, given a time period \bar{t} , define the "time- \bar{t} " continuation problem as:

$$\max_{\{d_t, s_t\}_{t=\bar{t}}^\infty} E_{\bar{t}} \sum_{t=\bar{t}}^\infty \delta^{t-\bar{t}} v(d_t s_t) \text{ st.}$$

$$(1)-(4) \text{ for all } t \geq \bar{t}$$

$$s_{\bar{t}-1} = s_{\bar{t}-1}^*$$

Denote the solution to this problem by $x_{\bar{t}}^{**} \equiv \{c_t^{**}, p_t^{**}, d_t^{**}, s_t^{**}\}_{t=\bar{t}}^\infty$. Note that this is the solution that would arise if, having followed the full commitment solution up to time \bar{t} , the manager decided to re-optimize and choose the best solution from then on, ignoring the plans that were involved in the solution x_0^* that was optimal from the standpoint of period zero. We have that, for any \bar{t} , the corresponding solution to the "time- \bar{t} " problem satisfies

$$\begin{bmatrix} c_t^{**} \\ p_t^{**} \\ d_t^{**} \\ s_t^{**} \end{bmatrix} = \begin{bmatrix} c_t^* \\ p_t^* \\ d_t^* \\ s_t^* \end{bmatrix} \quad \text{for all } t \geq \bar{t} \quad (26)$$

(ii) Consider the optimal full commitment solution $x_0^* \equiv \{d_t^*, s_t^*, k_t^*, c_t^*, p_t^*\}_{t=0}^\infty$ of a production economy where the capital allocation satisfies $u'(c_t^*) = \delta E_t [u'(c_{t+1}^*) (\theta_{t+1} f'(k_t^*) + 1 - \eta)]$. Consider the continuation problem where the manager follows full commitment up to time \bar{t} and decides to re optimize afterwards. Denote this solution by $x_{\bar{t}}^{**} \equiv \{d_t^{**}, s_t^{**}, k_t^{**}, c_t^{**}, p_t^{**}\}_{t=\bar{t}}^\infty$. We have that:

$$\begin{bmatrix} d_t^{**} \\ s_t^{**} \\ k_t^{**} \\ c_t^{**} \\ p_t^{**} \end{bmatrix} = \begin{bmatrix} d_t^* \\ s_t^* \\ k_t^* \\ c_t^* \\ p_t^* \end{bmatrix} \quad \text{for all } t \geq \bar{t} \quad (27)$$

Proof of Proposition 2.

(i) We start by proving the first part of the proposition. In an exchange economy, we can ignore the first order condition with respect to capital in (18). Further, the first

order conditions with respect to dividends and stocks corresponding to the full commitment solution imply that:

$$0 = E_t [(\gamma_t^* - \gamma_{t+1}^*) u'(c_{t+1}^*) (d_{t+1}^* + p_{t+1}^*)] \quad (28)$$

$$s_m v'(d_t^*) = \gamma_t^* s_{t-1}^* u'(c_t^*) - u'(c_t^*) \mu_{t-1}^* \text{ for } t > 0$$

$$s_m v'(d_0^*) = \gamma_0^* s_{-1}^* u'(c_0^*) \quad (29)$$

We argue now that these first order conditions imply that the first order conditions for the time- \bar{t} problem are satisfied for the same dividend and stock processes but for different multiplier variables γ and μ . The conditions characterizing the solution for the time- \bar{t} problem are:

$$s_m v'(d_t^{**}) = \gamma_t^{**} s_{t-1}^{**} u'(c_t^{**}) - u'(c_t^{**}) \mu_{t-1}^{**} \text{ for } t > \bar{t} \quad (30)$$

$$s_m v'(d_{\bar{t}}^{**}) = \gamma_{\bar{t}}^{**} s_{\bar{t}-1}^{**} u'(c_{\bar{t}}^{**})$$

$$0 = E_t [(\gamma_t^{**} - \gamma_{t+1}^{**}) u'(c_{t+1}^{**}) (d_{t+1}^{**} + p_{t+1}^{**})] \text{ for } t \geq \bar{t} \quad (31)$$

$$p_t^{**} = \delta E_t \frac{u'(c_{t+1}^{**})}{u'(c_t^{**})} [d_{t+1}^{**} + p_{t+1}^{**}] \text{ for } t > \bar{t} \quad (32)$$

$$s_{t-1}^{**} E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j}^{**})}{u(c_t^{**})} d_{t+j}^{**} = E_t \sum_{j=0}^{\infty} \delta^j \frac{u(c_{t+j}^{**})}{u(c_t^{**})} n_{t+j}^{**} \text{ for } t > \bar{t} \quad (33)$$

To show that we can find multipliers so that the previous first order conditions are satisfied for (26), we can set

$$\gamma_{\bar{t}}^{**} = -\frac{s_m v'(d_{\bar{t}}^*)}{s_{\bar{t}-1}^* u'(c_{\bar{t}}^*)}$$

$$\mu_{\bar{t}-1}^{**} = 0$$

and we can then derive γ_t^{**} and μ_t^{**} for $t > \bar{t}$ using (30) and the law of motion of μ in (15) for (26). Since, equations (32)-(33) are satisfied for (26), we only need to show that the following condition is satisfied so that (31) also holds for (26):

$$\gamma_t^{**} - \gamma_{t+1}^{**} = \gamma_t^* - \gamma_{t+1}^* \text{ for } t > \bar{t}.$$

To see that this is the case note that equation (30) implies that

$$\gamma_t^{**} s_{t-1}^* = \frac{s_m v'(d_t^*)}{u'(c_t^*)} + \mu_{t-1}^{**}$$

$$\gamma_{t+1}^{**} s_t^* = \frac{s_m v'(d_{t+1}^*)}{u'(c_{t+1}^*)} + \mu_t^{**}$$

It therefore follows that:

$$\gamma_t^{**} s_{t-1}^* - \gamma_{t+1}^{**} s_t^* = x_t - [\mu_t^{**} - \mu_{t-1}^{**}]$$

$$= x_t - \gamma_t^{**} [s_t^* - s_{t-1}^*]$$

where $x_t = \frac{s_m v'(d_t^*)}{u'(c_t^*)} - \frac{s_m v'(d_{t+1}^*)}{u'(c_{t+1}^*)}$ and the last equality uses the law of motion for μ . Finally, rearranging the previous equation, we obtain that:

$$\gamma_t^{**} - \gamma_{t+1}^{**} = \frac{x_t}{s_t^*} \quad (34)$$

Since the right hand side only depends on variables that are assumed to be the same in the two allocations, our initial claim is true.

(ii) To prove the second part of the proposition, assume that the value maximizing level of capital is actually implemented in the full commitment solution, that is, (25) satisfied for all t . We now show that, in this case, the manager will not want to re optimize even if given the chance.

To prove this statement, we observe the following. The first part of the proposition has shown that all the first order conditions of the "continuation problem" of an exchange economy are satisfied for different multipliers. Since the system of equations that characterizes the equilibrium in a production economy is the same, except that it includes also the capital Euler equation in (18), all that is left to show is that this last condition will be satisfied for the new gammas we have found. We can rewrite the optimality condition with respect to capital in the full commitment solution (equation (18)) as follows:

$$\gamma_t^* \delta E_t [u'(c_{t+1}^*) (\theta_{t+1} f'(k_t^*) + 1 - \eta)] = \delta E_t [\gamma_{t+1}^* u'(c_{t+1}^*) (\theta_{t+1} f'(k_t^*) + 1 - \eta)]$$

But this implies that:

$$E_t[(\gamma_{t+1}^* - \gamma_t^*) u'(c_{t+1}^*) (\theta_{t+1} f'(k_t^*) + 1 - \eta)] = 0$$

As we see, the series for γ that satisfy the first order conditions in the continuation problem of the exchange economy also satisfy this first order condition with respect to capital in the production economy as long as the capital is at the value maximizing level. The solution is therefore time consistent. ■

The previous proposition shows that the dividend policy in exchange economies is time consistent. Further, it shows that time consistency also arises in production economies where the value maximizing level of capital is chosen under full commitment. In general, however, the value maximizing level of capital is not the full commitment solution. In this case, time consistency requires that the gamma increments in (34) of the "continuation problem" are the same as in the full commitment case, since this always ensures that the first order conditions with respect to dividends and stocks of the "continuation problem" are satisfied. But it would seem that these gammas cannot satisfy the first order condition with respect to capital (equation (18)) in general. Some of the cases where the solution turns out to be time inconsistent are studied numerically in Sections 4 and 5. In what follows, we discuss some additional variants of the model with production where time consistency arises. For simplicity of exposition, these cases are discussed assuming that investors are risk neutral.

Example 1: Constant dividends. Assume that, for some reason, the full commitment solution implies that dividends are constant, that is,

$$d_t = \bar{d} \text{ for all } t$$

This would occur, for example, if there was no uncertainty. In addition, it would occur if we introduced a full array of contingent claims. If dividends are constant, it is easy to see from (16) that γ_t will be constant, and the first order condition for capital in (18) implies that it will be set at the value maximizing level. In this case, the arguments of Proposition 3 apply and we have time consistency.

Example 2: No uncertainty after T. Consider a special case, where productivity is previously known after T , that is,

$$\theta_t = \bar{\theta} \text{ for } t \geq T$$

for a predetermined constant $\bar{\theta}$. Using the previous arguments, it is easy to see that the value maximizing level of capital will be implemented from period T onwards. In particular, notice that the following solution satisfies all the first order conditions for $t \geq T$:

$$\begin{aligned}
k_t &= \bar{k}, \text{ where } 1 = \delta [\bar{\theta} f'(\bar{k}) + 1 - \eta] \text{ for } t \geq T \\
\gamma_t &= \gamma_T \text{ for } t \geq T \\
d_t &= \bar{d}(k_{T-1}, s_{T-1}) \\
&\equiv \frac{1 - \delta}{s_{T-1}} \left(\bar{\theta} f(k_{T-1}) - \bar{k} + (1 - \eta)k_{T-1} + \delta \frac{\bar{\theta} f(\bar{k}) - \eta \bar{k}}{1 - \delta} \right) \text{ for all } t \geq T
\end{aligned} \tag{35}$$

where the third equation gives the constant level of dividends that satisfies the budget constraint in (19) given the states k_{T-1} and s_{T-1} in that period. In addition, the levels for stocks after T will be given by:

$$s_t = s_T = \frac{\bar{\theta} f(\bar{k}) - \eta \bar{k}}{\bar{d}} \text{ for all } t \geq T$$

because this guarantees (19) for $t \geq T$. In other words, the value maximizing capital stock, dividends and stocks will be constant from period T onwards.

Here, we have used the quotes because these dividends are only "optimal" contingent on the state variables k_{T-1}, s_{T-1} , which are themselves random. This makes the dividends $\bar{d}(k_{T-1}, s_{T-1})$, in principle, random. Note also that the capital stock is set at the risk neutral level from period T onwards, as it would have been under complete markets. Note also that, even though the dividend is constant, it is not the one that would have been achieved with complete markets for the actual initial condition, since the shocks up to period T will influence the realized k_{T-1}, s_{T-1} . In other words, the long run level of d is stochastic and will be different from the one with complete markets.

By the previous arguments, it is clear that the model will be time consistent after T . In the appendix we provide a proof that the value maximizing capital will also be chosen at $T - 1$, implying that the model is also time consistent at $T - 1$.

Example 3: Constant Productivity after T. Consider another special case where randomness stops at period T , so that:

$$\theta_t = \theta_T \text{ for } t \geq T$$

In such a case, it is clear that the risk neutral capital level will be implemented from period $T + 1$ onwards. To see this, notice that the following solution satisfies all the first order conditions for $t \geq T$:

$$\begin{aligned}
k_t &= \bar{k} \text{ where } 1 = \delta [\theta_T f'(\bar{k}) + 1 - \eta] \text{ for all } t \geq T + 1 \\
\gamma_t &= \gamma_T \text{ for all } t \geq T + 1 \\
d_t &= \bar{d}(k_T, s_T) \\
&\equiv \frac{1 - \delta}{s_T} \left(\bar{\theta} f(k_T) - \bar{k} + (1 - \eta)k_T + \delta \frac{\bar{\theta} f(\bar{k}) - \eta \bar{k}}{1 - \delta} \right) \text{ for all } t \geq T + 1
\end{aligned} \tag{36}$$

As before, \bar{k} is random, since it is a function of θ_T . Using similar arguments to the ones used in the previous example, it is possible to show that the model is time consistent after period T , but again it is unlikely that it is the case in earlier periods.

Example 4: Finite Horizon and Full Capital Depreciation. Assume that the economy only lasts for a finite number of periods and capital depreciates fully. In this case, the risk neutral level of capital will be chosen and the solution will be time consistent. To see this, consider the last period. The first order condition with respect to dividends in (16) can be rewritten as:

$$u'(d_T) = v'(d_{T-1}) + s_{T-1}(\gamma_{T-1} - \gamma_T)$$

implying that:

$$\gamma_T = \gamma_{T-1} + \frac{v'(d_{T-1}) - v'(d_T)}{s_{T-1}}$$

Substituting for γ_T into the capital Euler equation in (18), which is given by:

$$\gamma_{T-1} = E_{T-1} [\gamma_T \delta \theta_T f'(k_{T-1})]$$

we obtain:

$$\gamma_{T-1} [1 - \delta \theta_T f'(k_{T-1})] = E_{T-1} \left[\frac{v'(d_{T-1}) - v'(d_T)}{s_{T-1}} \delta \theta_T f'(k_{T-1}) \right]$$

Note that just need to show that the right hand side of the previous equation is equal to zero, that is,

$$E_{T-1} \left[\frac{v'(d_{T-1}) - v'(d_T)}{s_{T-1}} \delta \theta_T f'(k_{T-1}) \right] = 0 \quad (37)$$

To see that this is the case, note first that the last period budget constraint is given by:

$$d_T s_{T-1} = \theta_T f(k_{T-1})$$

implying that

$$\theta_T f'(k_{T-1}) = \frac{d_T s_{T-1}}{k_{T-1}}$$

Replacing the previous expression in (37), it follows that we just need to show that:

$$E_{T-1} [v'(d_{T-1}) - v'(d_T) d_T] = 0$$

On the other hand, the first order condition with respect to stocks implies that:

$$E_{T-1}(\gamma_T d_T) = \gamma_{T-1} E_{T-1} d_T$$

and replacing γ_T in terms of γ_{T-1} we get

$$E_{T-1} \left[\frac{u'(d_{T-1}) - u'(d_T)}{s_{T-1}} \right] d_T = 0$$

This proves that:

$$1 = \delta f'(k_{T-1}) E_{T-1}(\theta_T)$$

Following the same steps for the case where depreciation is not equal to one, it is easy to show that we would also need to have that $E_{T-1} [v'(d_{T-1}) - v'(d_T)] = 0$, which is unlikely to be satisfied. Given that the solution is likely to be time inconsistent with partial depreciation, we study this case numerically in the next sections.

4. A THREE-PERIOD EXAMPLE

This section analyzes numerically a three-period version of the model, where financial policy is likely to be time inconsistent. As stated earlier, we compare the financial and real allocations as well as the prices for the cases with naive and fully rational firms. Even though our solution is the one under full commitment, we also discuss the time-inconsistency issues.

We make the following assumptions on the functional forms: $f(k) = k^\alpha$, $u(c) = \frac{c^{1-\gamma_h}}{1-\gamma_h}$ and $v(d) = \frac{d^{1-\gamma_m}}{1-\gamma_m}$, where γ_h and γ_m are the risk aversion values for the household and the manager respectively. Regarding the parameterization, we assume that $\gamma_h = 0.5$ (almost risk neutral investors)⁶, $\gamma_m = 5$, $\alpha = 0.4$, $\beta = 0.9$, $\delta = 0$ and $s_m = s_{h,-1} = 1$. Finally, we assume that there is no uncertainty in period $t = 1$, while the productivity shock θ can only take two possible values $\theta^L = 0.7$ and $\theta^H = 1.3$ in periods $t = 2$ and $t = 3$.

4.1. Exchange Economy. We begin by analyzing an exchange economy and then proceed to the economy with capital accumulation. To explain the equilibrium prices and allocations more clearly, it is best to begin by considering the role of the financial asset (the stock). In the absence of stock trading ($s_{h,t} = s_{h,t-1} = s_h$), investors and managers would get the following consumptions:

$$c_t = \frac{s_m}{s_m + s_h} \theta_t$$

$$d_t = \frac{1}{s_m + s_h} \theta_t$$

This implies that each agent gets a constant fraction of the earnings realization under autarky. On the other hand, the ability to trade in stocks allows managers and investors to smooth the consumption and dividend processes against earnings fluctuations. Here, it is important to note that the agents have conflicting objectives. For example, when earnings are low, both investors and managers would like to use stock trade to smooth c_t and d_t , but both cannot do so simultaneously. It turns out that the agent with the highest level of risk aversion obtains more insurance, while the autarkic equilibrium obtains if the two agents are equally risk averse.

With our benchmark parameterization, the manager is more risk averse, implying that he can smooth dividends using stock trade. In turn, investors are only willing to provide insurance and withstand a higher consumption *volatility* only if they receive a higher *level* of consumption as a compensation. In other words, there is a level versus smoothness trade-off in consumption. This results in the more risk averse party getting smoothing but less level, while the less risk averse party gets less smoothing and a higher level. With our parameterization, this implies that stocks will be issued when θ is low and repurchased when θ is high, since this is the profile that smooths dividend payments.

This basic idea is enough to understand the allocations in the 3-period model with naive firms, which is shown in Table 1. Throughout the Table, we denote the equilibrium with naive firms with N and the one with fully rational firms with FR . In the first period, the investor is allowed to consume more than the manager ($c_1 > d_1$). This way, the manager transfers some of his resources to the investor through stock repurchases and he pays a lower dividend per stock. As explained before, this is the payment for the insurance that the manager has bought. In period two, if bad times come (θ_2 is low), the investor ‘agrees’ not to try to smooth his consumption. On the contrary, we see that external funds are positive and consumption is hit especially hard by the shock. In contrast, the manager maintains a relatively stable level of dividends. The exact opposite happens in the case of good times (θ_2 is high).

⁶Similar results can be obtained with $\gamma_h = 0$. Since consumption turns out to be negative in this case, however, we have decided to report the results with almost risk neutral investors.

Finally, the last period allocations are dictated by the budget constraints and by the fact that $s_3 = p_3 = 0$. Since there is really no choice to be made in the last period, given any realization of internal funds θ_3 , total dividends are equal to this realization, while the dividends per stock d_3 are equal to $\frac{\theta_3}{s_2+s_m}$. As a result, whenever there is a lot of equity issued in period two, d_3 is low in the last period and vice versa. The previous observations allow us to explain price behavior using:

$$p_t = E_t \sum_{j=1}^T \beta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} \quad (38)$$

According to the previous equation, we expect prices to be high when expected future dividends are high. As a result, we expect period two prices to be high when equity is bought back and low when new equity is issued. The only thing that could reverse this relationship is a strong opposite movement in the intertemporal marginal rate of substitution⁷.

Table 1: Exchange Economy

| $(t, \theta) :$ | 1 | $(2, l)$ | $(2, h)$ | $(3, ll)$ | $(3, lh)$ | $(3, hl)$ | $(3, hh)$ |
|-----------------|---------------|---------------|---------------|--------------|--------------|--------------|--------------|
| N d_t | 0.456 | 0.394 | 0.550 | 0.338 | 0.628 | 0.439 | 0.816 |
| FR d_t | 0.458 | 0.391 | 0.560 | 0.340 | 0.631 | 0.433 | 0.804 |
| N c_t | 0.543 | 0.305 | 0.749 | 0.361 | 0.671 | 0.260 | 0.483 |
| FR c_t | 0.541 | 0.308 | 0.739 | 0.359 | 0.668 | 0.266 | 0.495 |
| N ef_t | -0.087 | 0.053 | -0.249 | | | | |
| FR ef_t | -0.082 | 0.048 | -0.227 | | | | |
| N s_t^h | 0.907 | 1.069 | 0.592 | | | | |
| FR s_t^h | 0.912 | 1.058 | 0.615 | | | | |
| N p_t | 0.950 | 0.330 | 0.792 | | | | |
| FR p_t | 0.943 | 0.334 | 0.766 | | | | |
| | U_c | U_m | PV_c | PV_d | std_c | std_d | |
| N | 3.815 | -17.28 | 1.378 | 1.331 | 0.173 | 0.106 | |
| FR | 3.813 | -17.23 | 1.375 | 1.334 | 0.167 | 0.107 | |

If we compare these results with the allocations under fully rational firms, all of the above mechanisms are in operation too. However, the manager realizes now the effect that dividends can have on the stock price. In this case, the questions of interest are: Can managers use this knowledge to improve their situation? What prices would they choose if they had this option? How could they actually implement these prices?

It turns out that the answer to the first question is positive. The FR allocation is different from the competitive equilibrium and it implies higher welfare for the manager and the same welfare for the investor. This can be seen in the last two rows of Table 1, reporting welfare values for the investor (U_c) and the manager (U_m), as well as the standard deviation of consumption (std_c) and dividends (std_d). In addition, we report a measure of the level of dividends and consumption that corresponds to their present value, PV_d and PV_c respectively, using risk neutral valuations. As we see, the welfare of the manager is increased through a higher level of dividends and despite their higher variability.

Proceeding to the second question, we observe the following. Whenever new stocks are issued, the manager would like the price to be high and whenever stocks are being repurchased the manager would like the price to be low. Comparing the naive and fully rational firm allocations clearly shows this pattern. Here, it is important to clarify that the manager does not really ‘choose’ prices as if it were a monopolist. In particular, the stock price has to

⁷This channel is very small because of our assumption on investors being almost risk neutral.

follow competitive market prices according to the price-dividend mapping in (38) and the manager cannot influence other firms with his actions. On the other hand, they can choose dividend payments and, in a sense, this means that they can choose the 'quality' of their stock, while the price is determined competitively given this quality choice. Nevertheless, the price-dividend mapping is exploited and it allows external funds to be raised with favorable prices. This is achieved by promising higher (lower) future dividends when they want to have higher (lower) stock prices.

Finally, recall that proposition 2 shows that the fully rational allocation is time consistent in the exchange economy. We postpone the discussion of this issue until the end of this section. In what follows, we discuss the production economy, where time inconsistency arises.

4.2. Production Economy. As in the exchange economy, we start by analyzing a simple benchmark where managers and investors have the same level of risk aversion. This implies that there is no scope for stock trading. In the absence of stock trading, the investor's budget constraint implies that consumption and dividends are always equal. In turn, by the manager's budget constraint, output net of investment is equally distributed between the investor and the manager. The only issue to be decided is therefore capital accumulation. Note also that the gamma multipliers are constant since,

$$\gamma_t = \frac{s_m v'(d_t s_m)}{(s_{h,t-1} + s_m) u'(c_t)} = \frac{1}{2}$$

where we have used the fact that $d_t = c_t$, $s_m = s_{h,t-1} = 1$ and u and v are the same. In this particular case, it also follows that capital accumulation is efficient, in the sense that it is equal to the value maximizing capital satisfying the following equation:

$$u'(c_t) = E_t [u'(c_{t+1})(\alpha \theta_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta)]$$

As usual in finite period models, the optimal choice of capital dictates that it is run down to zero in the last period. Further, the optimal allocation dictates choosing high capital/investment in periods of high productivity and low capital/investment in periods of low productivity. Capital is then 'optimal', in the sense that it is the one that investors would choose if they had control of investment. Here there is a perfect alignment of manager and investor objectives since they have the same risk aversion and dividends equal consumption, therefore the manager chooses investment 'optimally' in the above sense.

It is important to point out that the risk averse manager would like to smooth dividends but cannot do so without trading in stocks. We now move to a case where the investor is less risk averse and thus willing to provide some insurance to the manager. Inevitably, investment will move away from the value maximizing level in this case. Table 2 reports results for this case where the manager is more risk averse than investors. We begin by focusing on the naive firm, indexed by N . The manager is issuing equity both in the first and second periods which allows for dividend smoothing across time. In the second period, he issues more when the shock is low, which means that dividends are also smoothed across states of nature. Dividend smoothing is also enhanced by the use of investment, which is now less than the value maximizing level in the beginning (intertemporal smoothing). The overall effect is smooth dividends and volatile consumption, as can be seen in the last two rows of Table 2. The investor is compensated for this higher consumption volatility with more consumption 'level' compared to dividend levels (see PV_c and PV_d). Obviously, stock trade makes both agents better off, since after all they are not forced to trade. As in the exchange economy, prices are negatively related to contemporaneous stock issuance. To be more precise, the more stocks $s_{h,t}$, the less dividends per stock d_t are expected to be tomorrow so the lower the price.

Armed with a clear understanding of the naive firm equilibrium prices and allocations and the mechanisms that drive them, we are now in a position to explain how that equilibrium changes when the firm realizes the price-dividend mapping. Table 2 also reports the prices and allocations for the fully rational firm, indexed by *FR*.

Table 2: Production Economy

| $(t, \theta) :$ | 1 | $(2, l)$ | $(2, h)$ | $(3, ll)$ | $(3, lh)$ | $(3, hl)$ | $(3, hh)$ |
|-----------------|---------------|-----------------|---------------|---------------|---------------|---------------|--------------|
| N d_t | 2.371 | 2.232 | 2.564 | 2.073 | 2.525 | 2.388 | 2.874 |
| FR d_t | 2.359 | 2.230 | 2.564 | 2.089 | 2.545 | 2.404 | 2.893 |
| N c_t | 2.349 | 1.456 | 2.704 | 1.129 | 2.096 | 1.202 | 2.231 |
| FR c_t | 2.349 | 1.457 | 2.705 | 1.130 | 2.099 | 1.203 | 2.234 |
| N ef_t | 0.176 | 0.172 | 0.162 | | | | |
| FR ef_t | 0.161 | 0.160 | 0.150 | | | | |
| N s_t | 1.046 | 1.137 | 1.120 | | | | |
| FR s_t | 1.042 | 1.126 | 1.110 | | | | |
| N y_t | 2.349 | 1.456 | 2.704 | | | | |
| FR y_t | 2.349 | 1.457 | 2.705 | | | | |
| N k_t | 6.238 | 3.301 | 3.860 | | | | |
| FR k_t | 6.247 | 3.311 | 3.869 | | | | |
| N p_t | 3.861 | 1.879 | 2.187 | | | | |
| FR p_t | 3.876 | 1.896 | 2.205 | | | | |
| | U_c | U_m | PV_c | PV_d | std_c | std_d | |
| N | 8.4116 | -0.02133 | 6.5531 | 6.5259 | 0.3179 | 0.1641 | |
| FR | 8.4114 | -0.02132 | 6.5523 | 6.5275 | 0.3134 | 0.1674 | |

The first thing to notice is that the FR allocations lead to higher welfare for the manager but lower welfare for the investor. For the manager, this comes as a result of higher dividend level and despite higher dividend volatility (the opposite is observed for the investors). Compared to the naive equilibrium, capital is higher in the beginning and thus closer to the value-maximizing level of capital. This inevitably comes with less smoothing of available resources across time.

Allowing the manager to understand and exploit the price-dividend relationship leads him to ‘choose’ prices that are higher, since equity is being issued. The ‘choice of price’ here is only indirect, since the price increase is achieved through a promise of higher future dividends.⁸ So the idea is to reduce dividend payments now, increase investment instead and then use the proceeds from this investment to pay higher dividends tomorrow. It is best to increase dividends in the third period because they would affect both first *and* second period prices.

Perhaps a clearer explanation comes when one looks at the firm’s financing equality (budget constraint) to explain how this new dividend profile will affect policy. We have

$$\begin{aligned}
 \text{Internal} + \text{External} &= \text{Investment} + \text{Dividends} \\
 \theta_t k_{t-1}^\alpha + p_t(s_{h,t} - s_{h,t-1}) &= k_t - k_{t-1} + d_t(s_{h,t-1} + s_m)
 \end{aligned}$$

In the first period, internal funds are given by past history and the current productivity shock and, as a result, are outside the control of the firm. Suppose the firm is considering lowering dividends now and raising them in the future. A reduction in dividends can be used

⁸Dividends are indeed higher in the third period, but not so in the second. But the movement in second period dividend is very small and obviously dominated by the third period increase. Note also that there is an effect through the intertemporal marginal rates of substitution which we abstract from because it is small in this example.

either to increase investment or decrease external funds or both. For a firm that is naive, this means decreasing stocks and increasing investment (both will happen in an interior solution). Both the increase in investment and the decrease in stocks will allow the firm to have higher dividends per stock d_t in the future. The optimum is decided by weighing these benefits against the cost of low d_t today and the actual optimal value is reported in the above table. Now let us allow the firm to realize that the change in dividend policy will also have an additional effect through prices. The additional effect is that prices will be higher, so the marginal benefit of reducing dividends today is actually higher than the manager thought before. The resulting allocation will have the manager choosing even lower dividends d_t today, higher investment and lower stocks which will allow the payment of even higher dividends per stock in the future. This is borne out in the allocations in Table 2.

This discussion is also at the heart of the time inconsistency problem. The reason is that the above arguments rely on the fact that today's dividends do not affect past prices, simply because the past has already happened and the price has been paid. Looking at the dividend first order condition

$$s_m v'(d_t s_m) = -\mu_{t-1} u'(c_{h,t}) + \gamma_t u'(c_{h,t}) (s_{h,t-1} + s_m) \quad (39)$$

we can trace the above intuition. On the left we have the 'utility' cost of reducing d_t . On the right the second term is the benefit through the increase in today's resources. The first term is the effect of the decrease in d_t , that comes through reducing stock prices, on all previous periods' resources. This would be a cost in periods where equity is issued and a benefit in periods where equity is bought back. For periods two and three, the increase in dividends will have an additional benefit which is the increase in prices in period 1. The important asymmetry here is that for period 1, the reduction in d_1 has no effect since $\mu_0 = 0$. So, in a sense, today's dividend reduction comes 'for free' at least with regard to the price effect and that is why the dividend profile is tilted towards the future. But if the manager could re-optimize in period 2, it is now those dividends that can be freely reduced without affecting any price, since in re-optimizing we would now have $\mu_1 = 0$. So we would expect that, despite the promise of higher dividends tomorrow, when tomorrow comes dividends will be low and a new promise of high future dividends will be made. This will further increase the price as well as investment.

Note also that time inconsistency does not arise in exchange economies, precisely because the change in the value of resources reflected by γ is exactly offset by the effect of μ . In other words, firms cannot gain anything by deviating from the FR full commitment equilibrium.

The intuition for this time inconsistency is therefore very similar to the standard optimal taxation case. In that framework, what matters for current investment decisions are expected future capital taxes not current capital taxes and that creates the opportunity for manipulation of the level of investment through promises about the future. In our setup, it is dividends that determine the return to investing in the firm, but current decisions on buying stocks depend on promises about future dividends.

5. EXAMPLES WITH AN INFINITE HORIZON

As reflected by the three period economy, the fact that fully rational firms take the price mapping into account can affect the equilibrium allocations. This section analyzes the two economies (naive and fully rational) in the infinite horizon economy. Before studying the fully fledged model, we start by providing an example that illustrates how the two economies could differ. In addition, the example illustrates that time inconsistency is likely to arise.

5.1. A Simplified Analytical Example. The present example assumes that initial capital is relatively low with respect to the steady state capital, so that the firm is growing over time. Further, to be able to derive some results analytically, we assume that investors

are risk neutral, we introduce a maximum amount of stock issuance and we abstract from uncertainty. For simplicity, we study the economy that is described at the beginning of the paper where the managers are not modelled explicitly as different agents.

In this setting, the naive manager solves:

$$\max_{\{d_t, s_t, k_t\}} \sum_{t=0}^{\infty} \delta^t v(d_t) \quad \text{s.t.}$$

$$d_t s_{t-1} + k_t - (1 - \eta)k_{t-1} = p_t(s_t - s_{t-1}) + f(k_{t-1}) \quad (40)$$

$$s_t - s_{t-1} \leq \Delta \quad (41)$$

$$k_{-1}, s_{-1} \quad \text{given} \quad (42)$$

where $\Delta > 0$ is a fixed constant limiting the amount of stocks that can be issued. In the fully rational case, the manager also takes into account the following constraint:

$$p_t = \sum_{j=1}^{\infty} \delta^j d_{t+j} \quad (43)$$

As stated earlier, we assume that initial capital is much lower than the steady state capital. Formally, the steady state capital k_s satisfies:

$$1 = \delta [f'(k_s) + 1 - \eta] \quad (44)$$

and we assume that $k_{-1} < k_s$.

It is important to note that, in the absence of uncertainty, the firm would be able to achieve the complete market solution if the constraint (41) would not be present. That is, if $\Delta = \infty$, the manager would be able to issue a sufficiently large amount of stocks in the first period to finance the desired accumulation of capital at $t = 0$, achieving the first best in one step. In fact, the manager would be able to complete the markets with stock issuance and he/she would achieve the first best in one period, so that $k_t = k_s$ for all $t \geq 0$.

On the other hand, if the upper bound on stock issuance is tight enough, it will prevent this from happening. Given this, there exist a sufficiently low Δ and a sufficiently low initial capital such that the constraint (41) is binding in the first period and the first best cannot be achieved. In what follows, we consider the case where the bound on stock issuance is binding for two periods.

A few relations hold for both the fully rational and naive firm cases. First, the first period capital is less than steady state and we therefore have that $k_0 < k_s$. Second, our assumption of a decreasing marginal productivity implies that:

$$1 < \delta f'(k_0) + 1 - \eta.$$

Third, the first order conditions with respect to capital and stocks coincide in both cases and they are given by the following equations for all t :

$$\gamma_{t+1} \delta (d_{t+1} + p_{t+1}) \leq \gamma_t p_t \quad (45)$$

$$\gamma_{t+1} \delta (f'(k_t) + 1 - \eta) = \gamma_t \quad (46)$$

where $\gamma_t \geq 0$ is the multiplier on the budget constraint of the firm.

Note that the Kuhn-Tucker condition in (45) holds with equality if $s_t - s_{t-1} < \Delta$ and it holds with inequality otherwise. Further, the fact that $k_0 < k_s$ and (46) imply:

$$1 < \frac{\gamma_0}{\gamma_1}$$

and since $\gamma_t \geq 0$ we have that

$$\gamma_0 > \gamma_1 \quad (47)$$

The previous condition is the analogue to the one in a standard growth model with infinitely many periods, where the shadow price of additional cash flows (γ) goes down as capital grows towards the steady state.⁹ Fourth, if the upper limit (41) is binding for M periods, we have that:

$$s_t = s_{-1} + (t + 1)\Delta, \quad \text{for } 0 \leq t \leq M$$

Fully Rational Firms

Consider now the fully rational firm, a case that we index with the superscript FR . The first order conditions are given by:

$$\mu_t^{FR} = \mu_{t-1}^{FR} + \gamma_t^{FR}(s_t^{FR} - s_{t-1}^{FR}) \quad \text{with } \mu_{-1} = 0$$

$$v'(d_t^{FR}) = \gamma_t^{FR}s_{t-1}^{FR} - \mu_{t-1}^{FR}$$

along with (45) and (46). Combining these equations, we obtain:

$$v'(d_t^{FR}) = v'(d_{t-1}^{FR}) + (\gamma_t^{FR} - \gamma_{t-1}^{FR})s_{t-1}^{FR} \quad (48)$$

for $t > 0$, while the analogous condition at period zero is equal to:

$$v'(d_0^{FR}) = \gamma_0^{FR}s_{-1}^{FR} \quad (49)$$

Using equation (47), this implies that

$$v'(d_1^{FR}) - v'(d_0^{FR}) = (\gamma_1^{FR} - \gamma_0^{FR})s_0 < 0$$

$$d_1^{FR} > d_0^{FR}$$

In sum, dividends grow between period 0 and period 1 when firms understand the link between future dividends and current stock prices.

To evaluate the potential for having time inconsistency under fully rational firms, we now consider whether a re-optimization in future periods would lead the firm to deviate from the dividend plans announced in period zero. We use the superscript R to denote the solution if the firm re-optimizes in period $t = 1$. The first order conditions for capital and the stock are the same as before. On the other hand, we have

$$\mu_t^R = \mu_{t-1}^R + \gamma_t^R(s_t^R - s_{t-1}^R) \quad \text{with } \mu_0^R = 0 \quad \text{for } t \geq 1$$

$$v'(d_t^R) = \gamma_t^R s_{t-1}^R - \mu_{t-1}^R$$

This implies that the following equation holds for $t > 1$:

$$v'(d_t^R) = v'(d_{t-1}^R) + (\gamma_t^R - \gamma_{t-1}^R)s_{t-1}^R$$

In addition, for the initial period ($t = 1$), we have

$$v'(d_1^R) = \gamma_1^R s_0$$

Suppose that $d_1^R = d_1^{FR}$ and $s_1^R = s_1^{FR}$. In this case, we would have

$$v'(d_1^{FR}) = \gamma_1^R s_0 \quad (50)$$

$$\gamma_2^{FR} - \gamma_1^{FR} = \frac{v'(d_2^{FR}) - v'(d_1^{FR})}{s_1^{FR}} = \gamma_2^R - \gamma_1^R \quad (51)$$

⁹Notice that this may not occur in a finite-life model and we therefore choose the infinite period model for this example.

where we used the fact that the upper bound on stock issuance is binding for a few periods, implying that $s_1^{FR} = s_1^R = s_0^{FR} + \Delta$. In addition, for these choices of γ to be compatible with the same choice for capital in period 1, we need to check that the following equation is also satisfied:

$$\gamma_2^R \delta(f'(k_1^{FR}) + 1 - \eta) = \gamma_1^R$$

We now show that this cannot happen. First, if (51) holds, we have $\gamma_2^R = \gamma_1^R - \gamma_1^{FR} + \gamma_2^{FR}$ so that

$$\begin{aligned} \gamma_1^R &= \gamma_2^R \delta(f'(k_1^{FR}) + 1 - \eta) = (\gamma_1^R - \gamma_1^{FR} + \gamma_2^{FR}) \delta(f'(k_1^{FR}) + 1 - \eta) \\ &= (\gamma_1^R - \gamma_1^{FR}) \delta(f'(k_1^{FR}) + 1 - \eta) + \gamma_1^{FR} \end{aligned}$$

Second, the last expression can only be equal to γ_1^R if either $\delta(f'(k_1^{FR}) + 1 - \eta) = 1$ or $\gamma_1^R = \gamma_1^{FR}$. The first condition arises when capital is optimal, a case that we already have shown gives time consistency but that we have excluded above by the choice of a low initial capital and an upper bound on issuance Δ that is binding for at least two periods (period 0 and 1). Further, the second case can be excluded by the formula for γ_1^R in (50). Given this, we have provided an example where time inconsistency will arise.

Naive Firms

For comparison, consider now the naive firms, for which we use superscript N . In this case, in addition to (45) and (46), the first order condition for dividends is given by:

$$v'(d_t^N) = \gamma_t^N s_{t-1}^N \quad (52)$$

This, and the fact that $s_0 = s_{-1} + \Delta$ implies

$$v'(d_1^N) - v'(d_0^N) = (\gamma_1^N - \gamma_0^N) s_0^N + \gamma_0^N \Delta$$

Since the first term is negative and the second is positive, the sign of the right hand side is ambiguous. For very low initial capital stocks, the size of γ_0^N might be very large so that the positive term could dominate. In contrast to the fully rational case, this would imply that dividends may go down. More generally, even if the previous statement is not true, the above equation might imply that

$$v'(d_1^{FR}) - v'(d_0^{FR}) < v'(d_1^N) - v'(d_0^N)$$

In turn, this seems to indicate that the growth rate of dividends under fully rational firms will be larger. Intuitively, the fully rational firm understands that announcing high *future* dividends she can inflate the period zero price, allowing for faster accumulation of capital. Thus, the firm in this case tilts the dividend profile to give higher dividend payments in the future than in period zero. In this way it raises more funds in period zero, pays lower dividends in the initial period (relative to the future) and it allows for a faster growth through higher accumulation of capital in the initial period.

While we cannot provide an analytical proof of the previous statements, the example seems to suggest that some of the results that we have discussed in the three period economy might go through in the infinite horizon economy, at least along the growth path. To evaluate this quantitatively, the next section analyzes a stochastic version of the model numerically.

5.2. Numerical Examples. To be completed.

6. EXTENSIONS

The previous analysis has assumed that firms take into account the effects of financial policy on prices but not on consumption. A possible extension of our work is to study the Stackelberg leader firm, assuming that firms also internalize the consumption effects. In this sense, this firm is the closest to a Ramsey government in the optimal taxation literature.

Under this assumption, the solution is likely to be time inconsistent, even in the exchange economy. To see this, consider the more general equilibrium of the model with investors and managers that we have described earlier. The problem of a Stackelberg leader firm is given by:

$$\begin{aligned} \max_{\{d,s\}} E_0 \sum_{t=0}^{\infty} \delta^t v(d_t s_m) \text{ st.} \\ d_t s_{t-1} + \theta_t = p_t (s_t - s_{t-1}) \\ s_{t-1} = s_{ht-1} + s_m \\ p_t = \delta E_t \left(\frac{u'(c_{ht+1})}{u'(c_{ht})} [(p_{t+1} + d_{t+1})] \right) \equiv E_t \left(\sum_{j=1}^{\infty} \delta^j \frac{u'(c_{ht+j})}{u'(c_{ht})} d_{t+j} \right) \\ c_{ht} = \theta_t - d_t s_m \end{aligned}$$

As before, we can apply recursive contracts and introduce the co-state variable $\{\mu\}$, with law of motion given by:

$$\mu_t = \mu_{t-1} + \gamma_t (s_t - s_{t-1})$$

where γ_t is the multiplier on the budget constraint of the firm. The conditions that characterize the equilibrium of the previous problem are:

$$\begin{aligned} v'(d_t) &= -\mu_{t-1} [u''(c_{ht}) d_t s_m - u'(c_{ht})] \\ &\quad + \gamma_t [u''(c_{ht}) s_m (d_t s_{t-1} - \theta_t) - u'(c_{ht}) s_{t-1}] \\ \gamma_t u'(c_{ht}) p_t &= \delta E_t \gamma_{t+1} u'(c_{ht+1}) [p_{t+1} + d_{t+1}] \end{aligned}$$

It is easy to see that the proof of Proposition 3 does not apply to the present setup unless households are risk neutral ($u''(c) = 0$). Given this, the solution to this problem is likely to be time inconsistent.

7. CONCLUSIONS

We have provided a way to formulate and solve a stochastic general equilibrium dynamic model of dividend and stock policy. The aim was to provide a framework within which a number of important issues can be addressed. The model proposed makes explicit the distinction between dividends and stock issuance or repurchases. It is thus well suited to analyze payout policy. In addition, the framework is also available for the analysis of questions regarding the interplay between payout policy and investment.

As a first implication of the theoretical analysis presented in the main section of this paper, we highlight the behavior of growing firms with regard to dividend payments. Typically, startup firms pay little or no dividends, while they funnel resources towards the available productive projects that lead to firm growth. One obvious theoretical explanation of this observation points at financial frictions that do not allow for unlimited funds being raised from external sources. Our framework provides another, complementary mechanism that can explain this observation. The idea is that young firms lack the burden of past promises about dividends and can therefore pay little now, while promising a lot of dividends for the future. This strategy allows them to raise external funds at more favorable prices by inflating the price of their stock. Using the cheaper external funds, they can also grow faster.

Our framework also provides a rationale for why a firm would prefer to use dividends as opposed to repurchases if the full commitment solution is taken as the benchmark case. As mentioned above, the reason is that dividend promises can be used to influence prices towards achieving cheaper external finance, while the same objective cannot be achieved through announcements in stock repurchases.

Finally, our work identifies a potential for time inconsistency in financial policy even in the absence of asymmetric information of the type considered by Miller and Rock (1985). We point out the complications arising from the need for commitment and we provide examples where the full commitment policy is time consistent and others where it is not. This raises the question of how the time consistent policy would look like, its efficiency properties and the arrangements that can be used to implement more efficient policies. We leave these questions for future research.

8. APPENDIX

Consider the first order conditions in period $T - 1$. These imply that:

$$E_{T-1}[\gamma_T(d_T + p_T)] = \gamma_{T-1}E_{T-1}[d_T + p_T]$$

$$E_{T-1}[\gamma_T(\bar{\theta}f'(k_{T-1}) + 1 - \eta)] = \gamma_{T-1}$$

Since $d_T + p_T = \frac{\bar{d}(k_{T-1}, s_{T-1})}{1 - \delta}$, we have that $d_T + p_T$ is known at $T - 1$ and the first equation implies that:

$$E_{T-1}[\gamma_T] = \gamma_{T-1} \tag{53}$$

Further, the second equation implies that

$$E_{T-1}[\gamma_T(\bar{\theta}f'(k_{T-1}) + 1 - \eta)] = E_{T-1}[\gamma_T](\bar{\theta}f'(k_{T-1}) + 1 - \eta) = \gamma_{T-1}$$

implying that:

$$\bar{\theta}f'(k_{T-1}) + 1 - \eta = 1$$

As we see, it follows that $k_{T-1} = \bar{k}$, while equation (53) and the first order condition for dividends gives:

$$E_{T-1}v'(d_T) = v'(d_{T-1})$$

However, since d_T is known with information up to period $T - 1$, this implies that $E_{T-1}v'(d_T) = v'(d_T)$ and $d_T = d_{T-1}$. Thus, even one period before there is no uncertainty, dividends are constant and the risk neutral level of capital will be chosen. Plugging $k_{T-1} = \bar{k}$ into (35), we obtain a cleaner expression for the dividends:

$$\bar{d}(\bar{k}, s_{T-1}) \equiv \frac{\bar{\theta}f(\bar{k}) - \eta\bar{k}}{s_{T-1}}$$

Further, we can determine s_{T-1} by just plugging in (1) evaluated at $t = T - 1$, the risk neutral level for k_{T-1} and the dividends to find that

$$\bar{d}(\bar{k}, s_{T-1})s_{T-2} + \bar{k} - (1 - \eta)k_{T-2} = \frac{\delta\bar{d}(\bar{k}, s_{T-1})}{1 - \delta}(s_{T-1} - s_{T-2}) + \theta_{T-1}f(k_{T-2})$$

Given s_{T-2} , θ_{T-1} and k_{T-2} , the previous equation gives the solution for s_{T-1} . Simplifying

$$\frac{\bar{d}(\bar{k}, s_{T-1})}{1 - \delta}(s_{T-2} - \delta s_{T-1}) = \theta_{T-1}f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2} \tag{54}$$

which means that the jump to the risk neutral level of capital at $T - 1$ (from k_{T-2} to $k_{T-1} = \bar{k}$) is financed by stock issuance in periods $T - 1$ and T . To obtain a more explicit solution, note that the previous equations imply that:

$$\begin{aligned} \frac{\bar{\theta}f(\bar{k}) - \eta\bar{k}}{(1 - \delta)s_{T-1}}(s_{T-2} - s_{T-1}) &= \theta_{T-1}f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2} \\ \frac{s_{T-2}}{s_{T-1}} &= \left((1 - \delta) \frac{\theta_{T-1}f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2}}{\bar{\theta}f(\bar{k}) - \eta\bar{k}} + 1 \right) \\ s_{T-1} &= s_{T-2} \frac{\bar{\theta}f(\bar{k}) - \eta\bar{k}}{(1 - \delta) [\theta_{T-1}f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2}] + \bar{\theta}f(\bar{k}) - \eta\bar{k}} \end{aligned}$$

Consequently, the long run dividends are given by

$$\bar{d}(\bar{k}, s_{T-1}) \equiv \frac{(1 - \delta) [\theta_{T-1}f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2}] + \bar{\theta}f(\bar{k}) - \eta\bar{k}}{s_{T-2}}$$

The previous arguments imply that we actually will have time consistency after $T - 1$. However, we can not extrapolate this to previous periods. For example, while we also have that $d_{T-1} + p_{T-1} = \frac{\bar{d}(k_{T-1}, s_{T-1})}{1 - \delta}$, s_{T-1} is not known at $T - 2$, since it is determined by θ_{T-1} . Given this, we do not have an analog of (53). Instead, we have that

$$E_{T-2}[\gamma_{T-1}\bar{d}(k_{T-1}, s_{T-1})] = \gamma_{T-2}E_{T-2}[\bar{d}(k_{T-1}, s_{T-1})]$$

For $T > 2$, using the above formula for long run dividends, we obtain

$$\begin{aligned} &E_{T-2}[\gamma_{T-1}((1 - \delta) [\theta_{T-1}f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2}] + \bar{\theta}f(\bar{k}) - \eta\bar{k})] \\ &= \gamma_{T-2}[(1 - \delta) [E_{T-2}(\theta_{T-1})f(k_{T-2}) - \bar{k} + (1 - \eta)k_{T-2}] + \bar{\theta}f(\bar{k}) - \eta\bar{k}] \end{aligned}$$

In this case, the solution is likely to be time inconsistent up to period $T - 2$, since the gammas that satisfy (34) are not likely to satisfy the previous condition.

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